ELASTIC PROTON AND NEUTRON CROSS SECTIONS CALCULATED FROM A MICROSCOPIC OPTICAL MODEL POTENTIAL

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1. Introduction - The empirical optical-model potential varies smoothly with mass number and with energy. Therefore, it seems natural to relate the real and imaginary strengths of the optical model potential (OMP) to the microscopic properties of nuclear matter. The latter can be evaluated within the frame of many-body theories, the most known being Brueckner's theory in which the basic ingredient is the realistic nucleon-nucleon interaction. This approach does follow the present trend in nuclear reaction theories.

The theoretical construction of the OMP is reported in a series of papers[1-4]) in which we used the Reid hard core interaction. The results are, therefore, meaningful in the energy range 0-250 MeV.

Besides the evaluation of volume integrals and root mean square radii of the OMP, we recently calculated elastic cross-sections. The aim was twofold. Firstly, we wanted to see how good the differential elastic cross-sections derived from a microscopic OMP are. Secondly, was it possible to have informations on the shape of the OMP, on the density distribution of the target and on the existence of a neutron skin for the heavy nuclei.

2. Definitions and formulae - The theoretical OMP, M is, in general, complex, non local, and energy dependent.

$$M(\vec{r},\vec{r}\,';E) = -V(\vec{r},\vec{r}\,';E) - iW(\vec{r},\vec{r}\,';E). \qquad (1)$$

The identification between the mass operator (which appears in Dyson's equation for the Green's function) and the OMP, $M(\vec{r},\vec{r}\,';E)$ had been established by Bell and Squires[5]).

In the case of an infinite medium such as nuclear matter,

M depends on $|\vec{r}-\vec{r}'|$, E and the density ρ or, by Fourier transformation, on k , E and ρ . The optical-wave function has a physical meaning only for "on-shell" values of $M_\rho(k,E)$, i.e. for

$$E = e(k) = \frac{\hbar^2}{2m}k^2 - V_\rho(k,E). \qquad (2)$$

This equation establishes a functional dependence between k and E .

We note that additional complications arise in the evaluation of M when we take into account the neutron excess and the Coulomb interaction.

In the spirit of the Brueckner's theory for the binding energy of nuclear matter, the mass operator can be expanded in a power series of the density ρ , i.e. according to the number of independent summations over momenta from 0 to k_F . The leading term called the Brueckner-Hartree-Fock approximation is

$$M_{BHF_\rho}(k,E) = \sum_{j\leq k_F} <\vec{j},\vec{k}|g[E+e(j)]|\vec{j},\vec{k}-\vec{k},\vec{j}>, \qquad (3)$$

where $g[W]$ is the solution of the equation

$$g[W] = v+v \sum_{a,b>k_F} \frac{|\vec{a},\vec{b}><\vec{a},\vec{b}|}{W - e(a) - e(b) + i\delta} g[W]. \qquad (4)$$

The simplest approximation for constructing the optical-model potential in a finite nucleus from the potential $M_\rho(E)$ evaluated in nuclear matter is given by the local density approximation (LDA)

$$M(r,E) = M_{\rho(r)}(E), \qquad (5)$$

where $\rho(r)$ is the experimental target density distribution. However this LDA must be improved to take into account surface effects. We proposed the following definition and discussed its justification in Ref. [4] :

$$M(r,E) = (t\sqrt{\pi})^{-3} \int M_{\rho(r')}(E) \exp(-|\vec{r}-\vec{r}'|^2/t^2) d^3r', \qquad (6)$$

where the range t is an adjustable parameter.

3. Results and discussion - We used the MAGALI or JIB3 co-

des with the OMP (eq. (6)) given point by point as an input. We analyzed n and p scattering on various nuclei from ^{12}C to ^{208}Pb and within the range 10-70 MeV for protons and 2-15 MeV for neutrons. The parameter t was chosen equal to 1.2 fm in order to reproduce the volume integrals and the r.m.s. radii of the OMP and to agree with the value, generally used, for the "effective interaction"[6]).

The target density distribution, parameterized by Negele[7]) was used for intermediate and heavy nuclei. For nuclei lighter than ^{40}Ca, the density distribution given by Ref. [8] gave better agreement.

The existence of a neutron skin is suggested in the case of nuclei Sn, Pb and Bi. Indeed, the best results are obtained from a difference of 0.1 fm between the r.m.s. radii for neutron and proton.

The full curves on figure 1 show the results. Keeping in mind that the theory contains but one parameter (t) , the comparison with experimental data seems to be stimulatingly good.

The theoretical cross-sections can be improved as shown by the dashed curves. These have been obtained by decreasing the depth of the imaginary part by 6 to 20 % depending on targets and energy, the depth of the real part remaining nearly unchanged. This renormalization can actually be justified by the facts (i) most nuclei studied here are magic and properties derived from nuclear matter include neither shell effects nor centre of mass corrections ; (ii) inclusion of higher orders in the expansion of the mass operator should improve the results.

References

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6) W.D. Myers, Nucl.Phys. A201(1973)465.
7) J.N. Negele, Phys.Rev. C1(1970)1260.
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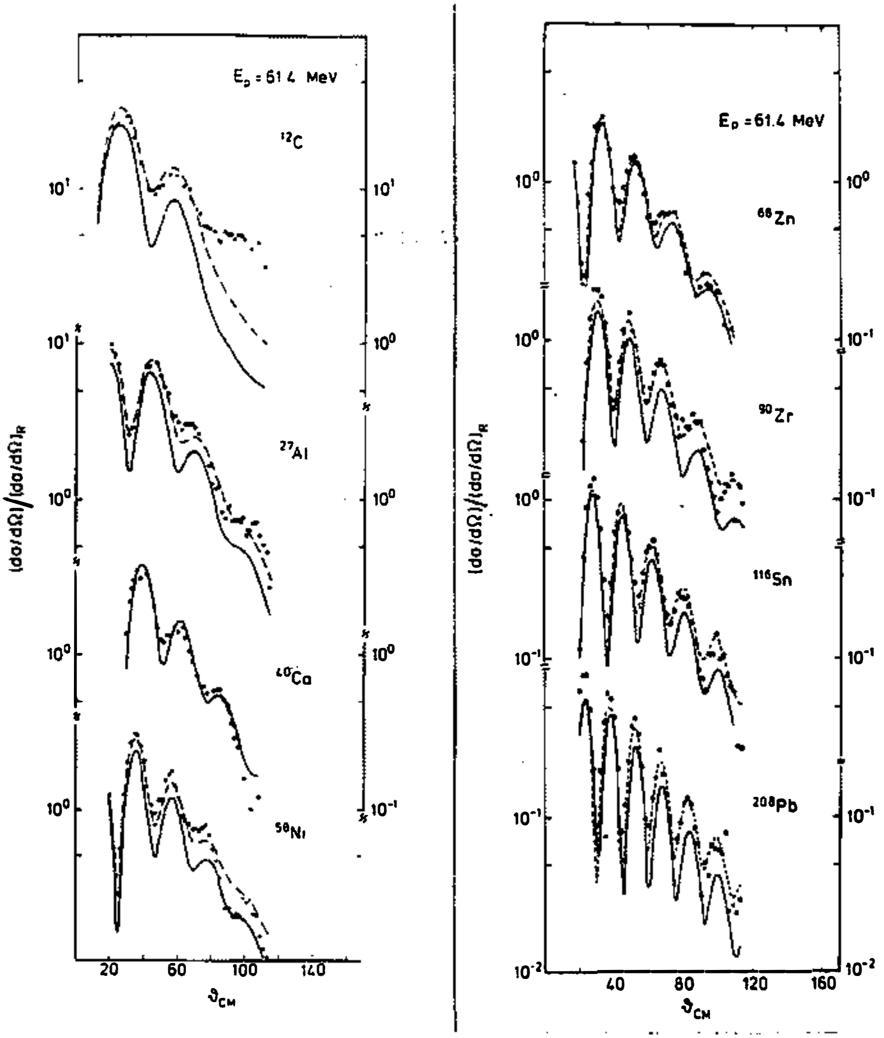

Fig. 1 - Differential elastic scattering cross-sections for 61.4 MeV protons on various nuclei. The full curves are obtained from the theoretical microscopic optical-model potential. The dashed curves resulted from the renormalization of the depth of the OMP.

FUSION BARRIER AT SUB-COULOMB ENERGIES

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The barrier seen by an incoming nuclei at sub-Coulomb energies is assumed to be pure Coulomb to the right of the maximum, and an inverted half parabola to the left of the top, namely

$$B_L(r) = \begin{cases} E_0 + \dfrac{\hbar^2 L(L+1)}{2\mu R_0^2} - \mu\omega_0^2 \dfrac{(r-R_0)^2}{2} & r < R_0 \\ \dfrac{Z_1 Z_2 e^2}{r} & r \geq R_0 \end{cases}$$

where E_0, R_0, $\hbar\omega_0$ are the height, the position, and the curvature of the s wave effective interaction. The Hill-Wheeler expression for the partial wave transmission probability for c.m. energy E is

$$P(L,E) = \{1+\exp[\pi(E_0-E)/\hbar\omega_0 + \pi\hbar^2 L(L+1)/2\pi R_0^2\hbar\omega_0 + C(E)]\}^{-1}$$

where the first two terms in the exponent are due to the penetration through the half parabola, while

$$C(E) = \pi\eta - 2\{(2\eta\rho_0 - \rho_0^2)^{\frac{1}{2}} + \eta \text{ arc } \sin[(\rho_0 - \eta)/\eta]\}$$

(with $\eta = \mu Z_1 Z_2/h^2 k$ and $\rho_0 = k R_0$), results from the penetration through the Coulomb part. The total reaction cross section σ_R can therefore be calculated exactly, i.e.

$$\sigma_R = \frac{\pi}{k^2} \sum_{L=0}^{\infty} (2L+1)P(L,E) = \frac{R_0^2\hbar\omega_0}{E} \ell n\{1+\exp[\pi(E-E_0)/\hbar\omega_0 - C(E)]\}$$

which is the natural extension of Wong's (1) expression to the low energy domain. The success of our expression is demonstrated in Fig. 1, where the nuclear S factor $S = \sigma_F E \exp(2\pi\eta)$ (σ_F = fusion cross section, and $\sigma_F = \sigma_R$ at very low energies) is compared with several experimental results reported by Stokstad et al. (2).

(1) C.Y. Wong, Phys. Rev. Lett. 31 (1973) 766.
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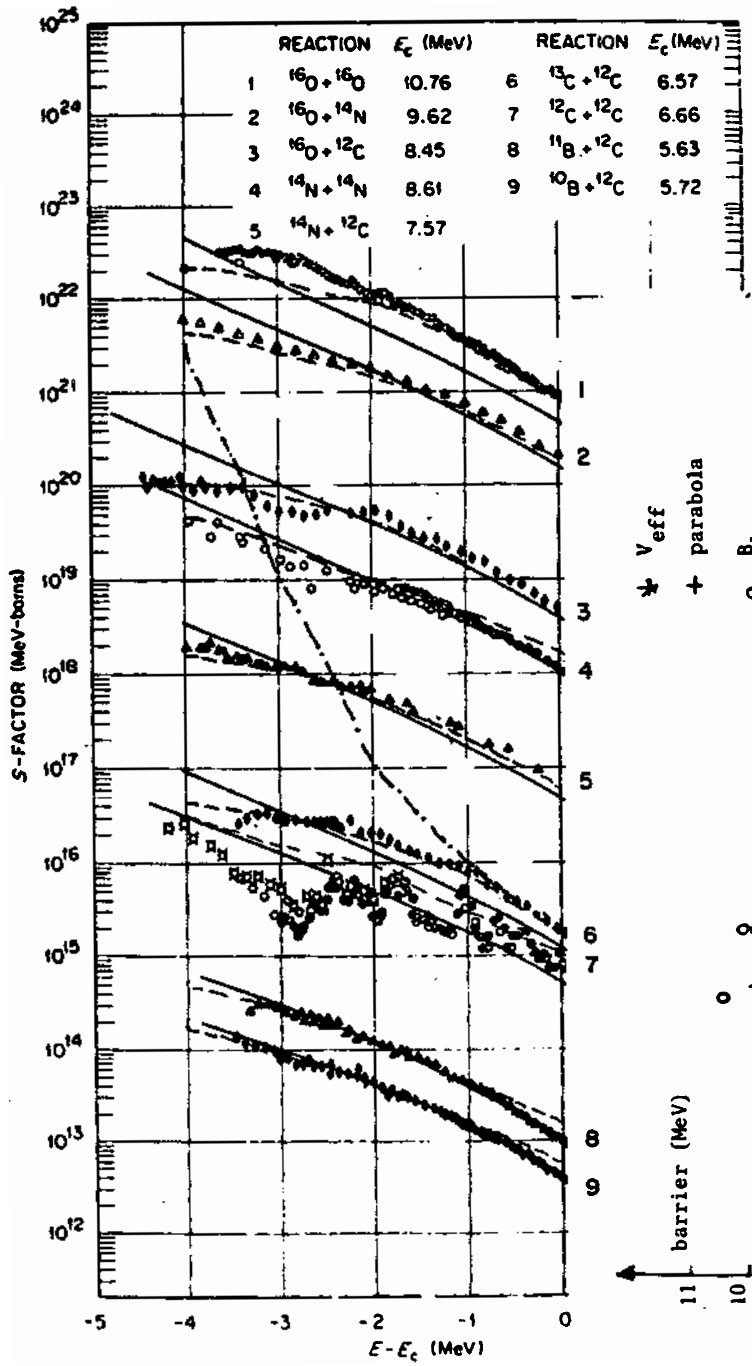
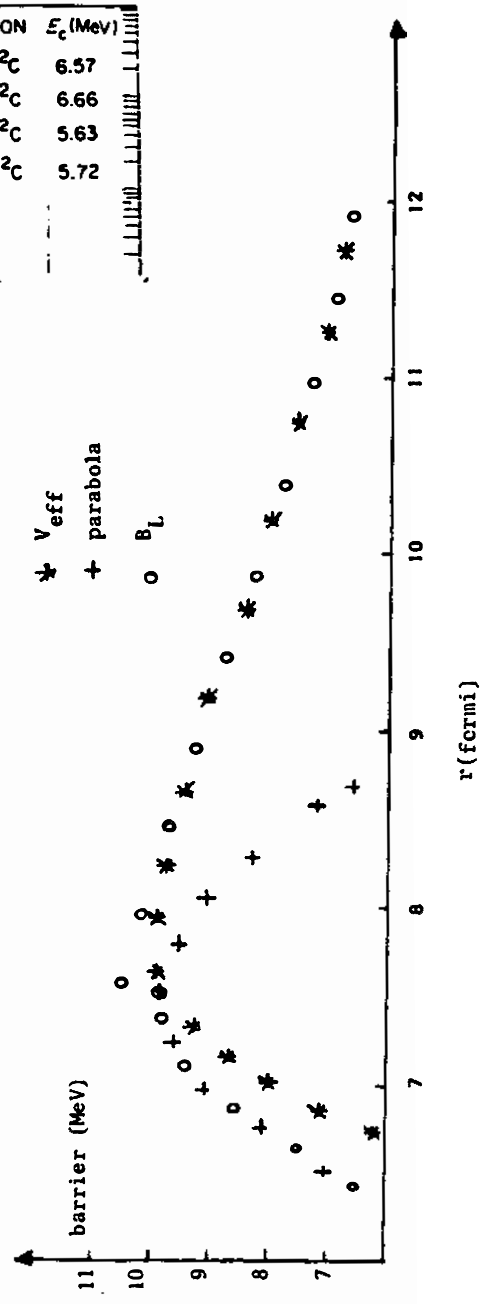
Figure caption

Fig. 1 - Left - Ecperimental results and optical model calculations (solid lines) from Ref. 1a, accompanied by our results (dashed-lines), in which R_0, E_0 and $\hbar\omega_0$ have been calculated from a Saxon-Woods Potential with $V_0 =$ 40 MeV, $r_0 = 1.23$ fm and $a = 0.4$ fm. The results of reactions 1, 2, and 4 are divided by 1000, 10, and 10 respectively. The dash-dot line represents the results for $C^{13} + C^{12}$ assuming pure parabolic barrier.

Right - Effective interaction, parabolic barrier and the barrier $B_L(r)$ for $L = 0$ $O^{16} + O^{16}$ reaction, with the potential defined above.


REACTION E_c (MeV)
1 $^{16}O + ^{16}O$ 10.76
2 $^{16}O + ^{14}N$ 9.62
3 $^{16}O + ^{12}C$ 8.45
4 $^{14}N + ^{14}N$ 8.61
5 $^{14}N + ^{12}C$ 7.57
6 $^{13}C + ^{12}C$ 6.57
7 $^{12}C + ^{12}C$ 6.66
8 $^{11}B + ^{12}C$ 5.63
9 $^{10}B + ^{12}C$ 5.72
S-FACTOR (MeV-barns)
$E - E_c$ (MeV)


V_{eff}
parabola
B_L
barrier (MeV)
r (fermi)