ON THE GENERAL FORM OF THE CROSS-SECTION OF DEEP-INELASTIC COLLISIONS

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We derive a general form of the cross-section of deep-inelastic (DI) collisions by proceeding as follows:

(i) We decompose the S-matrix into an energy-averaged part $\langle S_{\beta\alpha}\rangle$ describing the "direct" reactions, an S-matrix $S^{CN}_{\beta\alpha}$ for compound nucleus formation ("fusion") defined as that part of the fluctuating S-matrix whose phase is random with respect to all the quantum numbers β, and a remainder $S^{DI}_{\beta\alpha}$ which is to describe the incompletely randomized DI reactions:

$$S_{\beta\alpha} = \langle S_{\beta\alpha}\rangle + S^{CN}_{\beta\alpha} + S^{DI}_{\beta\alpha} \tag{1}$$

(ii) We evaluate the amplitude $f^{DI}_{\beta\alpha}(\vartheta)$ for DI reactions using semi-classical approximations for the Clebsch-Gordan coefficients and the Legendre functions and performing the integration over the orbital angular momenta in the "stationary phase approximation (SPA)". The Poisson representation serves to separate the amplitude for DI reactions from the one for fusion reactions. The use of the SPA implies that the S-matrix $S^{DI}_{\beta\alpha}$ depends smoothly on the orbital angular momentum.

(iii) THe case that DI reactions are confined to a limited and sharply bound range of orbital angular momenta ℓ

$$\Lambda_1 < \ell < \Lambda_2 \tag{2}$$

is treated by confining the ℓ-integration in the SPA to this range (2). The lower and upper cut-off might be produced by the set-in of fusion reactions and quasi-elastic reactions, resp. The cut-off results in a diffractive pattern which is typical for diffraction from a circular slit.

(iIV) The so resulting cross-section $\sigma_{\beta\alpha}^{DI}$ for DI reactions is integrated over the energy averaging interval and summed over the microscopic channels β which lie in a "coarse" interval of the measured "macroscopic" variables

$\{a_\nu\}$ ≡ scattering angle , relative kinetic energy, charge and mass of the fragments, etc.:

$$\frac{d^f Q^{DI}}{d\cos a_1 \, da_2 \ldots da_f} \Delta\cos a_1 \, \Delta a_2 \ldots \Delta a_f := \sum_{\beta\in\{a_\nu, a_\nu+\Delta a_\nu\}} \langle\sigma_{\beta\alpha}\rangle \tag{3}$$

Interference terms between compound and DI reactions as well as between different semi-classical paths disappear as a result of the averaging process. From the same reason, the difractive oscillations of the angular distribution are usually wiped out, and only a diffractive broadening of the peak of the DI reactions by an amount of the order $\Delta\vartheta = \frac{1}{\Lambda_2-\Lambda_1}$ remains.

(iV) If the macroscopic degrees of freedom are treated as classical variables, the general form of the average cross section for DI reactions is found to beoof the form

$$\frac{d^f Q^{DI}}{d\cos a_1 \, da_2 \ldots da_f} = \frac{1}{\sin a_1} \sum_{\nu=1}^{N'} P^{class}\left[\ell_\nu^s(a);\ a_2 \ldots a_f\right] \vartheta_\nu^{class} \quad (\text{?}$$

Here, P^{class} is a classical coarse grain distribution which depends on the scattering angle $a_1 \equiv \vartheta$ only through the "stationary angular momenta" $\ell_\nu^s(a)$, and $\mathcal{V}_\nu^{class}(a)$ is a classical average over a known function of the scattering angle. The summation extends over the different "branches" of the inverse classical deflection function[1]. In the case of 2 rainbow angles, we have N = 3. The prime means that branches without a real stationary point are to be omitted.

In the case that the DI reactions are confined to an angular momentum window (2), it is $\mathcal{V}_\nu^{class}(a)$ which contains the diffractive effects. If quantum diffraction is negligible, $\mathcal{V}_\nu^{class}$ is approximately given by

$$\mathcal{V}_\nu^{class} = \frac{1}{\left|\bar{\Theta}'\left[\ell_\nu^s(a); a_2 \cdots a_f\right]\right|} \tag{5}$$

where $\bar{\Theta}$ is an "average classical deflection function". It is defined by averaging over all the classical trajectories which are compatible with the macroscopic variables $a_1 \ldots a_f$. The detailed form of $\mathcal{V}_\nu^{class}$ is given in ref. 1. P^{class} may be deduced from a classical transport theory[2,3]. Analyses of experimental results on the basis of eq.(4) and results on P^{class} are in progress[4].

Footnotes and References

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