THE VALIDITY OF AN INTEGRAL REPRESENTATION OF THE SEMICLASSICAL S-MATRIX FOR MULTIPLE COULOMB EXCITATION OF HEAVY-IONS

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1. Introduction. - There have been several attempts to apply the so-called classical S-matrix theory developed in molecular physics /1,2/ to nuclear reaction problems.One uses either a representation of the S-matrix where the contributions of only a few stationary classical trajectories have to be summed up /1/, or an integral representation /2,3/ is applied where all classical trajectories are taken into account.

The first method has been investigated in the backward scattering (one intrinsic degree of freedom) Coulomb excitation problem /4,5/ and in the Coulomb excitation of classically allowed transitions for all scattering angles /6/ (two intrinsic degrees of freedom). In the latter case, the treatment was found to be very laborious, because the caustics had to be treated correctly, the search for the stationary points was difficult to automatize and the inconvenient Bessel uniform approximation in two dimensions had to be used in order to obtain sufficient accuracy.

The integral representation of the classical S-matrix /2,3/ is more straightforward to apply, because the search for the stationary trajectories is avoided, the uniform approximation is not necessary, and the classically allowed and forbidden transitions are treated on equal footing. Therefore we investigate in this paper a model study of the Coulomb excitation to all scattering angles within the integral representation. The backward scattering Coulomb excitation has already been treated successfully by this approach /7/.

2. The model.- Using the Sommerfeld parameter η , half the distance of closest approach a , the reduced mass μ , the velocity v, the intrinsic quadrupole moment Q_o,and the moment of inertia $\mathcal{J}$ of the rigid rotor target, and the monopole-quadrupole strength $q = Z_T e^2 Q_o/4\hbar va$, we introduce the dimensionless, physically meaningful parameters $\Lambda = 2q/\eta$, $\zeta = Q_o\mu\,/\mathcal{J}\,Z_T$ ($Z_{T,P}$ = charge number of the target (T) or projectile (p)) /8/ and $\bar{\varepsilon} = \ell_o/\eta$ (ℓ_o is the orbital angular momentum in the entrance channel), which govern the reaction dynamics. The quantity Λ is a measure of the recoupling of the intrinsic excitation to the trajectory of the projectile. ζ measures the adiabaticity of the reaction, and $\bar{\varepsilon}$ determines the orbit of the projectile in the entrance channel.

The Hamiltonian in dimensionless quantities ($\bar{H} = H/\mu v^2$) for the Coulomb excitation problem is given by

$$\bar{H}(\bar{p}_r,\bar{r},\bar{p}_\theta,\theta,\bar{p}_\beta,\beta,\bar{p}_\varphi,\varphi) = \frac{\bar{p}_r^{\,2}}{2} + \frac{\bar{p}_\theta^{\,2}}{2\bar{r}^2}\Lambda^2 + \frac{\bar{p}_\theta}{\bar{r}^2}\,\bar{\varepsilon}\,\Lambda + \frac{1}{2\bar{r}^2}\,\bar{\varepsilon}^2 \tag{1}$$

$$+ \bar{p}_\varphi^{\,2}\left(\frac{\Lambda^2}{2\bar{r}^2\sin^2\theta} + \frac{\zeta}{4}\frac{\Lambda}{\sin^2\beta}\right) + \frac{\zeta}{4}\Lambda\,\bar{p}_\beta^{\,2} + \frac{1}{\bar{r}} + \frac{\Lambda}{\bar{r}^3}\left\{\frac{3}{2}(\cos\theta\cos\beta + \sin\theta\sin\beta\cos\varphi)^2 - \frac{1}{2}\right\}.$$

The meaning of the variables $\bar{r} = r/a$, θ , β , φ and the dimensionless conjugate momenta $\bar{p}_r = p_r/\mu v$, $\bar{p}_\theta = (p_\theta - \ell_o)/2\hbar q$ (the Coulomb motion in the entrance channel is subtracted from p_θ), $\bar{p}_\beta = p_\beta/2\hbar q$ and $\bar{p}_\varphi = p_\varphi/2\hbar q$ is made clear in Fig.1.

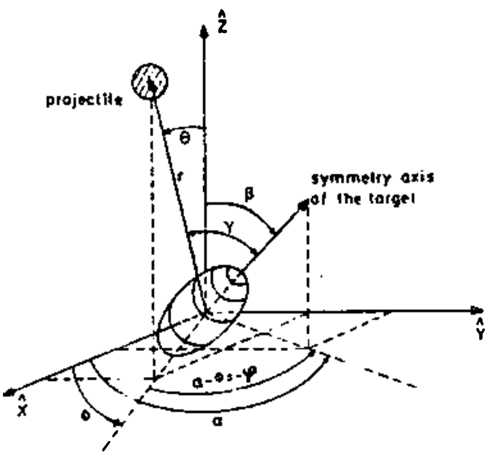


Fig.1:
The variables defining the Coulomb excitation problem

In the following we restrict the discussion to the sudden limit which we define by $\Lambda = 0$ and $\zeta = 0$ in the classical equations of motion. Physically this means that the coupling between the projectile and the target occurs so quickly that neither the projectile nor the target is disturbed from its unperturbed motion during this period. In the sudden limit all the expressions up to the final integral for the S-matrix (eq.2) can be calculated analytically. Moreover in this limit the exact solution is known which is a de Boer-Winther calculation with zero excitation energy and can be used as a reference for the present approach.

We found that it is convenient to do the calculations in the M - representation of the S-matrix because of the close connection of the target spin I and its projection M with the variables discussed above. In /6/ a l-representation of the S-matrix was used. The S-matrix in the M-representation reads

$$S^{I_{TOTAL}=\ell_0}_{I_f M_f, I_i=0.5\, M_i=0} = \frac{1}{2\pi^2}\int_{-\pi/2}^{\pi/2} dq_{I_i} \int_0^{2\pi} dq_{M_i} \left\{ \frac{\partial(q_{M_f}, q_{I_f})}{\partial(q_{M_i}, q_{I_i})} \right\}^{1/2} \exp\left\{ i\, \phi^{\ell_0}_{I_f M_f} \right\}, \quad (2)$$

where the phase ϕ is given by

$$\phi^{\ell_0}_{I_f M_f} = (M_f(q_{I_i}, q_{M_i}) - M_f)\, q_{M_f}(q_{I_i}, q_{M_i}) + (I_f(q_{I_i}, q_{M_i}) - I_f)\, q_{I_f}(q_{I_i}, q_{M_i}) - \int_{M_i}^{M_f(q_{I_i}, q_{M_i})} q_M\, dM - \int_{I_i}^{I_f(q_{I_i}, q_{M_i})} q_I\, dI - \int_{p_r(-\infty)}^{p_r(+\infty)} r\, dp_r \,. \quad (3)$$

Here q_I, q_M are the angle variables conjugate to I and M. We do the final integrations in (2) with respect to φ_i and β_i instead of (q_{I_i}, q_{M_i}) because the equations of motion are solved in these variables (i = initial, f = final). The F_2-generator which connects (q_I,q_M) with (φ,β) is found by introducing

$$I = (p_\beta^2 + p_\varphi^2/\sin^2\beta)^{1/2}\,, \quad M = -p_\varphi \quad \text{as}$$

$$F_2 = \int p_\varphi\, d\varphi + \int p_\beta\, d\beta = -M\varphi - M \operatorname{arctg}\left(\frac{\sqrt{I^2\sin^2\beta - M^2}}{M\cos\beta}\right) - I \operatorname{arctg}\left(\frac{\sqrt{I^2\sin^2\beta - M^2}}{I\cos\beta}\right) \quad (4)$$

and $q_M = \frac{\partial F_2}{\partial M} = -\varphi - \operatorname{arctg}\frac{\sqrt{I^2\sin^2\beta - M^2}}{M\cos\beta}$, $q_I = \frac{\partial F_2}{\partial I} = \operatorname{arctg}\left(\frac{\sqrt{I^2\sin^2\beta - M^2}}{I\cos\beta}\right)$.

The F_2-generator also helps to simplify the phase in (3)

$$-\int q_M\, dM - \int q_I\, dI = -F_2(t\to\infty) + F_2(t\to-\infty) = -M_f q_{M_f} - I_f q_{I_f} + M_i q_{M_i} + I_i q_{I_i}$$

Then the quantity to be calculated is

$$S^{\ell_0}_{I_f M_f} = \frac{1}{2\pi^2}\int_{-\pi/2}^{\pi/2} d\beta \int_0^{2\pi} d\varphi \left| \frac{\partial(q_{M_i}, q_{I_i})}{\partial(\varphi, \beta)} \right| \left\{ \frac{\partial(q_{M_f}, q_{M_i})}{\partial(\varphi, \beta)} \frac{\partial(\varphi, \beta)}{\partial(q_{M_i}, q_{I_i})} \right\}^{1/2} \exp\left\{ i\, \phi^{\ell_0}_{I_f M_f} \right\} \quad (5)$$

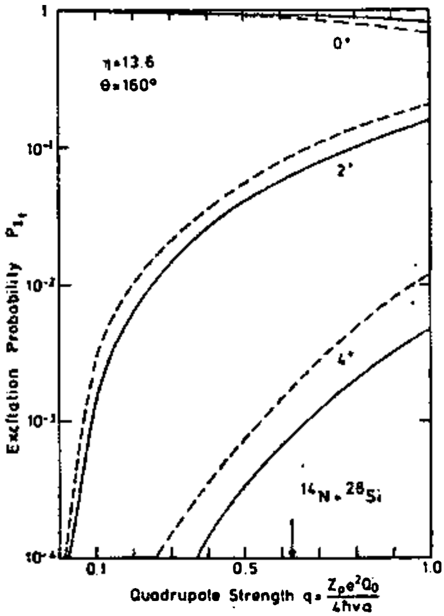
$$\phi^{\ell_0}_{I_f M_f} = -M_f q_{M_f}(\alpha,\beta) - I_f q_{I_f}(\alpha,\beta) + M_i q_{M_i} + I_i q_{I_i} - \int r\, dp_r \,.$$

The transition probabilities are given by

$$P_{I_f}(\theta = 2\operatorname{arctg}(\eta/\ell_0)) = \sum_{M_f} |S^{\ell_0}_{I_f M_f}|^2 . \quad (6)$$

3. Results.- Within the sudden limit we have calculated the transition probabilities for systems with $\eta = 13.6$, $\theta = 160^\circ$ ($\ell_0 = 2.5$) as functions of the strength of the interaction. This situation corresponds to light heavy-ion systems (the realistic case of $N^{14} + Si^{28}$ at E_{Lab} =18.0 MeV is indicated in Fig.2),

where all excitations are classically forbidden. In Fig.2 the comparison of the semiclassical result (dashed line) with the exact result (a deBoer-Winther calculation with zero excitation energy, solid line) is shown. For not too low transition probabilities there are systematic deviations of 20-30 % and for very low (10^{-4}) probabilities the agreement is still within a factor of two (see the 4^+ excitation). The same degree of accuracy of the integral representation applied to a system with two intrinsic degrees of freedom is reported in /3/ for a molecular example.



4. Discussion. - Whereas the results shown in Fig.2 are fairly good, we found that if q is increased beyond the values of Fig.2 (which means that the transitions are becoming less forbidden) the agreement with the exact result gets worse. This is connected with zeroes in the pre-exponential Jacobian in eq.5, which start to occur at a q-value of about 0.75. The same behaviour is observed in calculating P_{I+} for heavy systems as a function of the scattering angle. For small angles, where the transitions are strongly forbidden, there is no zero in the Jacobian and the agreement with the exact result is good. But with increasing scattering angles the transitions become more and more classically accessible, zeroes in the Jacobian appear and the agreement gets worse. When there are zeroes in the Jacobian, the validity of the integral representation is doubtful, because such zeroes are connected with infinities in the WKB-wave function from which it is derived. This point is also brought out in deriving this representation from a path integral /9/. At the moment it is an open question how the integral representation has to be refined to account for this situation.

Although for a system with two degrees of freedom the method presented here is easier to handle than the stationary phase method applied in /5/, it is generally less accurate. If one is interested in more accurate transition probabilities then there is need for a more quantal description.

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