HEAVY ION SCATTERING IN THE ADIABATIC APPROXIMATION

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1. Introduction. — Recently, a formalism has been developed[1] with the purpose of describing large amplitude collective vibrations of nuclei, on the basis of a restrictive dynamical parametrization of the many-body wave function by a suitable set of parameters. Here, we wish to discuss the application of this formalism to the description of heavy ion scattering.

2. Restrictive adiabatic approximation. — We consider a state vector (Slater determinant) $|\Phi(\alpha)\rangle$ which is a real function of n real variables α_i. The equations of motion for the parameters α_i may be derived with the help of the time dependent variational principle, which, for non normalized wave functions, is conveniently written in the form[2)]

$$\frac{i}{2}\left[\delta\frac{\langle\Psi|\partial\Psi/\partial t\rangle-\langle\partial\Psi/\partial t|\Psi\rangle}{\langle\Psi|\Psi\rangle}+\frac{d}{dt}\frac{\langle\delta\Psi|\Psi\rangle-\langle\Psi|\delta\Psi\rangle}{\langle\Psi|\Psi\rangle}\right]$$

$$-\delta\frac{\langle\Psi|\hat{H}|\Psi\rangle}{\langle\Psi|\Psi\rangle}=0\,. \qquad (1)$$

In order to allow for momentum to be acquired by the degree of freedom α_i, we replace the parameters α_i by complex quantities $\gamma_i=\alpha_i+i\beta_i$. The state vector $|\Phi(\gamma)\rangle$ becomes, therefore, a function of n complex quantities γ_i. The variational principle reduces to

$$i\left[\delta\gamma_j^* S_{ji}\,\dot{\gamma}_i-\dot{\gamma}_j^* S_{ji}\,\delta\gamma_i\right]-\delta H'(\gamma^*,\gamma)=0\,, \qquad (2)$$

where

$$S_{ji} = S_{ji}(\gamma^*, \gamma) = \frac{\langle \partial_j \Phi | \partial_i \Phi \rangle}{\langle \Phi | \Phi \rangle} - \frac{\langle \partial_j \Phi | \Phi \rangle \langle \Phi | \partial_i \Phi \rangle}{\langle \Phi | \Phi \rangle^2} \tag{3}$$

with

$$| \partial_j \Phi \rangle = \partial | \Phi \rangle / \partial \gamma_j$$

and

$$H'(\gamma^*, \gamma) = \langle \Phi | \hat{H} | \Phi \rangle / \langle \Phi | \Phi \rangle \ , \tag{4}$$

$\hat{H}$ being the microscopic Hamiltonian.

Finally we arrive at the equations of motion

$$i \, S_{ji} \, \dot{\gamma}_i = \partial H' / \partial \gamma_j^* \ . \tag{5}$$

In the adiabatic approximation, one assumes that the quantities $\dot{\alpha}$ and β are small. Expanding everything in powers of β we have

$$S_{ij}(\gamma^*, \gamma) = S_{ij}(\alpha) + i\beta \left[\frac{\partial S_{ij}}{\partial \gamma_k} - \frac{\partial S_{ji}}{\partial \gamma_k} \right]_{\beta=0} + \ldots \tag{6}$$

where

$$S_{ij}(\alpha) = S_{ij}(\alpha, \alpha) \tag{7}$$

and

$$H'(\gamma^*, \gamma) = V(\alpha) + \beta_i \, \beta_j \, M_{ij}(\alpha) + \ldots \tag{8}$$

where

$$V(\alpha) = H'(\alpha, \alpha) \tag{9}$$

$$M_{ij}(\alpha) = \langle \partial \Phi / \partial \alpha_i | \hat{H} - V(\alpha) | \partial \Phi / \partial \alpha_j \rangle - \langle \partial^2 \Phi / \partial \alpha_i \, \partial \alpha_j | \hat{H} - V(\alpha) | \Phi \rangle . \tag{10}$$

We have assumed that the quantities $S_{ij}(\gamma^*,\gamma)$ and $H'(\gamma^*,\gamma)$ are real functions of complex arguments.

In the adiabatic approximation the dynamical equations become

$$2\,S_{ij}(\alpha)\,\dot{\beta}_j = -\frac{\partial H'(\alpha,\beta)}{\partial\alpha_i} - 2\left[\frac{\partial S_{ik}(\alpha)}{\partial\alpha_j} - \frac{\partial S_{jk}(\alpha)}{\partial\alpha_i}\right]\beta_k\,\dot{\alpha}_j \;,$$

$$2\,S_{ij}(\alpha)\,\dot{\alpha}_j = \frac{\partial H'(\alpha,\beta)}{\partial\beta_i} \;. \tag{11}$$

Defining the momentum variables and the Hamiltonian H through the equations

$$p_i = 2\,S_{ij}(\alpha)\;\beta_j \;, \tag{12}$$

$$H(\alpha,p) = H'(\alpha,\frac{1}{2}S^{-1}p) = V(\alpha) + \frac{1}{4}\,p_i\,p_j\,(S^{-1})_{ki}\,(S^{-1})_{lj}\,M_{kl} \;, \tag{13}$$

where $(S^{-1})_{ij}$ is the matrix inverse of S_{ij}, the dynamical equations become, finally,

$$\dot{p}_i = -\frac{\partial H}{\partial\alpha_i} \;,$$

$$\dot{\alpha}_i = \frac{\partial H}{\partial p_i} \;. \tag{14}$$

3. Heavy ion scattering. — In order to describe heavy ion scattering, it seems reasonable to construct the ket $|\Phi(\alpha)>$ by filling up single particle states in two distinct potential wells, a and b, a refering to the projectile and b to the target nucleus. The variables α_1, α_2 are now the polar coordinates Θ, r defining the relative positions of the two potential wells. Since Θ is just a rotation angle, we may write

$$|\Phi(\Theta,r)> = \exp(-i\,\Theta\,J_z)\,|\Phi(0,r)> \;. \tag{15}$$

After some straightforward algebra, the Hamiltonian becomes

$$H(r, p_\theta, p_r) = V(r) + p_r^2/(2m) + p_\theta^2/(2I), \tag{16}$$

$$1/(2m) = M_{22}/(4S_{22}^2) ,$$

$$1/(2I) = <\Phi|J_z^2(\hat{H} - V(r))|\Phi> / <\Phi|J_z^2|\Phi>^2 .$$

Of course, during collisions, collective degrees of freedom of the ions may be excited, or transfer of particles may occur. For instance, if the projectile does not really penetrate the target, transfer of particles may be described by a wave function of the form

$$|\Phi(\theta, r, \alpha_3)> = \exp(i\alpha_3 W(\theta, r))|\Phi(0, r)> , \tag{17}$$

$$W(\theta, r) = \Sigma (w_{ij}\, a^+_{ai}\, a_{bj} + w^*_{ij}\, a^+_{bj}\, a_{ai}) \tag{18}$$

where a^+_{ai} creates a particle in well a, orbital i, and a^+_{bj} creates a particle in orbital j of well b. Since the wells are parametrized by θ, r, it is clear that these operators are also parametrized by θ, r. Moreover, it is natural to relate the operator $W(\theta, r)$ to the matrix elements of the potential barrier separating the projectile from the target, so the matrix elements w_{ij} may themselves be functions of r. When we wish to describe transfer of particles, the Hamiltonian becomes, therefore,

$$H(r, \alpha_3, p_\theta, p_r, p_3) = V(r, \alpha_3) + p_\theta^2/(2I) + p_r^2/(2m) + p_3^2/(2K) , \tag{19}$$

$$1/(2K) = <\Phi|W(\hat{H} - V)W + W^2(\hat{H} - V)|\Phi> / (2 <\Phi|W^2|\Phi>^2)$$

$$V(r, \alpha_3) = <\Phi|\hat{H}|\Phi> + i\alpha_3 <\Phi|[\hat{H}, W]|\Phi>.$$

The probability for transfer of particles will be given by $\alpha_3^2 <\Phi|W^2|\Phi>$.

The present formalism is flexible enough to allow for the excitation of collective degrees of freedom of the target and projectile, provided the wave function is properly parametrized.

References

1) T. Une et al., Prog. Theor. Phys. 55 (1976) 498;

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2) J. da Providência, Nucl. Phys. 46 (1963) 40; Portugal. Phys. 4 (1965) 171.