GENERIC CURVED SPACE SUPERSTRING AND $\overline{SL}(4,R)$ HADRON CLASSIFICATION

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A generic curved space superstring formulation involves the use of world spinors transforming with respect to the (infinite) unitary representations of $\overline{SL}(10,R)$, the double-covering of the linear group $SL(10,R)$. The $d = 10$ world spinors and tensors reduce, when a spontaneous compactification $R^{10} \to M^{1,3} \otimes K$ takes places, down to the $\overline{SL}(4,R)$ world spinors and tensors. All hadron resonances for each flavor are classified according to spinor and tensor infinite-component systems based on unitary irreducible representations of $\overline{SL}(4,R)$. The fit to observations is very good.

1. Superstrings in Generic Curved Space

It has been realized recently[1)] that superstrings may provide both the first mathematicaly consistent (renormalizable or even finite) quantum theory of gravity and the first truly unified theory of all fundamental interactions and matter. As for the quantum gravity, the crucial observation was that $\alpha' \to 0$ limit of string scattering amplitudes corresponds to appropriate scattering amplitudes of a point particle (string excitation) field theory[2,3)]. In order to establish this connection one has to perform the $\alpha' \to 0$ limit which yields the on-shell scattering amplitudes, and then to guess a covariant lagrangian from which these on-shell scattering amplitudes can be derived. An off-shell formulation which would provide us with a complete field theory effective action from a superstring theory path integral is lacking. Moreover, the quantum theory of a string is quite different from that of a point particle. A consistent field theory for a point particle can be formulated in any number d of space-time dimensions, while the string quantum theory is known to be consistent quantum theory only in the critical d_c flat space-time dimensions: $d_c = 26$ for bosonic string, and $d_c = 10$ for superstring. Indeed, in the conventional lagrangian formulation for strings or superstrings, the 2-dimensional curved (locally reparametrizable) string world-sheet R^2 is embedded in a flat d_c-dimensional Minkowski space-time. On the other hand, macroscopic gravity is described classically by Einstein's theory, corresponding to a curved Riemannian R^4 manifold. It is now clearly necessery first to find a generic curved space formulation for superstring, and then to study the theory on manifolds with some of the dimensions compactified. A Kaluza-Klein-like scenario for a superstring theory may be important in reduction of the theory (spontaneous compactification) down to d = 4 dimensions.

The attempts to embed the superstring in a curved manifold[4-6] have encountered three fundamental difficulities: (a) The fermionic $\theta(\xi)$ frame-fields required by supersymmetry and constructed at any point ξ^μ, $\mu = 0,1$ of the world-sheet as spinors in $M^{1,9}$, cannot be embedded in a curved generic Riemannian (i.e. Riemann-Cartan) manifold R^{10}, since there exist no finite-dimensional spinorial representations of $\overline{GL}(10,R)$ on the one hand, and on the other hand one cannot apply the usual tetrad formulation, as discussed below. (b) The critical dimensionality of the embedding space is modified[4, 5, 7] in the presence of generic curvature, thus destroying the essential condition for a ghost-free finite-theory. (c) Without spinors we also lose supersymmetry, essential as a constraint for the removal of a tachyon whose unwanted presence now makes the theory unphysical.

We have suggested recently solutions for all three difficulties, based upon the application of the doubly-covered groups of Diffeomorphisms and Superdiffeomorphisms in the "tangent" at ξ^μ (Ref. 8, 9). We have constructed explicitely the (infinite) spinorial and tensorial representations of the double covering $\overline{SL}(10,R)$ group[8] thus answering the quest in (a), while realizing the Principle of General Covariance. Further, we have solved problems (b) and (c) by embedding $\overline{GL}(10,R)$ in the double-

-covered real-form $\overline{\overline{GQ}}(10,R)$, a supergroup generated by the classical hyperexceptional superalgebra $q(10)$[9]. This way the curving of $M^{1,9} \rightarrow R^{10}$ preserves supersymmetry (i.e. no ghosts) both on and off mass shell.

In the Polyakov formulation[10] of the Green-Schwarz quantized superstring, every point ξ^{μ} of the evolving string carries a global 10-dimensional-Poincaré supersymmetric frame $(X^{m}(\xi),\ \theta^{\alpha a}(\xi))$, $m = 0, 1, \ldots, 9$ denoting the components of a bosonic 10-vector frame, $a = 1, 2, \ldots, 16$ standing for the fermionic components of a real (Majorana) chiral (Weyl) spinor frame, and $\alpha = 1,2$.

In going from $M^{1,9}$ to R^{10}, one replaces $\eta_{mn} \rightarrow g_{\hat{m}\hat{n}}$, a curved metric, and so on for all bosonic quantities. However, the method fails for the spinors. In the usual technique in General Relativity, a spinor would be defined in the local tangent (flat embedding, coordinate x^{m}) space at $x^{\hat{m}}$ (curved embedding), where a local Lorentz group can act on it. This is achieved by introducing the "decad" frames

$$e^{m}_{\hat{m}}(\hat{x}) = \partial\, x^{m} / \partial\, x^{\hat{m}}, \qquad \psi^{a}_{\hat{m}}(\hat{x}) = \partial\, \theta^{a} / \partial\, x^{\hat{m}}.$$

In the string formalism the original spinor field is defined as a local (fiber) frame-field on the tangent to the curved string (the base manifold) at the string coordinate ξ^{μ}. Indeed, the coordinates x^{m} (flat) or $x^{\hat{m}}$ (curved) do not appear in the formalism. As a result, the usual transition of the gamma matrices $\gamma^{n} \rightarrow \gamma^{\hat{n}}$ cannot be performed by an ordinary tetrad-like (decad) matrix. Our $X^{m}(\xi)$ are in the flat tangent at ξ^{μ}, and there are no "tangents to the tangents", frames over frames. Were it not for the spinors, generic curving could have been achieved by replacing $X^{m}(\xi)$ by $X^{\hat{m}}(\xi)$, a world-vector carrying finite linear representations of GL(10,R) and non-linear representations of the General Coordinate Transformation Group $\Delta(10)$. In other words changing the structure group of the bundle from $\overline{SO}(1,9)$ to $\overline{GL}(10,R)$ and $\overline{\Delta}$. For spinors, where the double-covering is required, one has only the infinite representations of $\overline{GL}(10,R)$ and $\overline{\Delta}$. We thus replace the $\theta^{a}(\xi)$ by $\psi^{A}(\xi)$, infinite frames transforming w.r.t. the unirreps of the linear subgroup $\overline{SL}(10,R)$. In order to preserve supersymmetry we additionally replace $X^m(\xi)$ by an appropriate infinite-component $\overline{SL}(10,R)$ bosonic frames $X^M(\xi)$.

2. $\overline{SL}(10,R)$ Unitary Irreducible Representations

Let Q_{mn}, m,n = 0, 1, ,,, , 9 be the $\overline{SL}(10,R)$ generators. In the configuration space they behave as $\eta_{mk}x^k p_n - \frac{1}{10}\eta_{mn}x^k p_k$, where μ_{mn} = disag (+1, -1, -1, ... , -1). The $\overline{SL}(10,R)$ commutation relations read

$$[Q_{mn}, Q_{kl}] = i\,(\eta_{nk}\,Q_{ml} - \eta_{ml}\,Q_{kn}) .$$

The maximal compact subgroup Spin(10) = $\overline{SO}(10)$ of the $\overline{SL}(10,R)$ group is generated by $L_{mn} = \frac{1}{2}(Q_{mn} - Q_{nm})$. The remaining generators $T_{mn} = \frac{1}{2}(Q_{mn} + Q_{nm})$ are noncompact, and transform as a 54-dimensional $\overline{SO}(10)$ irreducible tensor operator, i.e. as (20000) in the Dynkin notation.

The Wigner-Inönü contraction of the $\overline{SL}(10,R)$ group w.r.t. its $\overline{SO}(10)$ subgroup is the $T_{54} \circledS \overline{SO}(10)$ group, with the mutually commuting noncompact generators U_{mn},

$$U_{mn} = \lim_{\epsilon\to 0} \epsilon\, T_{mn} , \qquad [U_{mn}, U_{kl}] = 0.$$

We construct the $\overline{SL}(10,R)$ unitary irreducible representations (unirreps) in the $\overline{SO}(10)$ basis. A very convenient way to carry out an explicite $\overline{SL}(10,R)$ unirrep construction consists in i) the construction of the $T_{54} \circledS \overline{SO}(10)$ unirreps and ii) the lifting of these unirreps to the $\overline{SL}(10,R)$ ones by making use of the decontraction formula.

Let L_{mn} and U_{mn} be the generators of a given $T_{54} \circledS \overline{SO}(10)$ unirrep. The generators of the corresponding (decontracted) $\overline{SL}(10,R)$ unirrep are given by L_{mn} and T_{mn}, where

$$T_{mn} = pU_{mn} + \frac{i}{2}\,(U_{kl}\,U^{kl})^{-1/2}\,[L_{kl}\,L^{kl}, U_{mn}] , \quad p \in R .$$

The simplest spinorial and tensorial $\overline{SL}(10,R)$ unirreps belong to the set of multiplicity free representations. They contain in the $\overline{SO}(10,R)$ subgroup decomposition each $\overline{SO}(10)$ representation at most once. We list some $\overline{SL}(10,R)$ multiplicity free unirreps[8] and their $\overline{SO}(10)$ unirrep content. The spinoral "discrete series" unirreps contain:

$$D(16) = \{ 16, 144, 720, 2640, 7920, \dots \} ,$$
$$D(560) = \{ 560, 3696', 8800, 15120, \dots \} ,$$
$$D(672) = \{ 672, 1440, 11088, \dots \} , \quad \dots ,$$

and their conjugated unirreps

$$D(\overline{16}) = \{ \overline{16}, \overline{144}, \overline{720}, \overline{2640}, \overline{7920}, \dots \} , \quad \dots$$

The tensorial "ladder series" unirreps are:

$$D(1) = \{ 1, 54, 660, 4290, 19305, \dots \} ,$$
$$D(10) = \{ 10, 210', 1782', 9438, 37180, \dots \} .$$

3. Riemannian Space Supersymmetry

We have presented above a solution to the problem of constructing the spinor frame fields for the Green-Schwarz superstring embedded in a generic curved space R^{10}. Were it not for the spinors, generic curving could have been achieved by replacing $X^m(\xi)$ by $\hat{X}^m(\xi)$, a world-vector carrying finite representations of $\overline{GL}(10,R)$ and non-linear representations of the $\overline{\Delta}$ group (analytical diffeomorphisms). When embedding the superstring in a generic curved space, beyond the fitting in of the spinors, we have to preserve the supersymmetry, otherwise we cannot get rid of an unphysical tachyon state and preserve the critical dimension $d_c = 10$.

We posed several natural requirements on the curved space supersymmetry and found a solution - the $\overline{GQ}(10,R)$ supersymmetry which is generated by the classical hyperexceptional $q(10)$ superalgebra[9].

Finally, by making use of the $\overline{GQ}(10,R)$ supersymmetry representations, which contain the even subgroup $\overline{SL}(10,R) \subset \overline{GL}(10,R)$ representations $D(10)$ and $D(16) \oplus D(\overline{16})$ in the reduction, we should achieve a complete description of the superstring embedded in a generic curved space.

4. Field Equations

The infinite-component $\overline{SL}(10,R)$ unirreps define the corresponding infinite-component fields - "manifields". There are two crucial physical requirements to be fulfilled. First, the intrinsic Lorentz generators represented on manifields must be non-unitarily represented, the Lorentz boosts would otherwise excite a given state to other spins and masses contrary to experience. We resolve this by applying the deunitarizing automorphism[11,12,8] to the $\overline{SL}(10,R)$ unirreps. Second, in the absence of gravity, by the Principle of Equivalence, only the Lorentz group survives (or, in fact the double covering of the Poincaré group). Thus, we require our manifields to satisfy only the $\overline{SO}(1,9)$ covariant equations[8]. For the tensorial manifields the simplest choice is a Klein-Gordon-like equation

$$(\partial_m \partial^m + M^2) X = 0 ,$$

while for the spinorial manifields we take a Dirac-like equation

$$(i \Gamma^m \partial_m - M) \psi = 0 ,$$

where $\psi \sim D(16) \oplus D(\overline{16})$, and Γ^m is a $\overline{SO}(1,9)$ 10-vector.

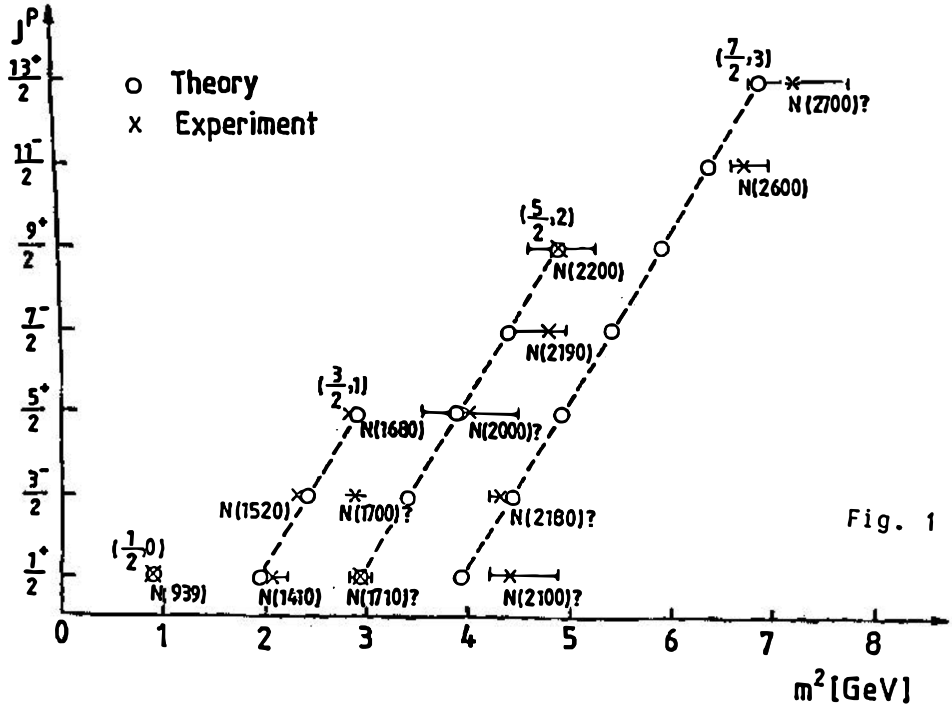
5. Spontaneous Compactification and $\overline{SL}(4,R)$ Hadron Classification

The ultimate test of any physical theory is not mathematical consistency, but confrontation with experiment. Owing to the fact that quantum gravitational effects are beyond the reach of present day experiment, the development of a quantum theory of gravity has been excedingly difficult.

Superstring theories appear to be consistent only if the dimension of space-time is 10 and the gauge group is $E_8 \otimes E_8$ or $SO(32)/Z_2$. These results are at the first sight discouraging since one certainly does not observe 10 dimensions or $SO(32)/Z_2$ or $E_8 \otimes E_8$ gauge symmetries. If these theories are to have any connection with nature one must find a way to reduce both the dimension of space-time and the gauge group. At this point one makes use of the Kaluza-Klein approach that some of the space-like dimensions are compactified and small enough to not be directly observable. In general relativistic field theories the geometry of space-time is not fixed, and one can expand around curved backgrounds. Phenomenologically interesting backgrounds are of the form $M^{1,3} \otimes K$, where K is some small 6-dimensional space and $M^{1,3}$ is the 4-dimensional Minkowski space. It is not at all obvious that there exists a consistent expansion around such backgrounds.

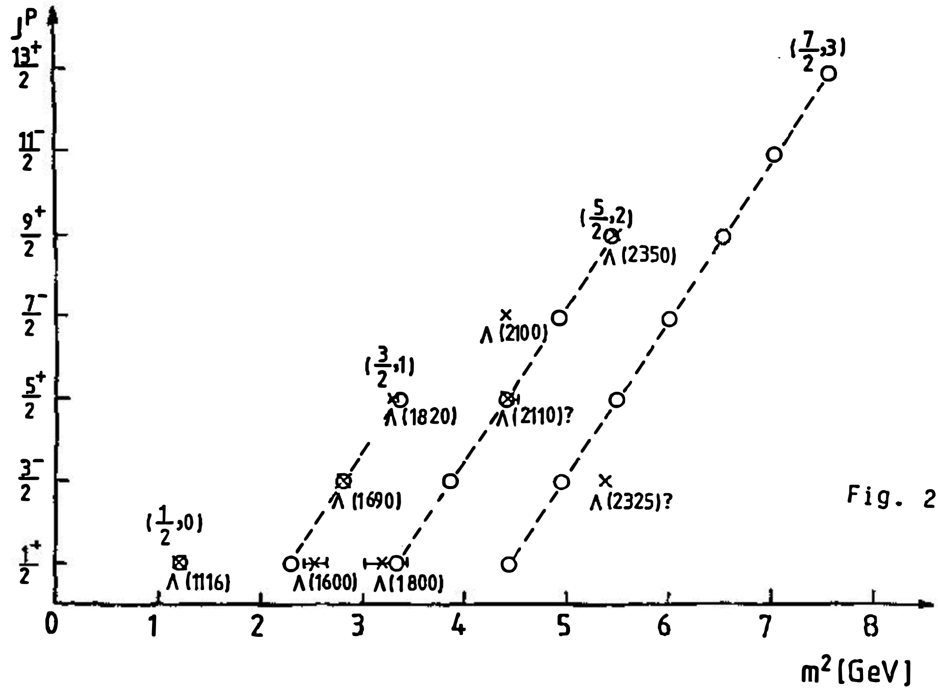
We will assume in the following that the $M^{1,3} \otimes K$ space--time configuration is a solution of our generic curved space superstring formulation, and we concentrate on the resulting Minkowski space field structure. As we have argued above, an embedding of the superstring in a generic curved space requires spinorial $\overline{SL}(10,R)$ manifields, while the supersymmetry requires in addition $\overline{SL}(10,R)$ tensorial manifields as well. In the flat $M^{1,3}$ background space-time, according to the Equivalence Principle one has a global Poincaré symmetry and the corresponding fields are Poincaré covariant. However, various of these Poincaré fields are actually components of a certain generic curved space manifields. Indeed, in going from a generic curved space R^{10} down to $M^{1,3}$ the $\overline{SL}(10,R)$ manifields reduce first to a sum of $\overline{SL}(4,R)$ manifields and then, in the absence of gravity, each of the latter manifields breaks up in an infinite sum of SL(2,C) fields. Thus, in the hadron (extended object) spectroscopy, we expect to find Poincaré objects, which are organized according to the $\overline{SL}(4,R)$ symmetry unirreps, and which in the flat Minkowski space limit fulfil the Poincaré invariance only. An intriguing possibility would be to associate the additional symmetry, beyond $\overline{SL}(4,R)$, in the $\overline{SL}(10,R) \rightarrow \overline{SL}(4,R) \otimes G$ reduction to the flavour degrees of freedom.

We have analysed recently[13)] the available data on meson and baryon resonances in the light of Poincaré wave equation projected $\overline{SL}(4,R)$ unirrep states. We have found that the complete spectrum of baryon and meson resonances for each flavour are organized according to these states, and we have suggested



J^P
13+/2
11-/2
9+/2
7-/2
5+/2
3-/2
1+/2
o Theory
x Experiment
(7/2,3)
N(2700)?
N(2600)
(5/2,2)
N(2200)
N(2190)
(3/2,1)
N(1680)
N(2000)?
N(1520)
N(1700)?
N(2180)?
(1/2,0)
N 939)
N(1440)
N(1710)?
N(2100)?
0
1
2
3
4
5
6
7
8
m² [GeV]

Fig. 1



J^P
13+/2
11-/2
9+/2
7-/2
5+/2
3-/2
1+/2
(7/2,3)
(5/2,2)
Λ (2350)
Λ (2100)
(3/2,1)
Λ (1820)
Λ (2110)?
Λ (1690)
Λ (2325)?
(1/2,0)
Λ (1116)
Λ (1600)
Λ (1800)
0
1
2
3
4
5
6
7
8
m² [GeV]

Fig. 2

certain Regge trajectory (J vs. m^2) generalized mass formulas. The agreement with experimental data is very good. The comparison with experiment is illustrated for the best known systems of N and Λ resonances (Fig. 1,2).

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