RADIATION EFFECTS IN THE QUANTUM FIELD THEORY OF ELECTRIC AND MAGNETIC CHARGE

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1. Introduction

The first idea about magnetic monopoles in the quantum-mechanical context was put forward by Dirac$^{1)}$. The quantum field theory of electric and magnetic charge (hereafter called QEMD, quantum electro-magnetodynamics) was formulated by Schwinger$^{2)}$ in a Hamiltonian operator framework. Schwinger's formulation is a non-local one. This motivated Zwanziger$^{3)}$ to introduce a local Lagrangian approach, which is based on two potentials. The one-potential formulation$^{4)}$ describes the physical content of the theory in the simplest manner. All the three formulations are shown to be equivalent$^{4)}$.

In order for the theory to be consistent it is necessary to confirm Poincaré invariance and solve the problem of infrared and ultraviolet divergences. The first problem was treated by Brandt, Neri and Zwanziger$^{5)}$. Using the one-potential formulation the second problem of the theory was formally solved$^{6)}$. The progress was also made in ultraviolet regularization$^{7)}$.

This article will give a short account of the one-potential formulation of QEMD, the solution of the infrared problem and the related superstrong radiation damping effects, that go beyond naive perturvation theory.

2. The one-potential formulation of QEMD

The formulation we are going to describe represents a natural generalization of Dirac's quantum-mechanical theory to quantum field theory. It is based on a Lagrangian and features only one four-potential$^{4)}$.

A. Classical Lagrangian formalism. We are after a Lagrangian that will yield the generalized Maxwell´s equations

$$\partial_\mu F^{\mu\nu} = J_e^\nu , \tag{1}$$

$$\partial_\mu {}^*F^{\mu\nu} = J_g^\nu . \tag{2}$$

Here, F means the tensor dual to F, J_e and J_g are the conserved electric and magnetic currents, respectively,

$$J_e^\nu = e\,\bar\psi\,\gamma^\nu\,\psi , \qquad J_g^\nu = g\,\bar\chi\,\gamma^\nu\chi , \tag{3}$$

and ψ and are spin 1/2 fields describing a pure charge and a pure monopole (the generalization to dyons is direct), respectively.

Since the usual way of introducing the electromagnetic potential by setting $F = \partial \wedge A$ leads to $\partial\,{}^*F = 0$, in contradiction to Eq. (2), we attempt$^{1)}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \varepsilon_{\mu\nu\lambda\sigma} G^{\lambda\sigma} . \tag{4}$$

Equation (2) suggests a simple ansatz for $G_{\mu\nu}$,

$$G_{\mu\nu} = h_\mu J_\nu^{\,g} , \tag{5a}$$

with $\partial\cdot h = -1$. We are free to choose h as

$$\begin{aligned} h^\mu(x) &= -\, n^\mu\,(n\cdot\partial)^{-1}(x), \\ (n\cdot\partial)^{-1}(x) &= \left[-\alpha\,\theta(n\cdot x) + (1-\alpha)\,\theta\,(-n\cdot x)\right]\,\delta_n(x), \end{aligned} \tag{5b}$$

where n^μ is a constant vector, $\theta(n)$ is the one-sided step function, α is an arbitrary real number, and $\delta_n(x)$ is a three-dimensional δ-function with support on the hypersurface through the origin, whose normal is n^μ.

The Lagrangian for the theory is

$$\mathcal{L} = -\frac{1}{4} F^2 + \bar{\psi} [\gamma \cdot (i\partial - eA) - m_1] \psi + \bar{\chi} (\gamma \cdot i\partial - m_2) \chi . \qquad (6)$$

There is an asymmetry here in the description of electric and magnetic variables, but this is only apparent. The Lagrangian (6) leads to the following equations of motion:

$$\partial_\mu F^{\mu\nu} = J_e^{\ \nu} , \qquad (7a)$$

$$[\gamma \cdot (i\partial - eA) - m_1] \psi = 0, \qquad (7b)$$

$$[\gamma \cdot (i\partial - gB) - m_2] \chi = 0, \qquad (7c)$$

where the potential B depends on other dynamical variables, $B = h^T \, {}^*F$. The equations (7 b, c) and the accompanying equations for ψ and lead to the conservation of electric and magnetic currents, which is the consistency condition for Maxwell´s equations.

We shall choose n^μ to be a spacelike vector so that we can have a choice $n^o = 0$ for which the theory is local in time.

B. Feynman rules. The perturbation expansion of QEMD possesses features that render it formal: (a) it contains two coupling constants which turn out not to be independent - the demand of Lorentz invariance relates one to the other$^{5)}$; (b) one of the coupling constants (the magnetic one) is very large; (c) to any finite order in the perturbation expansion the theory is not Lorentz invariant. Nevertheless, the (formal) Feynman rules will be usefull in giving us insight into the structure of the theory.

Using the Lagrangian (6) we can obtain the following Feynman rules, local in momentum space:

the photon propagator (in Feynman gauge):

$$D_F^{\ \mu\nu} = - i \, \eta^{\mu\nu} / k^2 , \qquad (8a)$$

the photon-charge vertex:

$$V_e^{\nu} = - i e \gamma^{\nu} , \tag{8b}$$

the photon-pole vertex:

$$\begin{aligned} \Gamma_{\nu} &= - i g \epsilon_{\mu\nu\lambda\sigma} k^{\mu} n^{\lambda} \gamma^{\sigma} / (n\cdot k)^2 \equiv \\ &\equiv - i g A_{\nu\sigma} \gamma^{\sigma} \end{aligned} \tag{8c}$$

The four-pole vertex Λ can be simulated by two naive vertices of the type $V_g^{\nu} = - i g \gamma^{\nu}$ connected by a spurion propatator D_s:

$$\begin{aligned} \Lambda &= V_g^{\mu} \left[- i (n^2 \eta_{\mu\nu} - n_{\mu} n_{\nu}) / (n\cdot k)^2\right] V_g^{\nu} \\ &\equiv V_g^{\mu} D^{s}_{\mu\nu} V_g^{\nu} . \end{aligned} \tag{8d}$$

In all considerations concerning the contribution of diagrams containing photon exchange between to monopole lines, calculations can be simplified substantionally. Two graphs involving photon and spurion exchange add up to a graph with naive vertices V_g connected by the effective propagator D_{EF},

$$\begin{aligned} &\Gamma D_F \Gamma + V_g D_s V_g = V_g D_{EF} V_g , \\ &D_{EF}^{\mu\nu} \equiv - \frac{i}{k^2} \left[\eta^{\mu\nu} - \frac{n^{\mu}k^{\nu} + n^{\nu}k^{\mu}}{n\cdot k} - \right. \\ &\qquad \left. - \frac{n^2}{(n\cdot k)^2} k^{\mu} k^{\nu}\right] . \end{aligned} \tag{9}$$

The n-dependent terms in D_{EF} can be gauged away[6], leading to $D_{EF}' = D_F$, in conformity with the dual symmetry of the theory.

3. The infrared problem and radiation effects

The resolution of the infrared problem in QED can be achieved by cancelling infrared divergences stemming from soft virtual photons against those coming from soft real photons. The work was made complete in the realm of perturbation theory by Jauch and Rohrlich[8], and Yennie, Frantschi and Suura[9]. It is seen diredtly that the infrared problem stems from the masslessness of the photon, and so it also appears in QEMD. The solution of the problem in QEMD is analogous to the one in QED[6]. The analogy is most explicit in the one-potential formulation.

A. Factorization of infrared divergences. We are going first to show that, in the realm of formal perturbation theory, the infrared divergent contribution of soft photons may be represented as a multiplicative factor.

Let G be the Feyman diagram corresponding to some physical process involving monopoles and charges, and let $\bar{G}$ denote the set of diagrams obtained from G by inserting one additional photon (real or virtual) into G in all possible ways. Ignoring, for the moment, the diagrams in $\bar{G}$ which contain closed loops, we divide the rest into two classes: $\bar{G}_1$ contains diagrams in which only one end of the additional photon line terminates on a given charge (or monopole) line, and $\bar{G}_2$ contains diagrams in which both ends of the additional photon line terminate on a given charge (or monopole) line.

We now turn our attention to the set $\bar{G}_1$ and assume, first, that the additional photon line is *virtual*. Then, the infrared factors associated with each charge or monopole line are of the respective forms

$$R_\mu^{\,e} = - i e I_\mu (p', p) = - i e \left(\frac{2p_\mu' - k_\mu}{2p'\cdot k - k^2} - \frac{2p_\mu - k_\mu}{2p\cdot k - k^2} \right), \tag{10}$$

$$R_\mu^{\,g} = - i g A_{\mu\nu}(k) I^\nu (p', p).$$

The expression for $R_\mu^{\,g}$ differs from the QED one ($R_\mu^{\,e}$) by the presence of the factor $A_{\mu\nu}$, originating from the different photon-pole vertex. The complete correction stemming from a virtual photon of momentum k which connects two different charge lines is given by

$$R_\mu^{\,e}(i)\ \tilde{D}_F^{\mu\nu}(k)\ R_\nu^{\,e}(j) = -\ e^2\ I_\mu(i)\ I^\mu(j)\ \tilde{\Delta}_F(k), \qquad (11)$$

where $\bar{D}_F = -\ i\ \eta\ \bar{\Delta}_F = -\ i\ \eta\ /\ (k^2-\lambda^2)$ and λ is a small photon mass regulator. If the additional photon line terminates on two different monopole lines, than, taking into account spurion exchange, one finds

$$\begin{aligned} &R_\mu^{\,g}(i)\ \tilde{D}_F^{\mu\nu}(k)\ R_\nu^{\,g}(j) + \text{spurion exchange} = \\ &= -\ g^2\ I_\mu(i)\ I^\mu(j)\ \tilde{\Delta}_F(k)\ , \end{aligned} \qquad (12)$$

which is the same as Eq. (11) up to the replacement $e^2 \to g^2$, in agreement with the dual symmetry between electric and magnetic sectors of the theory. Finally, when the additional virtual photon line connects an electric and a magnetic line, one obtains an expression whose contribution vanishes after integrating over the photon momenta.

In the case of *real* photons, the infrared factors associated with each charge or monopole line are

$$\begin{aligned} &\bar{R}_\mu^{\,e} = -\ i\ e\ \bar{I}_\mu(p',p) \equiv -\ i\ e\ \left(\frac{p_\mu'}{p'\cdot k} - \frac{p_\mu}{p\cdot k}\right)\ , \\ &\bar{R}_\mu^{\,g} = -\ i\ g\ A_{\mu\nu}(k)\ \bar{I}^\nu(p',p)\ . \end{aligned} \qquad (13)$$

To produce the corresponding emission cross-section we square $\bar{R}^e$ and $\bar{R}^g$ and find

$$\bar{R}_\mu^{\,e\,*}\ \bar{R}^{e\mu} = -\ e^2\ \bar{I}_\mu\ \bar{I}^\mu\ , \qquad (14)$$

$$\bar{R}_\mu^{\,g\,*}\ \bar{R}^{g\mu} = -\ g^2\ \bar{I}_\mu\ \bar{I}^\mu \qquad (15)$$

after using the mass-shell condition for real photons $k^2 = 0$, and the transversality of $\overline{I}_\mu$. Expressions (14) and (15) are of the same form up to the replacement $e^2 \to g^2$, which is, again, a simple consequence of the duality.

In the set $\overline{G}_2$, the contribution of the diagrams in which both ends of an additional photon line terminate on an electrically charged line is the same as in QED. When we go over to the monopole line then, by taking into account also the spurion exchange, we kind the result of the same form, with the replacement $e^2 \to g^2$. (The question of ultraviolet divergences is left untouched here.)

This completes the result for both real and virtual soft photon contributions, but with *fermion loops* disregarded. The presence of these loops, however, does not influence the previous results. Indeed, the diagrams with photon insertions on charged loops are infrared finite in the photon momenta - that is a known result in QED. We can prove a similar result for pole loops by separating out the factor $A_{\mu\nu}$ in the photon-pole vertex, and treating the rest in the same fashion. This is to within possible string divergences stemming from the form of $A_{\mu\nu}$. The n-dependent parts of these divergences may cancel only after summation of all diagrams$^{5)}$, but with a fortuitous regularization they may even disappear order by order in perturbation theory$^{6)}$.

The upshot of the above discussion is that the soft photon contributions in monopole processes are finite and factorizable. We can, therefore, proceed to show exponentiation in the same way as in QED.

B. Infrared regularization of monopole processes. Let us consider the *potential scattering* of a monopole (on a charged center) as the simpliest case necessitating infrared regularization. Using the factorization of soft-photon contributions, one obtains, as in QED$^{9)}$, the exponentiation of the infrared contribution to the cross-section, which takes the form

$$\frac{d\sigma}{d\varepsilon} = \exp\left[2\,\alpha_g\,(R_e\,B + \bar{B})\right]\frac{d\tilde{\sigma}}{d\varepsilon} \quad . \tag{16}$$

Here $d\tilde{\sigma} / d\varepsilon$ is proportional to the cross-section of the basic process without radiative corrections, $\alpha_g = g^2 / 4\pi$ and $d\sigma / d\varepsilon$ is the differential cross-section with energy loss ε and momenta p and p'. The exponential factor in Eq. (16) describes the contribution of infrared photons,

$$\begin{aligned} B &= \frac{i}{8\pi^3}\int \frac{d^4k}{k^2-\lambda^2}\, I_\mu(p',p)\, I^\mu(p',p) \equiv B(p',p) \ , \\ \bar{B} &= -\frac{1}{8\pi^2}\int \frac{d^3k}{\omega_\lambda}\, \bar{I}_\mu(p',p)\, I^\mu(p',p) \equiv \bar{B}(p',p) \ , \end{aligned} \tag{17}$$

where $\omega_\lambda = (\vec{k}^2 + \lambda^2)^{1/2}$. Re B and $\bar{B}$ can be calculated to be of the form

$$\begin{aligned} \mathrm{Re}\, B &= \frac{1}{2}\left(-\,K\, \ln\frac{m}{\lambda} + K_2\right) \ , \\ \bar{B} &= \frac{1}{2}\left(K\, \ln\frac{2\varepsilon}{m} + K_1\right) \ , \end{aligned} \tag{18}$$

where K, K_1 and K_2 are independent of λ . Then the complete cancellation of infrared divergences in the cross-section results from the expression

$$R_e\, B + \bar{B} = \frac{1}{2}\left(K\, \ln\frac{2\varepsilon}{m} + K_1 + K_2\right) \ . \tag{19}$$

For *charge-monopole scattering* one similarly obtains a cross-section of the form (16), but now B and $\bar{B}$ are given by different expressions:

$$\begin{aligned} \alpha_g\, B &\to \alpha_g\, B(p_4,p_2) + \alpha_e\, B(p_3,p_1) \ , \\ \alpha_g\, \bar{B} &\to \alpha_g\, \bar{B}(p_4,p_2) + \alpha_e\, \bar{B}(p_3,p_1) \ . \end{aligned} \tag{20}$$

The case of *monopole pair creation* in a time- and space-dependent external field can be treated analogously (this process lies in just another kinematic region of the former).

In solving the infrared problem in QEMD, we have followed the method of Yennie, Frautschi and Suura$^{9)}$, in which the infrared finitness of the physical cross-section is proved.

C. Radiation effects. We have seen that the cross-section, which is of the form (16), is infrared finite. As a byproduct of this analysis we have obtained a finite exponentiated piece (stemming from soft photons) in the cross-section. While in QED this factor is only a small correction to the naive result, due to the smallness of the fine structure constant, here in QEMD the effect is enormous since the relevant coupling constant is large.

In order to see the nature of this effect more closely, let us look at monopole *potential scattering*. In the relativistic kinematic region $|(p'-p)^2| \gg m^2$ and $\varepsilon \ll E$, we have$^{9)}$

$$\mathrm{Re}\, B + \bar{B} \sim - \frac{1}{4} K \ln \frac{EE'}{2} + \frac{1}{8} K , \qquad (21)$$

where $K \approx (2/\pi)(\ln 2p\cdot p' / m^2 - 1)$, E and E' are monopole energies before and after scattering. This result means a superstrong damping of the cross-section with a factor smaller than exp (-137).

In monopole-charge scattering and pair creation one can find damping factors of the same type$^{6)}$.

The presence of the factor $d\tilde{\sigma} / d\varepsilon$ in the expression for the cross-section demands some caution in the estimate of the complete cross-section. We cannot completely rule out the possibility that this factor might cancel, or even completely overshadow the damping in the oposite direction. Yet, since there is no physical reason for this to occur, the effect is likely to be genuine. These results are also supported by the results of Drukier and Nussinov$^{10)}$.

The physical implications of the phenomenon of superstrong radiation damping are the following. First, in scattering the damping is enormous in the relativistic kinematic region, therefore experimental searches should be geared toward detection of slow monopoles. The calculation of pair creation does not clarify the case of monopole creation in the early universe, as the kinematic region in this case is nonrelativistic ($M \simeq 10^{17}$ GeV, $T_c \simeq 10^{15}$ GeV). Finally, radiation damping has a bearing on the problem of dynamical origin of monopole confinement$^{11)}$.

4. Conclusion

In this paper we have studied the infrared behaviour of QEMD using the one-potential formulation. It is shown that the leading infrared contribution is factorizable and can be exponentiated with a cancellation of infrared divergent contributions of real and virtual soft photons.

The remaining infrared finite parts of real and virtual soft photon contributions yield a significant damping of the cross-section in the relativistic kinematic region, where, moreover, we can trust the approximations made. This phenomenon may be important for planing experiments of monopole detection, as well as for the problem of monopole confinement

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