TWIST-FOUR SPIN-ONE CORRECTIONS IN DEEP INELASTIC NEUTRINO SCATTERING

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In addition to perturbative Quantum Chromodynamic (QCD) corrections there are nonperturbative QCD corrections which behave like $1/Q^2$ relative to the $\ln Q^2$ leading log corrections. The twist-four spin-one corrections have been calculated in deep inelastic neutrino scattering on nucleon targets. It is shown that they violate by very small percentage Bjorken and Gross-Llewellyn-Smith sum rules.

1. Introduction

The deep inelastic lepton scattering on nucleon targets has been extensively used to test QCD. The twist-four corrections to the nucleon structure functions for electron and neutrino scattering have been investigated and compared with typical QCD perturbative corrections[1-6]. The systematic procedure for the study of higher twist effects in lepton deep inelastic scattering has been established[1, 2]: The bilocal product of the two quark current operators is expanded into local operators using Wilson operator product expansion[7]. The coeficient functions obey the renormalization group equation and the anomalous dimension of the operators are calculated using perturbative techniques.

The nucleon matrix elements of the local operators are model dependent and they can be determined using some quark confinement model for the nucleon wave functions[8]. Most of these calculations have included only contributions coming from four-quark operators, neglecting the two-quark, one-gluon operators. These two-quark, one-gluon operators[9] contribute corrections to the Bjorken and Gross-Llewellyn-Smith sum rules.

The first sum rule stands for the first Nachtmann moment of the first structure function $F_1(x,Q^2)$ in neutrino scattering on nonsinglet nucleon targets and the second sum rule stands for the first Nachtmann moment of the third structure function $F_3(x,Q^2)$ in neutrino scattering on singlet nucleon targets.

These twist-four spin-one corrections can affect the calculation of $\sin^2\theta_W$ from the ratio R^ν defined as the ratio of the total cross section of neutrino neutral current scattering and the cross section of neutrino charged current scattering on isoscalar targets. It is shown that the twist-four spin-one corrections are negligable comparing to the twist-four spin-two effects.

2. Results and discussion

The hadronic tensor is defined by

$$\begin{aligned} W_{\mu\nu} &= \int dz e^{iqz} < p \,|\left[J_\mu (z), J_\nu (0)\right]|\, p > = \\ &= (g_{\mu\nu} - q_\mu q_\nu / q^2)\left[\nu W_2 (\nu, Q^2) / 2x\right] - \\ &- \left[g_{\mu\nu} + p_\mu p_\nu q^2 / \nu^2 - (p_\mu q_\nu + p_\nu q_\mu) / \nu\right] \cdot \\ &\cdot \left[\nu W_2 (\nu, Q^2) / 2x\right] - i\varepsilon_{\mu\nu\alpha\beta}\, p^\alpha p^\beta / \nu \left[\nu W_3 (\nu, Q^2)\right] \end{aligned} \tag{1}$$

where the standard structure function are given by:

$$\nu W_{L,2,3} (\nu, Q^2) = F_{L,2,3} \tag{2}$$

To estimate the higher twist effects we use the Wilson operator product expansion[6)]

$$T_{\mu\nu} = i \int dz e^{iqz}\, T\left[J_\mu(z)\, J_\nu(0)\right] \tag{3}$$

$T_{\mu\nu}$ can be decomposed into scalar structure functions $T_{L,2,3}$ completely analogously to $W_{\mu\nu}$. In the physical region they are related by:

$$W_i = \frac{1}{2\pi} I_m < p \mid T_i \mid p > \qquad (4)$$

where an average over nucleon spins is understood.

Jaffe and Soldate[1, 2] have developed the very convenient basis of the local operators $O_i^{\mu_1 \dots \mu_n}$ which is completely symmetric and traceless, as well as free of contracted covariant derivatives. We use this basis for the operator product expansion of $T_{\mu\nu}$

$$T_{\mu\nu} = \sum_{n,i} \Big[(g_{\mu\nu} - q_\mu q_\nu / q^2)\, q_{\mu_1} q_{\mu_2} C_{L,n}^{i} (Q^2/\mu^2, g^2) - (g_{\mu_1\mu_2} g_{\nu\mu_2} q^2 - g_{\mu\mu_1} q_\nu q_{\mu_2} - g_{\nu\mu_2} q_\mu q_{\mu_1} + g_{\mu\nu} q_{\mu_1} q_{\mu_2})\, C_{2,n}^{i} (Q^2/\mu^2, g^2) - i\, \varepsilon_{\mu\nu}{}^{\alpha\beta} g_{\alpha\mu_1} \cdot q_\beta q_{\mu_2} C_{3,n}^{i} (Q^2/\mu^2, g^2)\Big] \cdot q_{\mu_3} \dots q_{\mu_2} \cdot \left(\frac{2}{Q^2}\right)^n \cdot O_i^{\mu_1 \dots \mu_n} \qquad (5)$$

The matrix elements of the local operators $O_i^{\mu_1\mu_2}$ must be of the form:

$$< p \mid O_k^{\mu_1\mu_2} (0) \mid p > = A_k (p^{\mu_1} p^{\mu_2} - \tfrac{1}{4} M^2 g^{\mu_1\mu_2}) \qquad (6)$$

for the spin-two operators and

$$< p \mid O_{1\mu}^{\pm} (0) \mid p > = A_1^{\pm} p_\mu \qquad (7)$$

for the spin-one operators.

The Bjorken and Gross-Llewellyn-Smith sum rules are then:

$$M_1^{\nu,NS} (1,Q^2) = \frac{1}{2} \left[1 - \frac{8}{3b \ln Q^2/\lambda^2} + \frac{2\, g\, (Q^2)}{9\, Q^2} A_1^- \right] \qquad (8)$$

$$M_3^{\nu,S} (1,Q^2) = \frac{1}{2} \left[1 - \frac{4}{3b \ln Q^2/\lambda^2} + \frac{2\, g\, (Q^2)}{27\, Q^2} A_1^+ \right] \qquad (9)$$

where $g(Q^2) = [b/4\pi \ln Q^2/\lambda^2]^{-1}$ is strong coupling constant, and $b = 11 - \frac{2N}{3}$.

For the calculation of matrix elements $A_1^{\pm}$ we use nucleon wave functions $|\tilde{p}\rangle$ normalized to unity, which represent the confined quark in the nucleon center of mass. Using these wave functions the matrix elements become:

$$\langle p \,|\, O^{\pm}_{1\mu} (0) \,|\, p \rangle = 2 M \langle \tilde{p} \,|\, \int d^3x \, O^{\pm}_{1\mu} (x) \,|\, p \rangle \quad (10)$$

Only the $\mu = 0$ component of the operator will have non--zero matrix element and $A_1^{\pm}$ is:

$$A_1^{\pm} = - 4 \langle \tilde{p} \,|\, \int d^3x \, \bar{q} (x) \, \vec{\gamma} \, \gamma^5 \, [I_-, I_+] \, (\lambda^a/2) \, q (x) \cdot \vec{B}^a (x) \,|\, \tilde{p} \rangle \quad (11)$$

where $\vec{B}^a$ is the color magnetic field[8]. The quark wave function for nucleon models with confinement radius R is of the usual spherically symmetric form:

$$\psi(\vec{r}) = \begin{vmatrix} f (r) \\ \vec{\sigma} \, \hat{r} \, g (r) \end{vmatrix} \quad (12)$$

Calculating the spin, flavor and color factors of the matrix elements for protons and neutrons one finds:

$$A_1^{+} = 3 A_1^{-} = - g (Q^2)/\pi \int_0^R \{ \, |f (r)|^2 \, [2 M (r) + \mu (R)/R^3] - \frac{1}{3} \, |g (r)|^2 \, [2 M (r) + \mu (R)/R^3 - 4 \mu (r)/r^3] \, \} \, r^2 \, d r \quad (13)$$

Here:

$$\mu (r) = - i \, 8\pi / 3 \int_0^r f^* (r') \, g (r') \, r'^3 \, d r' \quad (14)$$

$$(r) = - i\, 8\pi / 3 \int_r^R f(r') \, g(r') \, dr' \quad (15)$$

Using the standard MIT bag parameters[8] we find $A^+ = - 3.53 \times 10^{-4}\ GeV^2 \cdot g(Q^2)$. From the formulas (14) and (15) we see that in nonrelativistic case where $g(r) \to 0$, the overlap integrals vanish. Also we have found[9] that they are very model dependent but again quite small in all models considered.

The twist-four spin-one corrections can affect the $R^\nu = \sigma_{NC}/\sigma_{CC}$ for singlet targets:

$$R_\nu = \sigma_{NC} / \sigma_{CC} = \frac{1}{2} + \left[-1 + \left(\frac{512}{27} I_1 - \frac{5120}{81} I_2\right) \frac{\alpha_S(Q_o^2)}{Q_o^2}\right] \cdot \sin^2\theta_W + \left[\frac{20}{27} - \left(\frac{-20,768}{729} I_1 - \frac{49,408}{729} I_2\right) \frac{\alpha_S(Q_o^2)}{Q_o^2}\right] \cdot \sin^4\theta_W \quad (16)$$

where the twist-four spin-two correction is included. The integrals I_1 and I_2 and other relevant parameters are given in reference[10]. The twist-four spin-one corrections to these formulas are given by:

$$\delta\left[\sigma_{NC} / \sigma_{CC}^{parton}\right] = \left[\alpha_S(Q_o^2) / ME\right] \left[\ln \frac{2ME}{Q_o^2}\right] \cdot \left[\frac{2}{3} - \frac{4}{3}\sin^2\theta_W\right] \frac{A_1^+}{g(Q_o^2)} \quad (17)$$

and

$$\delta\left[\sigma_{CC} / \sigma_{CC}^{parton}\right] = \left[\alpha_S(Q_o^2) / ME\right] \left[\ln \frac{2ME}{Q_o^2} \frac{4}{3}\right] \cdot \frac{A_1^+}{g(Q_o^2)} \quad (18)$$

with $\sigma_{CC}^{parton} = \frac{G^2 ME}{2\pi}$ and therefore:

$$\delta R^\nu \doteq \alpha_S(Q_o^2) / ME \left[\ln \frac{2ME}{Q_o^2}\right] \frac{81}{80} \sin^4\theta_W \frac{A_1^+}{g(Q_o^2)} \quad . \quad (19)$$

Comparing the numerical values we find that these twist-four spin-one corrections are quite small, typically 1%, of the twist-four spin-two corrections, which can increase the value of $\sin^2\theta_W$ by only 1/2 % for the MIT bag model parameters.

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