PROCESSES $\gamma\gamma \to M^+M^-$ ($M = \pi, K$) IN PERTURBATIVE QUANTUM CHROMODYNAMICS

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1. Introduction

The framework for analyzing large-momentum-transfer exclusive processes in perturbative QCD has been developed by Brodsky and Lepage[1], and, independently, by Efremov and Radyushkin[2]. In this framework, the transition $\gamma\gamma \to M^+M^-$ ($M = \pi, K$) is, at high energies and large center-of-mass scattering angles $\theta_{c.m.}$, described by the helicity amplitudes

$$\mathcal{M}(\lambda\lambda';s,\theta_{c.m.}) = \int_0^1 dx \int_0^1 dy\ \phi_M^*(x,\overline{Q_x})\,\phi_M^*(y,\overline{Q_y})\ T_H(\lambda\lambda';x,y;s,\theta_{c.m.}) \ , \qquad (1.1)$$

where $\overline{Q_x} \sim \min(x,1-x)\sqrt{s}\,|\sin\theta_{c.m.}|$ (and similarly for $\overline{Q_y}$), λ and λ' are photon helicities, and $\sqrt{s} \equiv W_{\gamma\gamma}$ is the total center-of-mass energy of the $\gamma\gamma$ system.

The function $\phi_M(x,Q)$ in (1.1) is the meson distribution amplitude. It represents the amplitude for the meson to consist of a $q\bar{q}$ pair, with q and $\bar{q}$ collinear and on shell relative to the scale Q, and sharing fractions of x and $1-x$ of the meson's total momentum. $\phi_M(x,Q)$ is intrinsically nonperturbative; it contains all effects of collinear singularities, confinement, nonperturbative interactions and meson bound-state dynamics. At the present time there are no reliable ways known for calculating $\phi_M(x,Q)$.

The function T_H in (1.1) is the hard-scattering amplitude for producing collinear meson constituents from the initial photon pair. It is obtained by evaluating Feynman diagrams contributing to the subprocess $\gamma\gamma \to (q\bar{q})+(q\bar{q})$, with massless on-shell quarks collinear with outgoing mesons. By definition,

T_H is free of collinear singularities and has a well-defined perturbative expansion in $\alpha_s(W_{\gamma\gamma})$.

As is well known, unlike in QED, the lowest-order calculations in perturbative QCD do not have much predictive power. Consequently, in order to achieve complete confrontation between theoretical predictions and experimental results, it is crucial that higher-order corrections are obtained.

The leading-order perturbative QCD analysis of the reaction $\gamma\gamma \to M^+M^-$ ($M = \pi, K$) has been performed by Brodsky and Lepage[1]. In order to check the reliability of their predictions, we examine the next-to-leading order corrections to this process.

2. Leading-order predictions

There are 20 Feynman diagrams contributing to the lowest-order hard-scattering amplitude T_H for the process $\gamma\gamma \to M^+M^-$ ($M = \pi, K$). These diagrams can be obtained, by particle interchange, from four basic diagrams shown in Fig. 1. Evaluating these diagrams and making use of the similarity between the $\gamma\gamma \to M^+M^-$ process and the meson electromagnetic form factor ($\gamma^+ \to M^+M^-$), we find that the lowest-order spin-averaged differential cross section can be written in the form[1]

$$\left(\frac{d\sigma}{dt}\right)^{(o)} = 16\pi\alpha^2 \left|\frac{F_M(s)}{s}\right|^2 \left[\frac{\langle(e_1-e_2)^2\rangle}{(1-z^2)^2} + \frac{2\langle e_1e_2\rangle\langle(e_1-e_2)^2\rangle}{1-z^2}\,\rho(z;\phi_M) + 2\langle e_1e_2\rangle^2\rho^2(z;\phi_M)\right]. \quad (2.1)$$

All the $\phi_M(x,Q)$ dependence of $(d\sigma/dt)^{(o)}$ is contained in the factor $\rho(z;\phi_M)$ which is given by

$$\rho(z;\phi_M) = \frac{\int_0^1 dx \int_0^1 dy \, \frac{\phi_M^*(x,Q)\phi_M^*(y,Q)}{x(1-x)y(1-y)} \, \frac{a[x(1-x)+y(1-y)]}{a^2-b^2z^2}}{\int_0^1 dx \int_0^1 dy \, \frac{\phi_M^*(x,Q)\phi_M^*(y,Q)}{x(1-x)y(1-y)}}, \quad (2.2)$$

where $z = \cos\theta_{c.m.}$ and

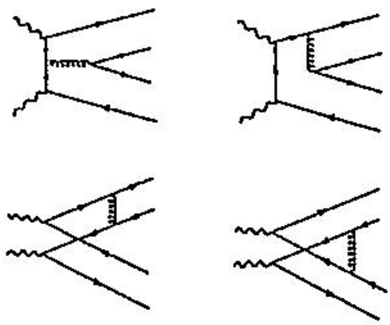

Fig. 1. Basic lowest-order diagrams contributing to the hard-scattering amplitude for the process $\gamma\gamma \to M^+M^-$ ($M = \pi, K$).

$$\left.\begin{matrix}a\\b\end{matrix}\right\} = (1-x)(1-y) \pm xy . \tag{2.3}$$

To get the final expression for $(d\sigma/dt)^{(o)}$, we need the x dependence of the meson distribution amplitude. In general, however, $\Phi_M(x,Q)$ is an unknown function. Nevertheless, it has been shown that the leptonic decay of each meson normalizes its distribution amplitude by the "sum rule"

$$\int_0^1 dx\,\Phi_M(x,Q) = \frac{f_M}{2\sqrt{3}} , \tag{2.4}$$

where f_M is the meson decay constant. In order to see to what extent the form of Φ_M influences the prediction for the $\gamma\gamma \to M^+M^-$ cross section, we use the normalized meson distribution amplitude of the general form

$$\Phi_M(x,Q) = \frac{f_M}{2\sqrt{3}} \frac{\Gamma(2+2\eta)}{\Gamma^2(1+\eta)} x^\eta (1-x)^\eta , \quad \eta > 0 . \tag{2.5}$$

The lowest-order spin-averaged cross section for $\gamma\gamma \to M^+M^-$ is plotted in Fig. 2 for three choices of the parameter η in (2.5). Curves a, b, and c correspond to $\eta = 1/4$, 1, and ∞, respectively. The pion electromagnetic form factor has been approximated by $F_\pi(s) = 0.4\ \mathrm{GeV}^2/s$. As is seen from Fig. 2, $(d\sigma/dt)^{(o)}$ for $\gamma\gamma \to \pi^+\pi^-$ is essentially independent of the choice of the parameter η.

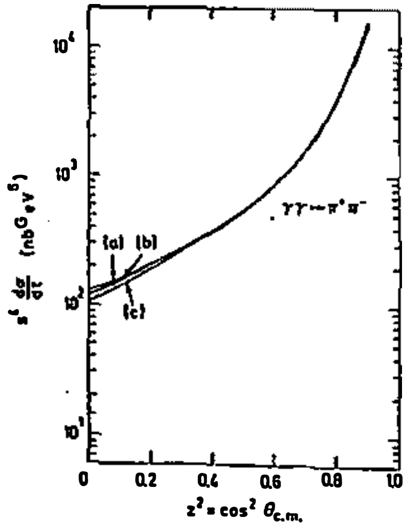

Fig. 2. Dependence of the lowest-order prediction for the $\gamma\gamma \to M^+M^-$ differential cross section on the form of the meson distribution amplitude.

Given the predictions for $\gamma\gamma \to \pi^+\pi^-$, predictions for $\gamma\gamma \to K^+K^-$ can be obtained using the relation

$$\left(\frac{d\sigma}{dt}\right)(\gamma\gamma \to K^+K^-) = \left(\frac{f_K}{f_\pi}\right)^4 \frac{d\sigma}{dt}(\gamma\gamma \to \pi^+\pi^-) , \qquad (2.6)$$

which is true to all orders in perturbation theory.

3. Next-to-leading-order predictions

In order to obtain the next-to-leading-order perturbative QCD predictions for the $\gamma\gamma \to M^+M^-$ process, one has to evaluate 448 one-loop Feynman diagrams. Some of them are shown in Fig.3. When computing these diagrams, one encounters ultraviolet and infrared (both soft and collinear) divergences. All these divergences can be controlled using the dimensional regularization method. Evaluating all the diagrams, taking into account the next-to-leading-order expression for the pion electromagnetic form factor[3], and using the model meson distribution amplitude $\phi_M \propto \delta(x - \frac{1}{2})$ (as a candidate form for the non-perturbative dynamical input), one finds that the next-to-

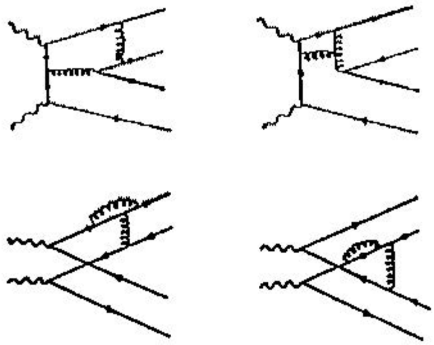

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Fig. 3. Few one-loop diagrams contributing to the hard-scattering amplitude for $\gamma\gamma \to M^+M^-$.

leading-order differential spin-averaged $\gamma\gamma \to M^+M^-$ cross section is, in the $\overline{MS}$ renormalization scheme, given by[4)]

$$\left(\frac{d\sigma}{dt}\right)^{(1)} = \left(\frac{d\sigma}{dt}\right)^{(o)} \left[1 + \frac{\alpha_{\overline{MS}}(W^*_{\gamma\gamma})}{\pi} h(\theta_{c.m.})\right] , \tag{3.1}$$

where $(d\sigma/dt)^{(o)}$ is given by (2.1) with $\rho(z;\Phi_M) = 1$, $W^*_{\gamma\gamma} = 0.21\, W_{\gamma\gamma}$, and

$$h(\theta_{c.m.}) = -\begin{bmatrix} 1.575(2) \\ 2.744(3) \\ 3.480(3) \\ 4.255(4) \end{bmatrix} , \tag{3.2}$$

where the numbers from top to bottom correspond to $\theta_{c.m.} = 45^o$, 60^o, 75^o, and 90^o, respectively.

In order to see how the correction to the lowest-order prediction for the $\gamma\gamma \to M^+M^-$ differential cross section depends on $W_{\gamma\gamma}$, in Fig. 4 we plot the ratio $R_{diff}=(d\sigma/dt)^{(1)}/(d\sigma/dt)^{(o)}$ for three different values of $W_{\gamma\gamma}$. Curves a, b, and c correspond to $W_{\gamma\gamma}$ = 2, 5, and 10 GeV, respectively. $\Lambda_{\overline{MS}}$ is taken to be 150 MeV. A glance at this figure reveals that the corrections to the lowest-order become sufficiently small (<25%) only for $W_{\gamma\gamma}$ > 10 GeV.

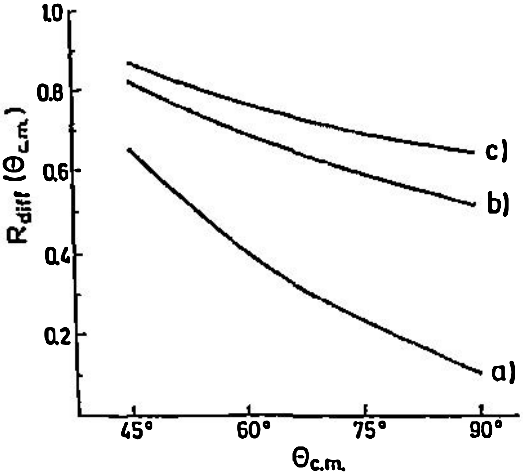
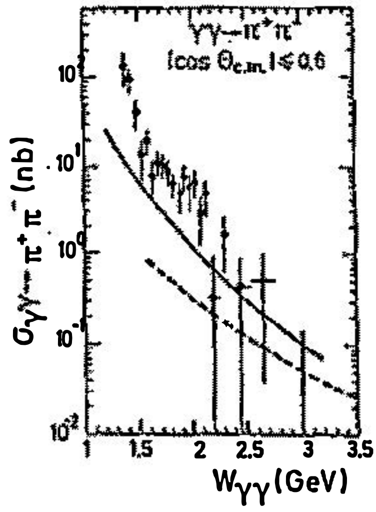
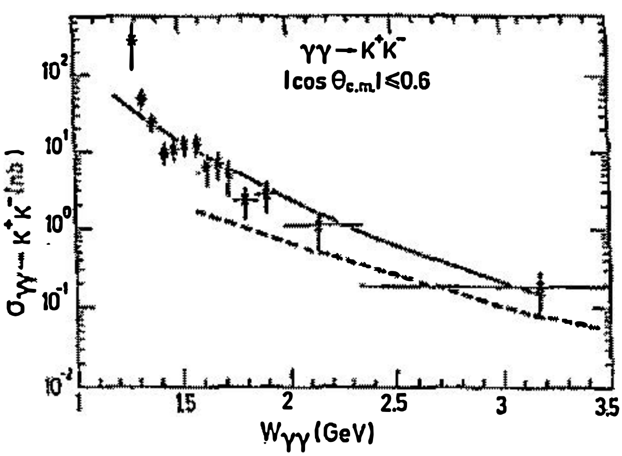


Fig. 4. Plot of R_{diff} vs $\theta_{c.m.}$ for three different values of $W_{\gamma\gamma}$. Curves a, b, and c correspond to $W_{\gamma\gamma}$ = 2, 5, and 10 GeV, respectively.

The process $\gamma\gamma \to M^+M^-$ ($M = \pi, K$) has been the subject of a number of experimental investigations. Measurements of $\sigma(\gamma\gamma \to \pi^+\pi^- + K^+K^-)$ in the region $1.6 \leq W_{\gamma\gamma} \leq 2.4$ GeV in the angular range confined to $|\cos\theta_{c.m.}| < 0.3$ have been made by the MARK II group at SPEAR[5]. These measurements have later been extended to include the region $2.4 \leq W_{\gamma\gamma} \leq 3.5$ GeV[6]. Using the PEP4/9 detector at PEP, separate contributions in the same region of $W_{\gamma\gamma}$ and the angular region given by $|\cos\theta_{c.m.}| < 0.6$ have now been obtained.[7] Figure 5 shows the PEP4/9 data for the $\gamma\gamma \to \pi^+\pi^-$ and $\gamma\gamma \to K^+K^-$ integrated cross sections, respectively, and their $W_{\gamma\gamma}$ dependence in comparison with the leading (solid curves) and next-to-leading-order (dashed curves) perturbative QCD calculations. From Fig. 5a one sees that the $W_{\gamma\gamma}$ dependence of the $\gamma\gamma \to \pi^+\pi^-$ cross section data is only in fair agreement with the calculation of Brodsky and Lepage. The lack of agreement can possibly be ascribed to the interference of the continuum with the f(1270) and other resonances. On the other hand, as is seen



a)



b)

Fig. 5. PEP4/9 data for
a) the $\gamma\gamma \to \pi^+\pi^-$ cross section,
b) the $\gamma\gamma \to K^+K^-$ cross section,
compared with the leading-order (solid lines) and next-to-leading-order (dashed lines) perturbative QCD predictions.

from Fig. 5b, the K^+K^- data are in good agreement, in both the $W_{\gamma\gamma}$ dependence and the absolute normalization with the lowest-order QCD prediction. As to the next-to-leading-order prediction, one sees that they deviate considerably from both the $\pi^+\pi^-$ and K^+K^- data, indicating unreliability of the perturbative calculation in the region of $W_{\gamma\gamma}$ in which data exist.

4. Conclusion

Perturbative next-to-leading-order analysis of the process $\gamma\gamma \to M^+M^-$ ($M = \pi,K$) shows that the corrections to the leading-order predictions become sufficiently small (<25%) only for $W_{\gamma\gamma}$ > 10 GeV, which is much larger than the highest $W_{\gamma\gamma}$ for which experimental data exist.

It should, however, be mentioned that although the leading-order predictions for the process under consideration are essentially insensitive to the precise form of ϕ_M, they should not necessarily hold true when the next-to-leading-order corrections are taken into account. Thus, because of the particular form of ϕ_M used in the calculation, some uncertainty has possibly been brought into the predictions obtained. A reasonable guess for the size of this uncertainty is that it should not be larger than a few percent.

In summary, one can say that reliable perturbative QCD predictions for the $\gamma\gamma \to M^+M^-$ ($M = \pi,K$) transition cannot be made until $W_{\gamma\gamma}$ of 10 GeV is reached or unless higher-order terms in the perturbative expansions are obtained.

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