EXTRA GAUGE SYMMETRIES IN POINCARÉ GAUGE THEORY OF GRAVITY*

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The Poincaré gauge theory always contains a ten-parameter symmetry gauge group, but there can be some extra symmetries depending on the choice of the gravity Lagrangian. In this review I will analyse the theory with the gravity Lagrangian linear in the curvature and quadratic in the torsion ($R+T^2$), which contains no other particles but the gravitation[1,2]. In the theory with R^2 terms, however, there are propagating tordions and the existence of extra gauge symmetries is related to their masslessness. At the end of this review I will mention some results concerning the linearized $R+T^2+R^2$ theory[3,4].

1. Primary constraints and Hamiltonian. Basic variables in this theory are tetrads $h_i{}^\mu$, spin-connections $A^{ij}{}_\mu$ and a matter field ψ. The gravity Lagrangian can be constructed from the gauge field strengths T_{ijk} (torsion) and R_{ijkl} (curvature). In the $R+T^2$ case it will be [1]

$$ {}_g = aR + A\,T_{ijk}\,T^{ijk} + B\,T_{ijk}\,T^{kji} + C\,T^k{}_{ki}\,T_s{}^{si} \qquad (1)$$

The constants A, B and C are arbitrary while $a \neq 0$. Using Poincaré gauge symmetry of the theory we can impose the time gauge condition $h_a{}^o = 0$. (Our convention is that the first letters of Latin and Greek alphabets a, b, c, ... ; α, β, γ, ... run over 1, 2, 3 while the rest of them i, j, k, ... ; μ, ν, λ, ... run over 0, 1, 2, 3.) This condition reduces our initial ten--parameter symmetry to a seven-parameter one.

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From the definition of the momenta $\pi^{o}{}_{\mu}$, $\pi^{a}{}_{\alpha}$, $\pi_{ij}{}^{\mu}$ and π (associated with $h_{o}{}^{\mu}$, $h_{a}{}^{\alpha}$, $A^{ij}{}_{\mu}$ and ψ) we get all the primary constraints in the theory[1]:

$$
\begin{aligned}
\phi^{o}{}_{\mu} &\equiv \pi^{o}{}_{\mu} \approx 0 \\
\phi_{ij}{}^{\mu} &\equiv \pi_{ij}{}^{\mu} \approx 0 \\
\phi &\equiv \pi - b\, \partial \mathcal{L}_M / \partial\phi_{,0} \approx 0
\end{aligned}
\tag{2a}
$$

$$
\begin{aligned}
&\phi^{a}{}_{a} \equiv \pi^{a}{}_{a} + 4\,K\,a\; A^{oa}{}_{a} \approx 0 \qquad (\beta = 0) \\
&\phi_{|ab|} \equiv \pi_{|ab|} - 2\,K\,(a - \alpha - 2\gamma/9)\; A_{o|ab|} \approx 0 \\
&(\alpha - 4\gamma/9 = 0) \\
&\bar{\phi}_{(ab)} \equiv \bar{\pi}_{(ab)} - 2\,K\,a\; A_{o\,(ab)} \approx 0 \quad (\alpha = 0)
\end{aligned}
\tag{2b}
$$

where α, β, γ are defined as follows: $A = \alpha/2 - \gamma/18$, $B = \alpha/2 + \gamma/9$ and $C = -\alpha/2 + \beta$. The constraints (2b) are the so-called if-constraints (they exist in the theory only if some constants vanish). After constructing the canonical Hamiltonian , we can write down the total Hamiltonian by adding all the primary constraints with their arbitrary multipliers:

$$
\mathcal{H}_T = \mathcal{H} + U_o{}^{\mu}\,\phi^{o}{}_{\mu} + U^{ij}{}_{\mu}\,\phi_{ij}{}^{\mu} + \phi U + (U\phi)_T \tag{3}
$$

where $(U\phi)_T$ is short for the sum of the if-constraints with their multipliers. In order to realize the programme for constructing all the gauge generators we need all first-class constraints (FC); so, we must examine the consistency conditions of all the primary constraints.

2. Secondary constraints. By demanding that the primary constrainrs be conserved during the time evolution

$$
d\,\phi / d\,t = \{\phi, H_T\} \approx 0 \tag{4}
$$

we can determine some multipliers, obtain new (secondary) constraints or the demand will be automatically satisfied. By a direct calculation we find 7 "sure" secondary constraints, which are related to the Poincaré gauge symmetry, and 21 if--constraints[2]. Among the if-constraints there are some which depend on the matter field only. In order to prevent the existence of these constraints we impose certyin conditions on the matter Lagrangian:

$$\overset{T}{\sigma}_{ijk} = 0 \qquad (\mu_2 = 0)$$
$$\overset{V}{\sigma}_{i} = 0 \qquad (\mu_1 = 0) \qquad (5)$$
$$\overset{A}{\sigma}_{i} = 0 \qquad (\mu_0 = 0)$$

where $\sigma_{ijk} \equiv (\partial \mathcal{L}_M / \partial A^{ij}{}_{\nu})\, b_{k\nu}$ and $\mu_o \equiv a - 2\gamma/3$, $\mu_1 \equiv a + 3\beta/2$, $\mu_2 \equiv a - 3\alpha/2$. After having got all the secondary constraints we must examine their consistency conditions, too. It turns out that some multipliers become determined but no other constraints appear. Consequently, the procedure of the examination of the consistency conditions is finished. The total Hamiltonian becomes[2]

$$\mathcal{H}_T = \mathcal{H}' + V_o{}^{\mu} P^o{}_{\mu} + V^{ab}{}_o P_{ab}{}^o + \\ + [1 - \lambda(\mu_2)] \overset{T}{V}{}^{ijk} \overset{T}{\pi}_{ijk} + [1 - \lambda(\mu_1)] \overset{V}{V}{}^{i} \overset{V}{\pi}_{i} + \\ + [1 - \lambda(\mu_o)] \overset{A}{V}{}^{i} \overset{A}{\pi}_{i} \qquad (6)$$

where $\mathcal{H}'$ contains all determined multipliers, V's are arbitrary multipliers and $\lambda(x)$ is defined by

$$\lambda(x) = \begin{cases} 1 & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

The constraints $P^o{}_{\mu}$, $P_{ab}{}^o$ and π_{ijk} (when $\mu_a = 0$) are the primary first class constraints (PFC).

3. The gauge generators. General form of a gauge generator is[5)]

$$G = \sum_{n=0}^{K} \varepsilon^{(n)} (t)\, G_n \tag{7}$$

where $\varepsilon^{(n)} \equiv d\, \varepsilon^n / d\, t^n$ and G_n are phase-space functions, which must satisfy the conditions:

$$\begin{aligned} G_K &= P\,F\,C \\ G_{K-1} + \{G_K, H_T\} &= P\,F\,C \\ &\dots \\ G_o + \{G_1, H_T\} &= P\,F\,C \\ \{G_o, H_T\} &= P\,F\,C \end{aligned} \tag{8}$$

This set of conditions also gives the procedure for the construction of the chain G_n. In our case the procedure is finished at the first step with the result:

$\overset{T}{\pi}_{ijk}$, $\overset{V}{\pi}_i$ and/or $\overset{A}{\pi}_i$ are symmetry gauge generators of the theory if the constants μ_2, μ_1 and/or μ_o vanish.

These generators act nontrivially only on the potentials A_{ijk} as shown in the following table:

	The generators	Number of parameters	δA_{ijk}
$\mu_2 = 0$	$\overset{T}{\pi}_{ijk}$	16	$\overset{T}{\varepsilon}_{ijk}$
$\mu_1 = 0$	$\overset{V}{\pi}_i$	4	$\overset{V}{\varepsilon}_i$
$\mu_o = 0$	$\overset{A}{\pi}_i$	4	$\overset{A}{\varepsilon}_i$

The constants μ_2, μ_1 and μ_o, which multiply the quadratic terms in the Lagrangian, do not represent any masses because neither of A_{ijk} propagates in this theory. In the theory with R^2 terms, however, the results of the examination of extra gauge symmetries are much more interesting.

5. $R + T^2 + R^2$ theory in the linear approximation. Except on a, μ_o, μ_1 and μ_2, this theory[3] depends on six more constants a_1, a_2, ... , a_6. As in $R+T^2$ case all the gauge symmetries can be found. To illustrate how the extra symmetries reflect the particle structure of the theory I shall give an example. Let the constants satisfy $\mu_o = a_1 + a_3 = 0$. Then the following symmetry transformations are obtained:

$$\delta \overset{A}{A}_i = \varepsilon_{,i} \quad , \quad \delta(\text{other fields}) = 0.$$

This symmetry reflects the fact that there is now one massless 1^+ particle in the theory instead of the massive 1^+ and 0^- ones.

5. Conclusion. The investigation of extra gauge symmetries presented here is based on Dirac's general method for constrained dynamical systems completed with Castellani's algorithm for construction of gauge generators. We have found that extra gauge symmetries in $R+T^2$ theory occur whenever the constants μ_o, μ_1, μ_2 vanish, a situation which has already been discussed in the literature[6]. In $R+T^2+R^2$ case the existence of extra gauge symmetries is related to the existence of massless tordions.

References

1) M. Blagojević and I. Nikolić, Nuovo Cim. 73 B, 258 (1983)
2) M. Blagojević and M. Vasilić, submitted for publication
3) I. Nikolić, Phys. Rev. D 30, 2508 (1984)
4) M. Blagojević and M. Vasilić, in preparation
5) L. Castellani, Ann. Phys. 143, 357 (1982)
6) K. Sundermeyer, Phys. Lett. 134 B, 415 (1984)
I. Bars and S.W. McDowel, Phys. Lett. 129 B, 182 (1983)