SUPERSYMMETRIC MAGNETIC MONOPOLES

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We present supersymmetric field theory of electric and magnetic charges using supersymmetric duality between electricity and magnetism.

There are two equivalent formulations of quantum electro-magnetodynamics (QEMD): the local-lagrangian formulation [1] and the one-potential formulation [2, 3]. The first one has a manifest dual symmetry, while the second one uses a smaller number of dynamical variables and has a canonical structure of the action. Both theories generalize ordinary quantum electrodynamics (QED to a dual invariant field theory.

There are two supersymmetric (SUSY) generalizations of this theory. In the first approach [4, 5] the supersymmetric lagrangians are postulated in such a way that their component presentations contain the corresponding nonsupersymmetric versions [1, 2, 3]. The second approach [6, 7] derives the same theory with a systematic pure superfield method. Here we present the elements of the second approach.

We start with a brief review of SUSY QED. The action is

$$A_{QED} = \frac{1}{4} \int dx d^2\theta W^2 + \frac{1}{4} \int dx d^2\overline{\theta} \overline{W}^2 - 2 \int dx d^2\theta d^2\overline{\theta} \overline{S} e^{-2\sigma_2 eV} S + \\ + m \left(\int dx d^2\theta \tilde{S} S + \int dx d^2\overline{\theta} \overline{S} \overline{\tilde{S}} \right) \tag{1}$$

where the gauge prepotential V is a generalization of the gauge field V_μ

$$V_\mu = - \frac{1}{4} (\overline{\sigma}_\mu)^{\dot{\beta}\alpha} \left[D_\alpha , \overline{D}_{\dot{\beta}} \right] /_{\theta=\overline{\theta}=0} \quad ; \tag{2}$$

and the supersymmetric field strengths

$$W_\alpha = - \frac{1}{4} \overline{D}^2 D_\alpha V \qquad \overline{W}_\alpha = - \frac{1}{4} D^2 \overline{D}_{\dot{\alpha}} V \tag{3}$$

are generalizations of the field strength V

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu = \frac{i}{4} \{ (\sigma_{\mu\nu})_\alpha{}^\beta D^\alpha W_\beta - (\bar{\sigma}_{\mu\nu})^{\dot\alpha}{}_{\dot\beta} \bar{D}_{\dot\alpha} \bar{W}^{\dot\beta} \} /_{\theta=\bar\theta=0} \,. \tag{4}$$

The matter superfields

$$S = \begin{vmatrix} S_1 \\ S_2 \end{vmatrix} \qquad \bar{S} = |\bar{S}_1 \; \bar{S}_2| \qquad (\bar{D}_{\dot\alpha} S = 0, \;\; D_\alpha \bar{S} = 0) \tag{5}$$

are generalizations of the Dirac spinor matter field ψ

$$\psi = \frac{1}{\sqrt{2}} \begin{vmatrix} D_\alpha (S_1 + iS_2) \\ \bar{D}^{\dot\alpha} (\bar{S}_1 + i\bar{S}_2) \end{vmatrix} /_{\theta=\bar\theta=0} \,. \tag{6}$$

Here, the tilde means transpose and σ_2 is the second Pauli matrix.

The equations of motion are given by

$$DW + \overline{DW} = J_e \,, \tag{7}$$

$$D^2 (e^{-2\sigma_2 eV} S) = - 4 m \tilde{\bar{S}} \qquad \bar{D}^2 (\bar{S} e^{-2\sigma_2 eV}) = - 4 m \tilde{S} \,, \tag{8}$$

where

$$J_e = 8 e \bar{S} \sigma_2 e^{-2\sigma_2 eV} S \tag{9}$$

is a conserved electric supercurrent

$$D^2 J_e = \bar{D}^2 J_e = 0 \,, \tag{10}$$

and the Bianchi identity reads

$$DW - \overline{DW} = 0 \,. \tag{11}$$

We shall refer to equations (7) and (11) as the Maxwell equations for SUSY QED.

The main idea in our presentation is to supersymmetrize dual transformation between electricity and magnetism and using such a concept to derive SUSY QEMD as a dual invariant generalization of SUSY QED.

It is well known that in nonsupersymmetric case the Maxwell equations in vacuum are symmetric under the duality transformations

$$ {}^{*}V_{\mu\nu} = \frac{1}{2}\,\varepsilon_{\mu\nu\rho\sigma}\,V^{\rho\sigma} \qquad {}^{*}({}^{*}V_{\mu\nu}) = -\,V_{\mu\nu}\,. \tag{12} $$

Using eq. (4) we can write

$$ V_{\mu\nu} + i\,{}^{*}V_{\mu\nu} = i\,(\sigma_{\mu\nu})_{\alpha}{}^{\beta}\,D^{\alpha}W_{\beta}\,/_{\theta=\bar\theta=0} $$

Then for supersymmetric dual field ${}^{*}W_{\alpha}$ we have

$$ i\,(\sigma_{\mu\nu})_{\alpha}{}^{\beta}D^{\alpha}\,{}^{*}W_{\beta}\,/_{0} = {}^{*}V_{\mu\nu} - i\,V_{\mu\nu} = -\,i\,(V_{\mu\nu} + i\,{}^{*}V_{\mu\nu}) = $$

$$ = (\sigma_{\mu\nu})_{\alpha}{}^{\beta}\,D^{\alpha}W_{\beta}\,/_{0} $$

We generalise the last equation to be valid for every θ

$$ {}^{*}W_{\alpha} = -\,i\,W_{\alpha} $$

Now, the supersymmetric Maxwell eqs. in vacuum, i.e.

$$ DW + \bar{D}\bar{W} = 0 $$

$$ D\,{}^{*}W + \bar{D}\,{}^{*}\bar{W} = 0 $$

are symmetric under the superduality transformations

$$ \begin{aligned} {}^{*}W_{\alpha} &= -\,i\,W_{\alpha} & \qquad {}^{*}({}^{*}W_{\alpha}) &= -\,W_{\alpha} \\ {}^{*}\bar{W}_{\dot\alpha} &= i\,\bar{W}_{\dot\alpha} & \qquad {}^{*}({}^{*}\bar{W}_{\dot\alpha}) &= -\,\bar{W}_{\dot\alpha} \end{aligned} \tag{13} $$

The SUSY QEMD is a generalization of the vacuum SUSY QED to the theory with matter which preserves superduality. The corresponding equations of motion are

$$DW + \overline{DW} = J_e$$

$$D^*W + \overline{D}^*\overline{W} = J_g$$

where we introduce conserved magnetic supercurrent J_g ,

$$D^2 J_g = \overline{D}^2 J_g = 0 \ . \qquad (15)$$

Our task is to find the action for eqs. (14) in the presence of conserved supercurrents (10) and (15). There are two possibilities to do it: following method of ref.[1] as in ref.[6], and methods of ref.[2, 3] as in ref.[7] . Here we present only the final result (for details of calculations see ref.[6, 7]).

The local-lagrangian two-prepotential theory reads

$$A_{QEMD} = \int dx d^4\theta \ \{ \frac{i}{2} W^{\alpha} \frac{n_{\alpha\dot{\beta}}}{n\partial} \overline{W}^{\dot{\beta}} - 2\,\overline{S}\, e^{-2\sigma_2 eV} S - 2\,\overline{R}\, e^{-2\sigma_2 gU} R \} + \qquad (16)$$
$$+ m_S \left(\int dx d^2\theta \tilde{S}S + h.c.\right) + m_R \left(\int dx d^2\theta \tilde{R}R + h.c.\right) ,$$

where the variables g, U, R, $\overline{R}$, m_R have the same meaning for magnetism as the e, V, S, $\overline{S}$, m_S for electricity,

$$W_\alpha = - \frac{(n\partial)^2}{4n^2\partial^2} \overline{D}^2 \, T_\alpha \, (V + iU), \quad \overline{W}_{\dot{\alpha}} = (W_\alpha)^+ \qquad (17)$$

are field-strengths, and

$$T^\alpha = \frac{\partial^\mu n^\nu}{n\partial} (\sigma_\mu \, \overline{\sigma}_\nu)_\beta{}^\alpha D^\beta \qquad \overline{T}_{\dot{\alpha}} = (T_\alpha)^+ \qquad (n\partial)(n\partial)^{-1}(x) = \delta^4(x) \ . \qquad (18)$$

We have to introduce fixed four vector n^μ as well as in the nonsupersymmetric case of QEMD [1, 2, 3] .

The component formulation of the SUSY QEMD (16) presented here contains both the spinor [1] and the scalar theory [3] .

References

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