

ON VACUUM SOLUTIONS IN KALUZA-KLEIN THEORIES WITH ARBITRARY
SIGNATURE

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Vacuum solutions in Kaluza-Klein theories with an arbitrary space-time signature are considered. Solutions for 11-dimensional supergravity and for the corresponding 10-dimensional theory are obtained. In particular solutions with two 4-dimensional universes are constructed. Also, vacuum solutions with zero cosmological constant in $N=2$, $D=10$ supergravity are found.

1. Introduction

Kaluza-Klein theories have become very significant candidates in the construction of realistic unified field theory¹⁾. In these theories it is usually supposed that the extra dimensions are space-like. However, one can also introduce compactified time-like dimensions²⁾. Moreover, it seems very natural to start from arbitrary metric signatures and make choice among them according to reasonable vacuum solutions. Introduction of compactified time dimensions may resolve some of standard problems in Kaluza-Klein theories. It is already shown that it leads to Minkowski space in the 10-dimensional Green-Schwarz effective field theory³⁾. Here, it will be shown that it solves the cosmological constant problem in the nonchiral $N=2$, $D=10$ supergravity. The problems which usually appear introducing compactified time-like dimensions result in an appearance of ghosts and tachyons in the effective 4-dimensional theory, but these problems can be avoided in some low energy models⁴⁾.

We shall first consider vacuum solutions in the bosonic sector of $N=1$, $D=11$ supergravity and then in the corresponding $N=2$, $D=10$ supergravity theory.

2. Vacuum solutions in 11-dimensional supergravity

One of the most attractive approaches to realistic unified field theory is through supergravity in the maximum dimension, i.e. N=1 supergravity in D=11 dimensions¹⁾. Vacuum solutions in the standard signature are considered in a number of papers (for a review see Refs. ^{1, 5)}) using the Freund-Rubin ansatz⁶⁾.

In N=1, D=11 supergravity the equations of motion for a ground state (the VEV of any fermion field is zero) are

$$R_{MN} - \frac{1}{2}g_{MN}R = T_{MN} = \frac{1}{3} \left[F_{MPQR} F_N{}^{PQRS} - \frac{1}{8}g_{MN} F_{PQRS} F^{PQRS} \right] \quad (1)$$

$$\nabla_M F^{MPQR} = \frac{1}{576\sqrt{g}} \epsilon^{M_1 \dots M_8 PQR} F_{M_1 \dots M_4} F_{M_5 \dots M_8} \quad (2)$$

where $F_{MNPQ} = 4\partial [M A_{NPQ}]$ and A_{NPQ} is an antisymmetric three-index gauge field. Starting from an arbitrary space-time signature and using the Freund-Rubin compactification ansatz, we are looking for vacuum solutions of the direct product form

$$M_T^{11} = M_t^4 \times M_{T-t}^7 \quad (3)$$

where $T(t)$ denotes total (partial) number of time-like dimensions. According to the ansatz of Freund and Rubin, only one of all possible F_{MNPQ} tensors in a ground state is non-vanishing and we set

$$F_{\mu\nu\rho\sigma} = \sqrt{(-1)^t g_{(4)}} \lambda_t \epsilon_{\mu\nu\rho\sigma} \quad (\mu, \nu, \rho, \sigma = 0, \dots, 3) \quad (4)$$

where λ_t is a constant of mass dimension and $\epsilon_{\mu\nu\rho\sigma}$ is the antisymmetric Levi-Cevita tensor. Substituting (4) into the equations of motion we see that (2) is trivially satisfied while (1) yields the energy-momentum tensors

$$\begin{aligned} T_{\mu\nu} &= (-1)^t \lambda_t^2 g_{\mu\nu} \\ T_{\hat{\mu}\hat{\nu}} &= (-1)^{t+1} \lambda_t^2 g_{\hat{\mu}\hat{\nu}} \end{aligned} \quad (5)$$

where indices $\hat{\mu}, \hat{\nu}$ belong to the complementary 7-dimensional space-time. From (1) and (2) we obtain the product of the Einstein space-times

$$R_{\mu\nu} = (-1)^t \frac{4}{3} \lambda_t^2 g_{\mu\nu} \quad (6)$$

$$R_{\hat{\mu}\hat{\nu}} = (-1)^{t+1} \frac{2}{3} \lambda_t^2 g_{\hat{\mu}\hat{\nu}} \quad (7)$$

Note that signs in (6) and (7) are opposite and depend only of the signature of 4-dimensional space-time. Solutions of equations (6) and (7) we shall denote by S_t^d when they have positive signs and by L_t^d for the negative signs (e.g. $S_0^d = S^d$ is d-dimensional sphere and $S_d^d = L_0^d$ is a manifold of the constant negative curvature).

Instead of writing all possible solutions of (6) and (7) in the form (3) we shall rather point out some of them. There are two solutions which could be significant

$$M_2^{11} = L_1^4 \times S_1^7 \quad (8)$$

$$M_5^{11} = L_1^4 \times S_4^7 \quad (9)$$

Note that the 7-dimensional Einstein manifolds can be treated as direct products of lower-dimensional space-times but not only of 1-dimensional spheres (because the curvature tensor of S_t^1 is zero). Among different solutions which follow from (8) and (9) there are

$$= \text{AdS}^4 \times \text{dS}^4 / \Gamma \times S^3 \quad (10)$$

$$M_5^{11} = \text{AdS}^4 \times \text{dS}^4 / \Gamma \times S_3^3 \quad (11)$$

where we substituted $L_1^4 = \text{AdS}^4$ (anti-de Sitter space) and $S_1^4 = \text{dS}^4$ (de Sitter space), and Γ is a discrete group of

isometry in dS^4 . Solutions (1) and (11) we shall call products of two universes: AdS^4 has macroscopic time and dS^4 has microscopic (compactified) time. According to these times two different dynamics will exist—one macroscopic and other microscopic.

3. Vacuum solutions in 10-dimensional theory

We shall now consider the corresponding $N=2, D=10$ non-chiral supergravity which can be obtained from $N=1, D=11$ supergravity by dimensional reduction of one space-like dimension. This 10-dimensional theory has more free parameters and may lead to better agreement with phenomenology than 11-dimensional supergravity. Vacuum solutions with standard signature have been considered⁷⁾ and the obtained 4-dimensional manifolds are of an anti-de Sitter type.

Since 3 new bosonic fields (γ, A_M, A_{MN}) appear, the equations of motion are more complicated and hence we shall write only two of them (other three will be trivially satisfied)

$$\begin{aligned}
 R_{MN} = & 2\partial_M \gamma \partial_N \gamma + e^{3\gamma} \frac{r^2}{2} (F_{MP} F_N^P - \frac{1}{16} g_{MN} F_{PQ} F^{PQ}) + \\
 & + e^{-2\gamma} (F_{MPQ} F_N^{PQ} - \frac{1}{12} g_{MN} F_{PQR} F^{PQR}) + \\
 & + e^\gamma (\frac{1}{3} F_{MPQR} F_N^{PQR} - \frac{1}{32} g_{MN} F_{PQRS} F^{PQRS}),
 \end{aligned} \tag{12}$$

$$\begin{aligned}
 \nabla_M \nabla^M \gamma = & \frac{3}{16} r^2 e^{3\gamma} F_{MN} F^{MN} - \frac{1}{6} e^{-2\gamma} F_{MNP} F^{MNP} + \\
 & + \frac{1}{48} e^\gamma F_{MNPQ} F^{MNPQ}
 \end{aligned} \tag{13}$$

where r is a radius of compactification of the eleventh dimension and $F_{MNPQ} = F_{MNPQ} + 4r A_{[M} F_{NPQ]}$. We shall hold in vacuum $\gamma(x) = c = \text{const}$. Analyzing different possibilities we concluded that three types of vacuum solutions exist

$$M_T^{10} = M_t^4 \times M_r^2 \times M_{T-t-r}^4 \tag{14}$$

$$M_T^{10} = M_t^4 \times M_{\tau_1}^2 \times M_{\tau_2}^2 \times M_{\tau_3}^2 \quad (15)$$

$$M_T^{10} = M_t^4 \times M_{\tau_1}^3 \times M_{\tau_2}^3 \quad (16)$$

We find solutions in the form (14) starting from

$$\begin{aligned} F_{\mu\nu\rho\sigma} &= \sqrt{(-1)^t} g_{(4)} \lambda_t \epsilon_{\mu\nu\rho\sigma} \\ F_{mn} &= \sqrt{(-1)^\tau} g_{(2)} \frac{1}{r} l_\tau \epsilon_{mn} \end{aligned} \quad (17)$$

what leads to

$$\begin{aligned} R_{\mu\nu} &= (-1)^t \frac{4}{3} e^c \lambda_t^2 g_{\mu\nu} \\ R_{mn} &= (-1)^\tau e^{3c} l_\tau^2 g_{mn} \\ R_{ab} &= (-1)^{t+1} \frac{2}{3} e^c \lambda_t^2 g_{ab} \end{aligned} \quad (18)$$

where indices a, b correspond to the manifold $M_{T-t-\tau}^4$ for which $F_{abcd} = 0$. From (13) it follows a condition $(-1)^t + (-1)^\tau = 0$, i.e. when t is even then τ is odd and vice-versa. The corresponding solutions of (10) and (11) are

$$M_2^{10} = \text{AdS}^4 \times S^2 \times dS^4 / \Gamma \quad (19)$$

$$M_4^{10} = \text{AdS}^4 \times S_2^2 \times dS^4 / \Gamma \quad (20)$$

Solutions of the form (15) can appear making 2-dimensional products of $M_{T-t-\tau}^4$ in (14), as well as giving non-vanishing values to $F_{\mu\nu\rho\sigma} (M_t^4)$ and $F_{mn} (M_{\tau_1}^2)$.

$$\begin{aligned} F_{mnk} &= \sqrt{(-1)^t} g_{(3)} f \epsilon_{mnk} \quad (m, n, k = 4, 5, 6) \\ F_{abc} &= \sqrt{(-1)^\tau} g_{(3)} f \epsilon_{abc} \quad (a, b, c = 7, 8, 9) \end{aligned} \quad (21)$$

we obtain Ricci tensors

$$\begin{aligned}
 R_{\mu\nu} &= 0 \\
 R_{mn} &= (-1)^{\tau_1} 2 e^{-2c} f^2 g_{mn} \\
 R_{ab} &= (-1)^{\tau_2} 2 e^{-2c} f^2 g_{ab}
 \end{aligned}
 \tag{22}$$

with the condition $(-1)^{\tau_1} + (-1)^{\tau_2} = 0$. Note that Einstein space-times (22) cannot exist in the standard signature since for the extra dimensions $\tau_1 = \tau_2 = 0$ and it does not satisfy the above condition. Solutions of equations (22) are manifolds of the form (16). For $t = 1$ we get very important solutions

$$M_2^{10} = M^4 \times S^3 \times L_1^3 \tag{23a}$$

$$M_4^{10} = M^4 \times S^3 \times L_3^3 \tag{23b}$$

$$M_4^{10} = M^4 \times S_2^3 \times L_1^3 \tag{23c}$$

$$M_4^{10} = M^4 \times S_2^3 \times L_3^3 \tag{23d}$$

where M^4 is a Minkowski space. Hence, in this approach, we find solutions of the vanishing cosmological constant. Solution (23b) has compactified extra manifold if one takes a hyperbolic dodecahedral space for L_3^3 and then massless ghosts will not appear⁴⁾.

4. Concluding remarks

The existence of vacuum solutions in 11 and 10-dimensional theories with various signatures is shown. Note that Majorana spinors on M_t^{11} manifolds can exist for $t = 1, 2 \pmod{4}$, i.e. for our solutions (10) and (11), too. Construction of supersymmetric field theories on space-time manifolds of arbitrary signature is very desirable. Obtained solutions deserve further

considerations, particularly solutions with two 4-dimensional universes and 10-dimensional solutions of zero cosmological constant.

Investigations on this subject will be published in a more complete form in *Theor. Math. Physics* (Moscow).

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