B 5 A Lower Bound of Triton Binding Energy

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The variational principle applied to parameters of a trial wave function gives an upper bound of the ground state energy. A lower bound of the ground-state energy can be obtained by varying the density matrix elements.

The energy of an N-particle system can be written in the form[1)]

$$E=\sum_{a}\varrho_{aa}\,\varepsilon_a .$$

Here, ε_a are all eigenvalues of the "associated two-body Hamiltonian"

$$H=(T_1+T_2)/(N-1)+V_{12},$$

and ϱ_{aa} are the diagonal elements of the two-body density matrix in the eigenbasis of H. Using the variational principle for the parameters ϱ_{aa}, the exact ground state would be obtained if all subsidiary conditions[2)] were taken into account. The necessary and sufficient conditions are not yet known. Here, only the necessary conditions after Sasaki[2)] will be considered so that a lower bound will be obtained,

$$0\leqq\varrho_{aa}\leqq N/2, \quad \text{for even } N,$$

and

$$0\leqq\varrho_{aa}\leqq (N-1)/2, \quad \text{for odd } N.$$

In the lowest solution of the variational problem only the lowest a are occupied and they are given the maximum allowed "weight" ϱ_{aa}:

$$E\geqq N/2\cdot\sum_{a=1}^{N-1}\varepsilon_a, \quad \text{for even } N, \qquad \text{(Bopp-Sasaki formula[2)])}$$

and

$$E\geqq (N-1)/2\cdot\sum_{a=1}^{N}\varepsilon_a, \quad \text{for odd } N.$$

As an example let us take triton, whose lowest eigen-value ε_1 is three-fold degenerate, so $E\geqq 3\times\varepsilon_1$. This estimate is too crude and Hall and Post[3)] improved it by demanding that triton have a definite isospin ($T=1/2$). Then

$$E\geqq\frac{3}{2}(\varepsilon_1+\varepsilon_2).$$

This formula is used to test the Tabakin[4)] potential. If a potential representing the two-particle interaction leads to a higher lower bound than the experimental value, it may be discarded as false. For Tabakin potential we obtained a lower bound of the ground-state energy of triton —12.6 MeV. So the Tabakin potential stands this test.

It is planned to strengthen the lower bound formula and to test some other potentials. An attempt will also be made to estimate the ground-state energy of larger systems ($N = 16$ or more). The formulae available at present are inadequate because they give a nonsaturating N-dependence ($E \propto N^2$).

References

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B6 The Electromagnetic Formfactor for ^{3}He and ^{4}He

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B7 An Exact Solution of the Three-Body Problem and the Triton

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An exact solution of the quantum mechanical three body problem has been found in a closed analytical form for harmonic oscillator forces acting between pairs of three particles[1]. The solution can easily be applied to various nuclear physics problems involving the bound state of three-particle systems, where the particles could represent nucleons (^{3}H and ^{3}He) or some other composite particles (alpha and two nucleons in the case of ^{6}Li), etc. The solution involving harmonic oscillator forces should be regarded only as approximative due to the simplified nature of the forces involved.

The simplest nuclear system for the application of this general three-body solution is the triton. Using triton data alone one can obtain information on the intensities of the three forces involved. The harmonic oscillator force does not include the interaction range parameter, but it is possible to introduce such a parameter artificially by interpreting the results of the parabolic potential as if it were a cut-off potential. In our calculations we used three physical parameters of the triton: (i) the mass radius, $R_m = 1.780$ fm, (ii) the charge radius, $R_c = 1.70$ fm, and (iii) the binding energy, $B = 8.492$ MeV. These parameters do not determine uniquely the intensities of the three forces involved. If one varies the depth of the potential well for the neutron-proton interaction from, for example, 10 MeV to 16 MeV, it follows that the depth of another neutron-proton interaction ranges from 26 MeV to 20 MeV, while the neutron-neutron interaction shows only a slight variation from 13 MeV to 11.6 MeV. In addition to potential depth, potential ranges were also taken into acount. Their