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C. NUCLEAR STRUCTURE

C1 The Generator Coordinate Theory of Nuclear States

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The generator coordinate method after Hill and Wheeler[1] gives Hamiltonians for collective motion if the Hill-Wheeler trial wave function

$$\psi(x) = \int f(p)\, \Phi(x, p)\, dp, \qquad (p = p_1, p_2, \dots, p_m)$$

satisfies the following conditions

$$O(p^*, p') = \langle \Phi(x, p^*),\ \Phi(x, p')\rangle = \exp\big(\sum_i p_i^* p_i'\big),$$

$$H(p^*, p') = \langle \Phi(x, p^*),\ H\Phi(x, p')\rangle =$$

$$= O(p^*, p')\left\{E(O) + \frac{1}{2}\sum_{i,j}\left[B_{ij} p_i^* p_j^* + 2A_{ij} p_i^* p_j' + B_{ij}^* p_i' p_j'\right]\right\}.$$

In some physical systems the above approximations for overlaps are either no good at all, or are satisfactory only when many parameters are introduced. As an alternative, it is proposed in both cases to (i) abandon the Gaussian overlap approximation, and (ii) introduce a new single parameter t through the relations $p_a(t) = p(\varepsilon_a, t)$, $(a = 1, 2, \dots, m)$, where ε_a's are parameters defining the system. For appropriate choice of the $p_a(t)$, the one-parametric family $\Phi(x, p_1(t), \dots. p_m(t)) \equiv \Phi(x, t)$ generates the same subspace H_Φ as the m-parametric family $\Phi(x, p_1, \dots. p_m)$. In the case of Gaussian overlaps the spectrum corresponds to several coupled harmonic oscillators. The proposed model describes, however, the existing degrees of freedom by a single "anharmonic" vibration.

The basis $\Phi(x, t)$ is nonorthonormal and the orthogonalization of this basis may give some vectors with a relatively small norm. As an approximation it was decided to reject vectors with very small norms. This truncation to a smaller subspace $H_{\Phi'}$ was tested on some simple systems using as the basis the set of functions $\Phi(t_i)$ corresponding to the values of the parameter t_i in the region of minima of $<\Phi(t), H\Phi(t)>/<\Phi(t), \Phi(t)>$. It was found that low lying states are quite well described in the remaining subspace $H_{\Phi'}$ even though it is much smaller than H_{Φ}.

The method was applied in the description of three types of states:

(1) Pairing vibrations. — The low-lying states of nuclei with large matrix elements for two-body transfer reaction are described by the trial function $\psi(x) = \sum_{\Delta_i=\Delta_0}^{\Delta_n} f_{\Delta_i} \Phi_{\mathrm{BCS}}^N(x, \Delta_i)$, where Φ_{BCS}^N is the N-projected BCS function and Δ is the gap parameter describing the degree of pairing. The BCS relations are supposed between the parameters describing the occupation of levels $p_a(\Delta) = (v_a/u_a)/(v_1/u_1)$ and the parameter Δ, in spite of the fact[4] that Δ might not generate the whole subspace $H_{\Phi'}$.

The different aspects of the proposed approach were tested on two[3] and three-level[4] systems. In both cases the proposed method describes the energies and the cross section for two-body transfer reaction very well in the whole region of the interaction strength. (There is no breakdown anywhere).

(2) Quadrupole vibrations in light nuclei. — The proposed trial function is

$$\psi_{JM}(x) = \sum_{\beta, k} A_{\beta k}^J \,|\, \beta k J M \rangle.$$

The function $|\beta k J M\rangle$ is obtained by projecting the angular momentum J from a Nilsson determinant $\Phi_k(x, \beta)$ corresponding to the deformation parameter β. The variational principle $\delta[\langle\psi^J, H\psi^J\rangle / \langle\psi^J, \psi^J\rangle] = 0$ led to the secular problem

$$H^J A^J = E_J O^J A^J,$$

where the matrix elements of H^J and O^J are

$$H_{\beta k, \beta' k'}^J = \langle \beta k J M \,|\, H \,|\, \beta' k' J M \rangle,$$

$$O_{\beta k, \beta' k'}^J = \langle \beta k J M \,|\, \beta' k' J M \rangle.$$

The central problem is the calculation of matrix elements. A tractable method has been developed to calculate the linear combinations from the matrix elements

$$\langle \Phi_k(\beta) \exp(\theta^* J_-) \,|\, H \,|\, \exp(\theta J_+) \Phi_{k'}(\beta) \rangle$$

and

$$\langle \Phi_k(\beta) \exp(\theta^* J_-) \,|\, \exp(\theta J_+) \Phi_{k'}(\beta') \rangle,$$

where $J_\pm = J_x \pm i J_y$ and θ is a complex parameter.

The proposed method is being applied in a study of low-lying states in ^{12}C and ^{16}O.

(3) Coupling of pairing and quadrupole vibrations. — Trial wave function can be expressed by means of a generator wave function depending on two parameters, β and Δ. It is proposed that the overlap integrals $O(\beta\Delta;\ \beta'\Delta')$ and $H(\beta\Delta;\ \beta'\Delta')$ be approximated with the Gaussian functions of $(\beta-\beta')$ and the method described above be used to treat the pairing vibrations.

References

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C2 The Pairing Vibrations

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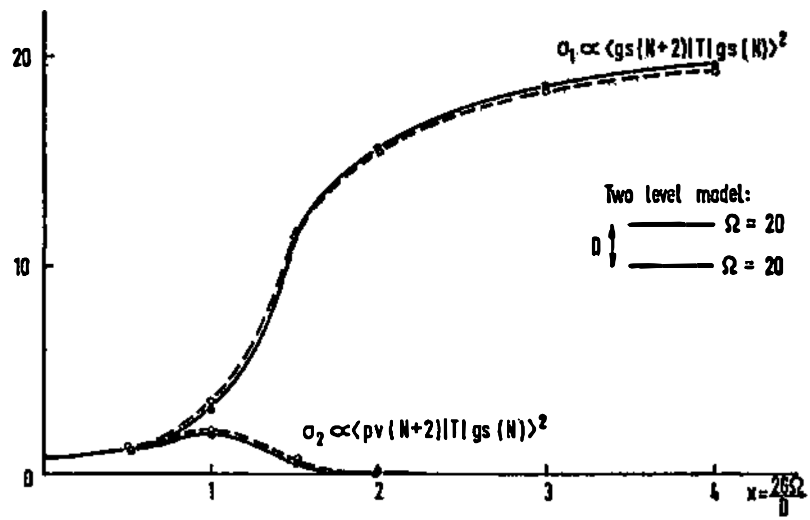
A new approach in calculating pairing vibrational states proposed earlier[1] is tested on a two-level model. The approximate eigenfunctions were constructed with the generator coordinate method

$$\Psi(x) = \int \Phi_N^{\mathrm{BCS}}(x, \Delta)\, f(\Delta)\, d\Delta.$$



The N-projected BCS wave functions $\Phi_N(x, \Delta)$ were taken as basic functions and the quantity Δ as the generator coordinate. The matrix elements for the two body transfer reactions were calculated and compared with the exact