an additional approximation is used which drastically reduces the size of calculations. In the calculation of matrix elements the sums over the finite numbers of single particle states are replaced by infinite ones.

TABLE 1

The ground-state energy E relative to the energy of the lowest configuration. $\langle N\rangle$ and $\langle \Delta N\rangle^2=\langle N^2\rangle-\langle N\rangle^2$ is calculated with the N-projected BCS(PBCS), BCS and the proposed approximation (AP) for 22 particles in doubly degenerate single-particle levels.

$$H=\sum_k \varepsilon_k a_k^+ a_k - G\sum_{\substack{k>0\\ l>0}} a_k^+ a_{-k}^+ a_{-l} a_l; \quad G=0.415\,D, \ \varepsilon_k=kD.$$

method	E/D	N	ΔN
PBCS	−7.5718	22	0
BCS	−5.9011	22	5.56
AP	−6.4151	22	7.1×10^{-2}

TABLE 2

Ground state energy (MeV) relative to the energy of the lowest configuration for Ni isotopes; $G=0.331$ MeV and $\varepsilon(p_{3/2})=0$, $\varepsilon(f_{5/2})=0.78$ MeV, $\varepsilon(p_{1/2})=1.56$ MeV, $\varepsilon(g_{9/2})=$ $=4.52$ MeV.

A	$2n$	E_{PBCS}	E_{BCS}	E_{AP}
58	2	−1.4808	−1.1279	−1.4808
60	4	−2.0815	−1.5135	−1.8837
62	6	−3.2837	−2.6459	−2.9878
64	8	−3.5656	−2.9029	−3.1545
66	10	−2.9291	−2.2020	−2.6571

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C4 The Characterization of Collective States with Density Matrices

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1. Pairing vibrations

States with a high degree of pairing are characterized by having a large eigenvalue of the two-body density matrix $\varrho_{abcd}=\langle\Psi|a_b^+ a_a^+ a_c a_d|\Psi\rangle$. For example, the BCS wave function has one large eigenvalue. The large eigenvalue will be denoted by λ and the corresponding eigenvector by g_{ab}. Suppose that the operator $\hat{g}^+ = \sum_{ab} g_{ab} a_a^+ a_b^+$ is very similar to the physical 2-body transfer operator $\hat{T}$ and that the ground state $\Psi_{N+2} \approx \hat{g}^+ \Psi_N$. This is a reasonable assumption because the additional two particles are likely to be added in the two-particle state g which is anyway energetically the lowest. The two-body transfer then has an enhanced transition amplitude: $\langle \Psi_{N+2} | \hat{T} | \Psi_N \rangle \approx \langle \Psi_N | \hat{g}\, \hat{g}^+ | \Psi_N \rangle = \lambda$.

Pairing vibrations[1] can be characterized by two alternatives:

(i) The 2-body density matrix has two or more large eigenvalues (with eigenvectors $g_1, g_2, \ldots$, respectively). Then there are enhanced transitions to the ground state $\Psi^0_{N+2} = \hat{g}_1^+ \ \Psi_N$ and to the pairing vibrational states $\Psi^{pv}_{N+2} = = \hat{g}_2^+ \Psi_N, \ldots$.

(ii) The two-body density matrix has only one large eigenvalue, but g for N particles is rather different from that for $N+2$ particles. This means that the degree of pairing changes rapidly when more particles are included. Then the additional two particles are not likely to be added in the two-particle state g to obtain the $N+2$ ground state. The state $\hat{g}^+ \Psi_N$ represents a band of several physical states ("pairing vibration"), among which the enhanced transition amplitude is distributed.

It has not yet been determined whether these two alternatives are independent, and if so, which occurs in realistic nuclei.

2. Rotations and Vibrations.

Collective states like rotations and shape vibrations are characterized by large eigenvalues of the "particle-hole matrix"[2]

$$G_{abcd} = \langle \Psi | (a_a^+ a_b - \varrho_{ab})^+ (a_c^+ a_d - \varrho_{cd}) | \Psi \rangle .$$

If there is only one collective degree of freedom (rotation, vibration or an intermediate one), there is one large eigenvalue of the G matrix which may be degenerate with respect to magnetic quantum number. The presence of two degrees of freedom is characterized by two different large eigenvalues (which may both be degenerate with respect to magnetic quantum number).

References

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