C5 Antilinear Operators in the Theory of Nuclear Structure

F. HERBUT and M. VUJIČIĆ, *Institute "Boris Kidrič", Beograd*

Lately, self-consistent field methods (Hartree-Fock theory and Hartree--Bogolyubov theory) have been applied with great success to the study of nuclear structure. To simplify these variational problems a new mathematical formalism utilizing the algebra of antilinear operators[1] has been developed.

This formalism generated new formulations of the Hartree-Bogolyubov theory in one-particle space[2] and quasiparticles space[3]. This permitted construction of original methods for solving the dynamical equations of the theory, which were based on antilinear symmetries. On the assumption that the pairing operator is a given symmetry operator of the nuclear Hamiltonian, two versions of the dynamical equations were obtained. They consist of two interdependent parts, a Hartree-Fock and a Bardeen-Cooper-Schrieffer part[2]. To solve both versions canonical forms of antilinear operators were used[1]. It was further shown that general complex Hartree-Bogolyubov equations in quasiparticle space can be solved by iterative diagonalization of a real symmetric matrix[3]. For this purpose an antilinear matrix characteristic for the theory itself was utilized.

These substantial simplifications in solving the dynamical equations should stimulate and enable more ambitious and elaborate computations in nuclear structure theory. Computations of this kind are in progres.

References

1) F. Herbut and M. Vujičic, Journ. Math. Phys. **8** (1967) 1345;
2) F. Herbut and M. Vujičic, Phys. Rev. **172** (1968) 1031;
3) M. Vujičic and F. Herbut, to be published.

C6 Phenomenological Limit of Semimicroscopic Description

L. ŠIPS, *Institute "Ruđer Bošković", Zagreb*

C7 On the BCS-Model in Nuclear Theory

J. HENDEKOVIĆ, *Institute "Ruđer Bošković", Zagreb*

The simple vibrational model failed to describe adequately the complex structure of the low-lying spectrum of some spherical vibrational nuclei, such as Sn isotopes. The main problem in the microscopic description of these nuclei arises from the large number of active particles moving in a limited number of orbitals. In some cases satisfactory results can be obtained by introducing particle excitations coupled to phenomenological phonons[1]. The natural limitation of this model lies in the fact that the ground state and phononic excitations are structureless. Thus, in complex situations the Pauli exclusion principle cannot be properly taken into account. The most important pairing correlations and the basic structure parameters of the ground state, namely the occupation probabilities of single particle orbitals, can be expressed simply through the parametrization of the BCS wave function

$$|\tilde{o}\rangle = \prod_{\alpha>0} \left(u_a + v_a s_\alpha c_\alpha^+ c_{-\alpha}^+\right) |0\rangle, \quad s_\alpha = (-1)^{j_a - m_a}.$$

Here c_α^+ are the creation operators of the nucleon in the orbit α, and $\alpha \equiv (a, j_a, m_a)$. v_a is the probability amplitude for the occupation of the level a, and $|0\rangle$ is the physical vacuum.

Many extensive calculations in the framework of the BCS-model have recently been performed with considerable success[2]. Nevertheless, there are many objections to the BCS-model, the most serious referring to its particle number nonconservation. In addition, there are examples of serious disagreement between the results of the shell-model and the BCS-model calculations[3]. Arguments which might explain this discrepancy are given below.

The following is an intuitive "justification" of the BCS-model in nuclear spectroscopy: The Bogoliubov-Valatin quasi-particle (qp) transformation approximately diagonalizes the strong short-range component of the nucleon--nucleon interaction. The residual interaction between quasiparticles is very much reduced and should be easily diagonalized. The quasiparticle method is then understood as an approximation method in shell-model calculation. Below the above arguments are shown to be incorrect.

A simple model of two levels with spacing ε and one kind of particles interacting through the pairing force of strength G is analysed. The first level with $j = 13/2$ is occupied in the normal state, the second one with $j = 15/2$ is empty. Fig. 1 compares the structure of the shell-model seniority zero ground state wave function, expressed in the particle-hole representation (P), and that of the very same wave function, expressed in terms of quasiparticles (Q). The number n refers to the n particle pairs and n hole pairs in the first case, or to the $4n$ and $(4n-2)$ qp excitation modes in the second case. Total weight (probability) of the components of a given type is drawn as a function of n for several ε/G. The features of Fig. 1 are:

a) The weight of the BCS ground state $|\tilde{o}\rangle$, (value of Q for $n = 0$) in the shell-model ground state is very small in both caces.

b) The weight of components with a large number of quasiparticles is not negligible, and is sometimes even dominant.

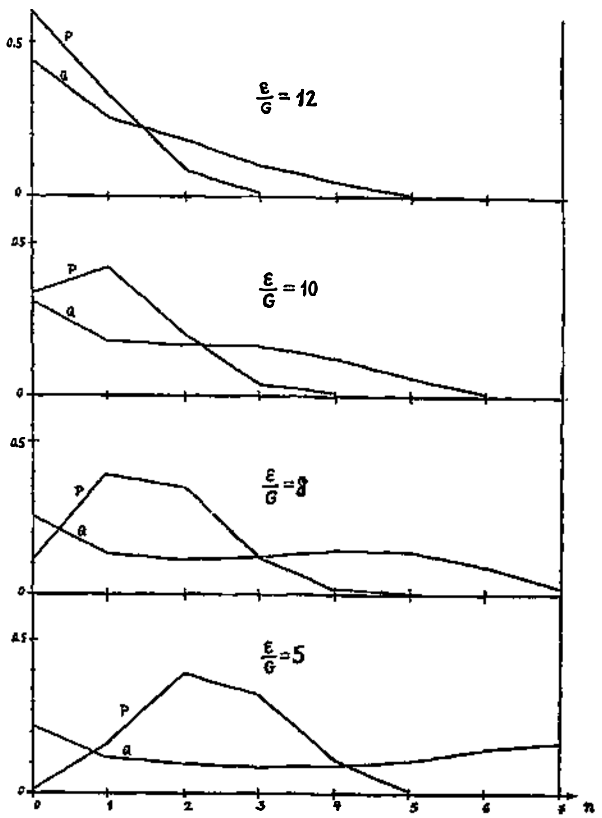
0.5
P
a
ε/G = 12
0
P
a
ε/G = 10
P
a
0
1
2
3
4
5
6
n
ε/G = 5

Fig. 1

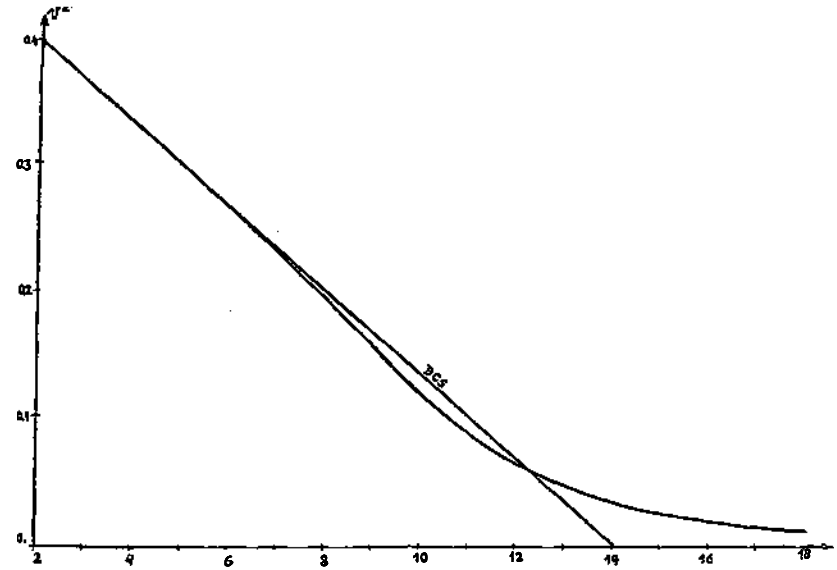
0.4
0.3
0.2
0.1
BCS
2
4
6
8
10
12
14
16
18

Fig. 2

c) The structure of the ground state wave function in the particle-hole representation is more convenient for various approximations (truncation) than in the qp representation.

This illustrates that the BCS-model cannot be interpreted as an approximation method in shell-model calculation.

In Fig. 2 the occupation probability for the second level $j=15/2$ is drawn as a function of ε/G. The straight line corresponds to the BCS ground state $|\bar{o}\rangle$ and the curved line to the shell model seniority zero ground state. It is seen that the BCS-model nicely reproduces the ground state distribution of particles. This suggests the interpretation of the BCS-model as a semi--phenomenological model, simulating the ground state particle density distribution by violating the total number of particles.

Analysis of more realistic calculations shows that the variational procedure for finding the amplitudes v_a sometimes does not reproduce the true ground state occupation probabilities[3], which is the source of disagreement with the shell-model results. Therefore, a new method for calculating the coefficients v_a should be developed, otherwise additional validity criteria for the BCS-model must be established.

References

1) G. Alaga, F. Krmpotić and V. Lopac, Phys. Lett. 24B (1967) 537, and references therein;
2) J. Sawicki, Supplemento Nuovo Cimento 6 (1968) 696, and references therein;
3) B. Lorazo, Phys. Lett. 29B (1969) 150.

C8 Some Open Problems in the 2s 1d Shell

M. KREGAR, *Institute "Jožef Štefan" and University of Ljubljana, Ljubljana*

1. Lifetime measurements of the low excited states in ^{35}Cl and ^{29}Si provide evidence for strongly inhibited $E1$ transitions, with a transition strength of 10^{-4}—10^{-8} of the $E1$ Weisskopf single particle unit (Fig. 1)[1,2]. These transitions cannot be accounted for by any of the known selection rules. It is proposed that other similar transitions in the $2s\,1d$ region should be explored to obtain additional information on the $E1$ transitions. So, e. g. there are $7/2^-$ and $3/2^-$ states in ^{33}S with unknown lifetimes at 2.970 and 3.22 MeV excitation energy, respectively. The ^{32}S(d, p)^{33}S reaction can be used to reach these levels. In ^{33}Cl there is a $3/2^-$ state at 2.98 MeV, which can easily be reached via ^{32}S(p, γ)^{33}Cl reaction. In ^{35}Ar the low energy spectrum is completely unknown, but it can be studied via ^{36}Ar(^{3}He, α)^{35}Ar reaction.