

ISING MODEL OF REGULARLY DISTRIBUTED  $S = 1$  AND  $S = 3/2$   
 SPINS ON A HONEYCOMB LATTICE

Viktor Urumov

Institut za Fizika, PMF, p.fah 162, Skopje 91001, Yugoslavia.

Abstract The critical point for an Ising model of regularly mixed spins of  $S = 1$  and  $S = 3/2$  on a honeycomb lattice is determined exactly along a line in the plane of interaction constant versus temperature.

Various Ising type models are of interest in the study of cooperative phenomena in different branches of physics and the literature on the subject is vast /1/. The two-dimensional Ising model of  $S = 1/2$  spins on a square lattice was first solved by Onsager /2/. Very few exact results are available for higher spins /3/. Recently Horiguchi /4/ obtained exactly the critical point for  $S = 1$  Ising model with bilinear and biquadratic exchange on a honeycomb lattice for some particular values of the parameters and the author /5/ within the same restrictions calculated the magnetization. In the following we extend the calculations to the case of regularly distributed spins of magnitude 1 and  $3/2$ . Systems of mixed spins are relevant in the study of ferrimagnetism. They were also studied /6/ within the context of verifying the universality hypotheses, which establishes that the critical exponents describing the behaviour near the critical points are independent on the spin magnitude.

We examine the following model

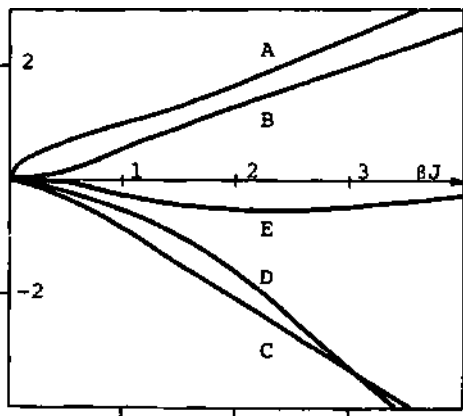
$$H = - J \sum_{i,j} S_i S_j + K \sum_{i,j} S_i^2 S_j^2 - \Delta_1 \sum_i S_i^2 - \Delta_2 \sum_j S_j^2 \quad (1)$$

where  $J$  and  $K$  are the bilinear and biquadratic exchange constants,  $\Delta_{1,2}$  are crystal-field single-site energies for the two triangular sublattices, the first composed of  $S = 1$  spins only that have three possible states ( $S = 0, \pm 1$ ) and the second entirely composed of  $S = 3/2$  spins with four possible orientations ( $S = \pm 1/2, \pm 3/2$ ). The first two summations are carried over each pair of nearest neighbours and the other two over each of the sublattices separately. Each 1-spin is surrounded by three 3/2-spins and vice versa. Together they form the honeycomb lattice.

To proceed, several transformations of the model are performed. At the beginning each factor in the partition function for the model, corresponding to the contribution of a single bond is represented by

$$e^{\beta J S_i S_j - \beta K S_i^2 S_j^2} = 0.5 \sum_{\sigma_{i,j}=\pm 1} e^{A[(S_i + S_j) + B S_{i,j}^3] \sigma_{i,j} + C S_i^2 + D S_j^2 + E S_j^4} \quad (2)$$

This is equivalent to the introduction of a new  $\sigma = \pm 1$  spin in the middle of each bond between two neighbouring 1 and 3/2 spins. By substitution of all the possible values of couples  $S_i, S_j$  in (2)



**Fig.1** The effective interaction parameters in (2) as functions of the inverse temperature.

we obtain a system of equations from which the effective temperature dependent interaction parameters  $A, B, C, D, E$  are to be obtained. The equations are compatible under the condition  $K/J = \ln(2 \cosh \beta J - 1) / 2\beta J$  (3) that binds the interaction constants with the temperature.

The system of equations for the effective interactions  $A - E$

was solved numerically and their dependence on the inverse temperature is shown in Fig.1. With the transformation (2) we have actually obtained a decorated honeycomb lattice with three spins lying on each bond in the order  $1 - 1/2 - 3/2$  or the inverse. Afterwards with the star-triangle transformation [3,5] the model is reduced to the generalized Kagomé lattice of  $1/2$  spins only, where by generalized we mean a Kagomé lattice on which the interaction parameters on neighbouring triangles change alternately between two different values. With the inverse star-triangle transformation the model is again reduced to a decorated honeycomb lattice but now of  $1/2$ -spins only. Finally by partial summation over the decorated sites (decoration-iteration transformation) the model is reduced to the usual honeycomb lattice of  $1/2$ -spins. All the transformation steps are represented in Fig.2, where the letters represent some effective interactions. At each

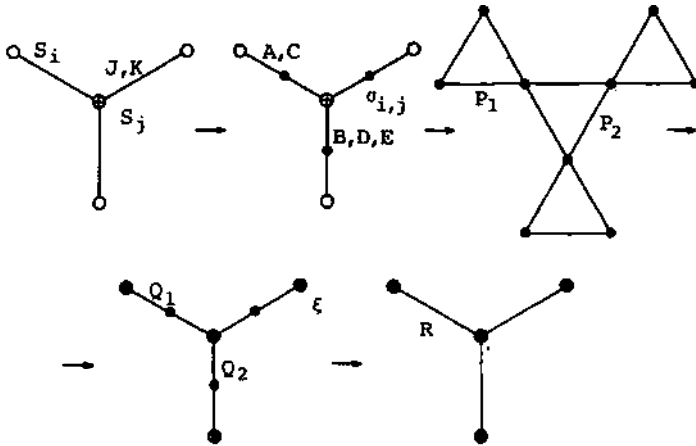


Fig.2 The transformations from the original  $1-3/2$  model to the  $1/2$ -spin honeycomb lattice.

step the effective interactions between the neighbouring spins change in a complicated fashion too cumbersome to be given here.

The knowledge of the effective interaction at the final stage of the transformations and the knowledge of the criticality condition  $e^{2R_c} = 2 + \sqrt{3}$  for the honeycomb lattice /3/ per-

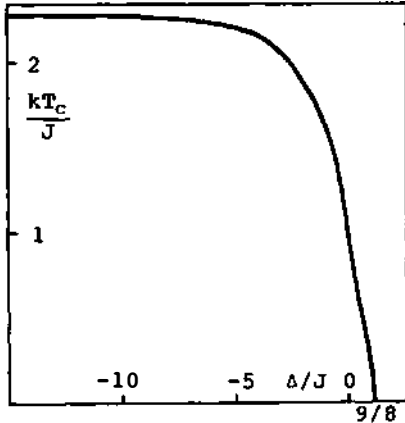


Fig.3 The critical temperature as a function of the crystal-field energy.

restrictive condition (3), will be given elsewhere. Generalization of the method for the case of other higher spins in mixtures or not appears to be possible.

#### References

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mit to determine the critical point for the model (1) under the restriction (3). In Fig.3 the dependence of the critical temperature on the single-site energy parameters  $\Delta_i$  for the case  $\Delta_1 = \Delta_2 = \Delta$  is given.

Further details concerning also the magnetisation of both sublattices, which can be calculated exactly within the same