THE COMPOUND NUCLEUS

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ABSTRACT

We survey the main progress accomplished in the field of Compound Nucleus Theory during the last ten years. The theory now has a microscopic basis and encompasses the limits of strong as well as of weak absorption, the effect of direct interactions, of preequilibrium decay, of intermediate structure and of isospin mixing.

The present talk is largely based on a review written in collaboration with Hans A. Weidenmüller, and which will appear in the Annual Review of Nuclear Science (1980). Until the last decade, compound nuclear reactions were described in the framework of phenomenological models. These derived expressions for the average compound nucleus cross sections from the basic assumption made by N. Bohr, who postulated in 1936 that the breaking up of the compound system has "no immediate connexion with the first stage of the encounter". When one takes into account conservation of angular momentum and of parity, this yields the familiar Hauser-Feshbach formula (1952)

$$\langle \sigma_{ab}^{CN} \rangle = T_a T_b / \Sigma_c T_c \quad . \tag{1}$$

Here, a, b, c, generically denote the channel quantum numbers. The quantity T_a is the transmission coefficient in channel a :

$$T_a = 1 - |\langle S_a \rangle|^2 \quad , \tag{2}$$

where $\underset{\sim}{S}$ is the scattering matrix.

In general, the Hauser-Feshbach formula turns out to be in good agreement with the empirical data. Since 1952, much effort was thus devoted to derive it from nuclear reaction theory, and to find how it should be modified in order to include the influence of direct reactions, of precompound processes, of intermediate structure, of isospin conservation, etc.

Intuitively, one expects the Compound Nucleus Model to work best in the thermodynamic limit, when the compound system nucleus rapidly reaches statistical equilibrium. This is accomplished when the recurrence time $2\pi\hbar/D$, becomes large. Here, D is the average distance between neighbouring compound nuclear levels. Nevertheless, the first derivation of the Hauser-Feshbach formula (1) was limited to the opposite limit, i.e. to the case of weak absorption ($T_a << 1$). Even then, eq. (1) has to be multiplied by a correction factor (Lane and Lynn 1957)

$$\sigma_{ab}^{CN} = \frac{T_a T_b}{\Sigma_c T_c} W_{ab} \quad , \tag{3}$$

where W_{ab} is the "width fluctuation correction", whose main origin is the unitarity property of the scattering matrix. In the limit $T_a \ll 1$, one has $W_{aa} \simeq 3$, and $W_{ab} < 1$.

In the thermodynamic limit $D \to 0$, a theoretical derivation of the Hauser-Feshbach formula (1) -and of appropriate extensions thereof- was found by Agassi, Weidenmüller and collaborators (1975). These authors worked in the framework of the shell-model theory of nuclear reactions. They introduced statistical assumptions on the matrix elements of the residual interaction between nuclear configurations containing different numbers of excitons (particles plus holes). These assumptions are the same as those used in the statistical theory of nuclear spectra. Agassi and Weidenmüller's theory encompasses preequilibrium processes, for which it gives expressions close to those of the master-equation approach. The theory also includes the influence of direct reaction processes, and yields

$$\langle S^{FL}_{ab} S^{FL*}_{cd} \rangle = X_{bd} X_{ac} + X_{bc} X_{ad} \quad , \tag{4}$$

where $\underset{\sim}{S}^{FL}$ is the fluctuating part of the S-matrix. This result had been conjectured by Vager (1971), and first proved by Kawai, Kerman and McVoy (1973). In their derivation, the latter authors use for the S-matrix an approximation which is not exactly unitary. Kerman and Sevgen (1976) have argued that this unitarity condition may be somewhat relaxed. In the limit $D \to 0$, Agassi and Weidenmüller also justify the central result of Ericson's fluctuation theory, namely that the auto-correlation function has a Lorentzian shape. The correlation width is given by

$$\Gamma_{corr} = \frac{D}{2\pi} \text{ trace } \underset{\sim}{T} \quad , \tag{5}$$

where

$$T_{ab} = \delta_{ab} - \sum_d \langle S_{ad} \rangle \langle S^*_{db} \rangle \tag{6}$$

is Satchler's generalized transmission matrix (1963).

Compound nucleus theory is thus in a satisfactory state in the two extreme limits $D \to 0$ on the one hand (strong absorption) and $T_a \to 0$ on the other hand (weak absorption). For intermediate values of T_a, one must still resort to approximations derived from extensive numerical studies. These start from the following K-matrix parametrization

$$\underset{\sim}{S} = \frac{1+i\underset{\sim}{K}}{1-i\underset{\sim}{K}} \quad , \tag{7}$$

$$K_{ab} = K^{(o)}_{ab} + \sum_\mu \frac{\gamma_{\mu a} \gamma_{\mu b}}{E_\mu - E} \quad . \tag{8}$$

One generates $\underset{\sim}{S}$ from sets of values for $\gamma_{\mu a}$ and E_μ, and one calculates the energy or the ensemble average $\langle S^{FL}_{ab} S^{FL*}_{cd} \rangle$. In the absence of direct reactions, Weidenmüller et al. (1975) find that

$$\langle \sigma_{ab}^{CN} \rangle = V_a V_b [1+\delta_{ab}(W_{aa}-1)]/\Sigma_c V_c \quad , \tag{9}$$

with

$$W_{aa} = 1+2/(1+T_a^{1/2}) \quad . \tag{10}$$

Moldauer (1975) gives an alternative prescription. He writes $T_a = \langle |t_{\mu a}|^2 \rangle$ and finds that

$$\langle \sigma_{ab}^{CN} \rangle = \langle |t_{\mu a}|^2 \, |t_{\mu b}|^2 / \Sigma_c \, |t_{\mu c}|^2 \rangle \tag{11}$$

is a good approximation, provided that one assumes that the quantities $|t_{\mu a}|^2$, $|t_{\mu b}|^2$ are uncorrelated random variables, with effective degrees of freedom ν_a , ν_b . The values of ν have been plotted by Moldauer for some cases. In practice, recipes (9) and (11) are in good agreement.

A general procedure has been found for including direct reactions in compound nucleus theory. It is based on a theorem due to Engelbrecht and Weidenmüller (1973) who showed that the matrix

$$\langle \tilde{S} \rangle = \underset{\sim}{U} \, \langle S \rangle \, \underset{\sim}{U}^T \tag{12}$$

is diagonal if $\underset{\sim}{U}$ is the unitary matrix specified by the requirement that $\underset{\sim}{U} \, T \, \underset{\sim}{U}^+$ is diagonal. Weidenmüller et al. (1975) and Moldauer (1975) have shown that one can calculate $\langle |S_{ab}^{FL}|^2 \rangle$ from the expressions (9) or (11).

In conclusion, compound nucleus theory has recently reached a satisfactory state, except perhaps in the case of intermediate absorption for which one must still use approximations based on numerical investigations. The major remaining problem is common to the statistical theories of nuclear spectra and of nuclear reactions : what are the properties of the nucleon-nucleon interaction which justify the use of random-matrix models for the nuclear Hamiltonian ?