STATISTICAL AND NON-STATISTICAL BEHAVIOUR IN THE POPULATION OF NUCLEAR MAGNETIC SUBSTATES

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ABSTRACT

Conditions under which statistically random (Gaussian) feeding of nuclear magnetic substates M occurs are examined. A mélange of γ-decay cascades offers the most likely means of achieving the statistical (Gaussian) filling of substates conjectured in level-density theory. In general particle transitions, though populating low-M substates preferentially, yield a non-Gaussian trend. The compound-nucleus reaction mechanism generally comes closer to promoting Gaussian feeding than do direct interactions.

INTRODUCTION

Although nuclear spectroscopy has hitherto been directed chiefly to the identification of nuclear states and transitions, it can be carried to a more penetrating quantum sublevel in the examination of substates and their populations. This microscopic substructure inherently establishes the characteristics of angular distributions, correlations, and polarizations. This feature has latterly been obscured through the use of Racah algebra to sidestep summation over magnetic quantum numbers in the evaluation of distribution functions, but fundamentally the course of a transition sequence in a nuclear reaction is conditioned by the substates. A study of substate populations should provide insight into the fine details of nuclear transitions and their outcome.

MAGNETIC SUBSTATE POPULATIONS

The population P(M) of the substates M belonging to a state of spin J is influenced directly by the preceding transitions that promote feeding of the residual state J, and is therefore influenced by the reaction mechanism as well as by the other quantum characteristics of the interaction process. For each state J there are (2J + 1) substates (M = -J, -J+1,.., +J) whose relative populations P(M) are normalized to

$$\sum_{M=-J}^{M=+J} P(M) = 1 \,, \qquad (1)$$

and are symmetric in the sense that P(+M) = P(-M).

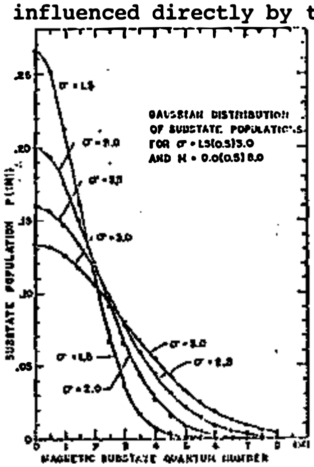

Figure 1. Gaussian distribution of magnetic substate populations *P(M) versus M* according to Eq. (2) for four representative values of the spin cut-off parameter σ.

It would be unnatural to expect substates to be filled uniformly in the course of a normal transition [i.e. usually P(M) ≠ const.]. A more reasonable conjecture, substantiated almost without exception in practice, is of a fairly monotonic decrease in P(|M|) with increasing |M|; indeed, it is customary to assume in the evaluation of level densities[1] that there is a perfectly random statistical distribution - e.g., a Gaussian trend - in the occupancy of substates, of the form

$$P(M) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{M^2}{2\sigma^2}\right) , \qquad (2)$$

with σ a spin cut-off parameter (Gaussian root-mean-square deviation) having a numerical value typically around $\sigma \simeq 2$. The substates extend only to $|M| \leq J$, and hence for any state J one has

$$P(|M|) = 0 \text{ for all } |M| > J. \qquad (3)$$

A plot of Eq.(2) for four representative values of σ is shown in Fig. 1 up to $|M| = 8$, and a quantitative tabulation of the P(M) for states J up to $J = 13/2$ under the statistical (Gaussian) conditions (1) - (3) is presented in Table I for $\sigma = 2$.

TABLE I. Substate populations $P(|M|)$, with $P(-M) = P(+M)$, for $|M| \leq J$, having the Gaussian distribution (2) with the normalization (1), calculated for a spin cut-off parameter $\sigma = 2$.

J	P(0)	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)
0	1.0000						
1	0.3617	0.3191					
2	0.2514	0.2218	0.1525				
3	0.2161	0.1907	0.1311	0.0702			
4	0.2042	0.1801	0.1238	0.0663	0.0276		
5	0.2006	0.1769	0.1216	0.0651	0.0271	0.0088	
6	0.1997	0.1762	0.1211	0.0649	0.0270	0.0088	0.0022
J	**P(1/2)**	**P(3/2)**	**P(5/2)**	**P(7/2)**	**P(9/2)**	**P(11/2)**	**P(13/2)**
1/2	0.5000						
3/2	0.2810	0.2190					
5/2	0.2221	0.1730	0.1049				
7/2	0.2021	0.1574	0.0954	0.0451			
9/2	0.1956	0.1524	0.0924	0.0436	0.0161		
11/2	0.1938	0.1510	0.0915	0.0432	0.0159	0.0045	
13/2	0.1934	0.1507	0.0914	0.0431	0.0159	0.0045	0.0010

EXAMINATION OF SUBSTATE POPULATIONS

The expectation that a mélange of γ-cascade transitions could bring about a Gaussian distribution in the feeding of substates is, for instance, borne out by the findings of Simms et al.[2], who studied the $^{99}Ru(\alpha,n)$ reaction ($Q = -5.34$ MeV) at incident energies around 17 MeV feeding high-lying states of ^{102}Pd. Their decay via γ-cascades produced a statistical filling of the substates in successive low-lying vibrational levels - particularly those of the ground-state band [J^π (E* MeV) = 2^+ (0.5564), 4^+ (1.2758), 6^+ (2.1112) and 8^+ (3.0129)]. The analysis of the γ-distributions between these levels, of Legendre-polynomial form,

$$W(\theta_\gamma) = \Sigma_\nu\, a_\nu \cdot P_\nu(\cos\theta_\gamma), \qquad (4)$$

yielded best-fit Legendre coefficients a_ν that in their turn, as described in detail by Sheldon and Rogers[3], enable one to derive the substate populations P(M) explicitly. For the three lowest states these prove to be

$$P(0)/P(1)/P(2)_{2^+} = 0.290/.233/.122 \ , \qquad (5)$$

$$P(0)/P(1)/.../P(4)_{4^+} = 0.196/.176/.124/.065/.038 \ , \qquad (6)$$

$$P(0)/P(1)/.../P(6)_{6^+} = 0.177/.163/.125/.072/.022/.000/.031 \ , \qquad (7)$$

in close conformity with the Gaussian values of Table I for $\sigma = 2$ [the "best fit" mean parameters are[2] σ = 1.52, 2.12 and 2.22 respectively], while the authors' "spreading parameter" σ/J = 0.29 for $J^{\pi} = 8^+$ indicates that

$$P(0)/P(1)/.../P(8)_{8^+} = 0.172/.157/.119/.075/.039/.017/.006/.002/.001. \qquad (8)$$

Within the two-figure accuracy of the authors' data, the agreement provides a convincing demonstration of random statistical substate feeding by γ-cascades.

As the Legendre coefficients a_ν in Eq.(4) bear a complicated linear functional relationship to the P(M), their numerical distribution is non-Gaussian even if the P(M) are Gaussian. If all the a_ν (except a_0) vanish, as in an isotropic γ-distribution, the substates are uniformly populated [P(M) = const.]. Constraints upon feeding of substates, particularly when occasioned by particle transitions, may arise from quantum selection rules, such as the "Bohr Theorem", which prevents[4,5,6] the population of $M = \pm 1$ substates in the excitation of a 2^+ level from a 0^+ target state by spinless particles [$J_0 = s = s' = 0$; $J = 2$, $\pi = +$], as verified in the study of the $^{12}C(^{16}O,\alpha)^{24}Mg(2^+)$ reaction at incident energies near 20 MeV by King et al.[6], or the "Litherland-Ferguson Theorem"[7], which allows the feeding of only those residual substates M with

$$M \leq J_0 + s + s' \qquad (9)$$

when the emergent particles (of spin s') following the bombardment of a target state J_0 with particles of spin s are observed (in coincidence with de-excitation γ-rays whose angular distribution is measured and Legendre-analyzed) at 0° or 180° to the incident beam (quantization) direction in an angular-correlation detector arrangement.

The restriction (9) can be mild if J_0 is fairly high [as in the case of the $^{99}Ru(\alpha,n\gamma)^{102}Pd$ reaction considered previously, for which $M \leq 3$] or stringent, as in the illustrative cases tabulated in Table II ($M_{max} = 3/2$).

When simple γ-distributions (integrated over all particle emergence angles and thus not in coincidence) are measured following the filling of residual substates by particle transitions, the Litherland-Ferguson restriction no longer applies. The feeding probability depends upon the particle emergence angles[5,6] and the substate populations are thus the integral over all angles. The tendency toward preferential population of low-M substates implied by the cut-off (9) is relieved, particularly in the case of reactions mediated by a compound-nucleus (CN) mechanism, and the trend in the P(M) is toward a more gentle decrease with increasing M - a dropoff usually more gradual and monotonic than Gaussian. With a direct-interaction (DI) mechanism, one may in some instances discern a tendency to preferentially populate low-M substates[8-12], but this is not consistent[9] and has invariably been found to be non-Gaussian.

Some examples of Gaussian trends have been observed following inelastic neutron scattering or (p,n) reactions mediated by a CN mechanism; e.g., the populations of the 2^+ (1.368 MeV) substates in ^{24}Mg fed via inelastic neutron scattering at 14 MeV (an energy that would favour a DI process) are[9]

$$P(0)/P(1)/P(2)_{CN\ 2^+} = 0.247/.210/.166 \qquad (10)$$

on evaluation with CN theory, whereas from DI theory one finds a trend that is skewed more toward the feeding of the lower-M substates, viz.

$$P(0)/P(1)/P(2)_{DI\ 2^+} = 0.417/.166/.125\ . \qquad (11)$$

Clearly, the values in Eq.(10) tally with those of Table I for Gaussian feeding. Moreover, in studying the energy variation of substate feeding for levels in ^{23}Na and ^{55}Mn [$^5/_2{}^+$ (0.440 MeV), $^7/_2{}^+$ (2.038 MeV), and $^7/_2{}^-$(0.126 MeV), $^3/_2{}^-$ (1.528 MeV) respectively] populated through inelastic neutron scattering from threshold up to incident energies of 4.5 MeV and beyond, Donati et al.[13,14,15] found a generally Gaussian trend of P(M) *versus* M to prevail at the higher incident energies, consonant with values furnished by CN theory while incompatible with those predicted by DI theory.

An illustration of Gaussian substate feeding in a (p,n) reaction mediated by the CN mechanism is furnished by findings from the experimental data of Birstein et al.[16] for the $^5/_2{}^-$ (0.190 MeV) first excited state in ^{63}Zn populated by the $^{63}Cu(p,n)^{63}Zn^*$ reaction at incident energies around 5 MeV. The result,

$$P(1/2)/P(3/2)/P(5/2)_{(p,n)\ ^5/_2{}^-} = 0.22/.18/.10\ , \qquad (12)$$

closely matches the values in Table I. It is interesting to note that similar analysis of the data[16] for population of the same level under comparable conditions but via the $^{60}Ni(\alpha,n)^{63}Zn$ reaction (Q = -7.91 MeV) at an incident energy of 9.25 MeV indicates that

$$P(1/2)/P(3/2)/P(5/2)_{(\alpha,n)\ ^5/_2{}^-} = 0.32/.21/(-0.03)\ , \qquad (13)$$

which bears no resemblance to a Gaussian trend. In the latter instance, even though the Litherland-Ferguson restriction is inoperative here, the incident channel spin of zero (whereas in the former instance it is 2) would tend to favour the population of low-M substates in the residual nucleus as not only the projectiles but also the emergent neutrons have rather low energy and hence small angular momenta (with low magnetic quantum numbers). An analogous, but non-Gaussian, outcome is found, for analogous reasons, in the case of the substate feeding of the $^5/_2{}^-$ (0.343 MeV) excited state in ^{59}Ni, as studied also by Birstein et al.[16] by way of the $^{59}Co(p,n)^{59}Ni^*$ CN reaction at an incident energy of 3.1 MeV and the $^{56}Fe(\alpha,n)^{59}Ni^*$ companion CN reaction at an incident energy of 6.57 MeV.

Practically all other findings for substate population behaviour prove to be non-Gaussian in distribution.

CONCLUSIONS

There is a general tendency for the feeding of magnetic substates to be non-random, i.e., not statistically Gaussian, except in isolated instances. It may, indeed, be forbidden by restrictive quantum selection rules.

The relative populations P(M) of the substates M belonging to a level of spin J in a residual nucleus can be determined from theoretical[3,10,13,17] and/or experimental data under a diversity of conditions involving the study of particle-particle correlations[18,19,20], particle-γ correlations[7,21,22,23], γ-γ correlations[7,24-32], and γ-distributions (with feeding that preserves symmetry about the incident [quantization] direction)[3,21,22,30,32]. While a mélange of γ-cascades or certain particle transitions (see above) may produce Gaussian feeding, heavy-ion transitions do not[6,11,12,33].

TABLE II. Substate Populations determined from parallel (0°) or antiparallel (180°) particle-gamma coincidence correlation measurements, illustrating the operation of the Litherland-Ferguson Theorem (9).

Reaction	Residual State J^{π} (E^* [MeV])	Incident energy [MeV]	M_{max}	Experimental P(0)	Substate P(1)	Population P(M>1)	Ref.
^{19}F(p,180°α-γ)^{16}O	2^- (8.88)	7.4	1	.342±.022	.329±.011	0	a
^{19}F(α,180°p-γ)^{22}Ne	2^+ (1.28)	5.4	1	.159±.020	.421±.040	0	b
	2 (4.47)	"	"	.152±.027	.424±.054	0	"
^{30}Si(d,180°α-γ)^{28}Al	3^+ (1.014)	5.36	1	.10 or .90	.45 or .05	0	c
^{40}Ca(h,0°p-γ)^{42}Sc	1^+ (0.61)	10 & 15	1	.36 ± .02	.32 ± .01	0	d
	3^+ (1.43)	"	"	.20 ± .43	.40 ± .22	0	"
	2^+ (1.59)	"	"	.95 ± .06	.03 ± .03	0	"
^{50}Ti(t,180°p-γ)^{52}Ti	2^+ (1.047)	2.9	1	1.006±.094	0	<5%	e
	2 (2.259)	"	"	.730or.878	.135or.061	"	"
	2^+ (2.428)	"	"	.927or1.001	.037or 0	"	"
	1 (4.230)	"	"	.913or.772	.044or.114	"	"
^{58}Ni(p,180°p'-γ)^{58}Ni	4^+ (2.46)	8.0	1	.21	.40	<1%	f
	2^+ (2.78)	"	"	.42 or .50	.29 or .25	"	"
	2^+ (3.04)	"	"	.61 or .54	.20 or .23	"	"
	2^+ (3.26)	"	"	.26	.37	"	"

Reaction	Residual State	Incident energy	M_{max}	$P(\frac{1}{2})$	$P(\frac{3}{2})$	$P(M>\frac{3}{2})$	Ref.
^{26}Mg(p,180°α-γ)^{23}Na	various [$\frac{5}{2}, \frac{7}{2}, \ldots$]	9.3-10.5	$\frac{1}{2}$	.50	0	0	g
^{24}Mg(t,180°α-γ)^{23}Na	"	2.8	"	.50	0	0	h
^{26}Mg(α,0°n-γ)^{29}Si	$\frac{5}{2}^+$ (2.03)	4.74&4.94	"	.50	0	0	i
^{26}Mg(α,nγ)^{29}Si:γ-dist.	"	"	-	.36 ± .04	.12 ± .04	.02 ± .02	†
^{26}Mg(α,0°n-γ)^{29}Si	$\frac{3}{2}^+$ (2.43)	"	$\frac{1}{2}$	.50	0	0	i
^{26}Mg(α,nγ)^{29}Si:γ-dist.	"	"	-	.41 ± .04	.09 ± .04	0	†
^{32}S(d,0°p-γ)^{33}S	$\frac{5}{2}^+$ (1.965)	2 - 3	$\frac{3}{2}$	.21 ± .06	.29 ± .06	0	j
	$\frac{3}{2}^+$ (2.313)	"	"	.06 ± .06	.44 ± .22	0	"
	$\frac{7}{2}^-$ (2.937)	"	"	.35 ± .03	.15 ± .03	0	"
	$\frac{3}{2}^-$ (3.224)	"	"	.38 ± .04	.12 ± .04	0	"
^{30}Si(α,0°n-γ)^{33}S	$\frac{5}{2}^+$ (1.965)	7.52	$\frac{1}{2}$	.50	0	<5%	k
	$\frac{3}{2}^+$ (2.313)	"	"	.50	0	"	"

† Note the remission of the Litherland-Ferguson Restriction for populations determined in simple γ-distribution geometry, omitting the emergent-particle coincidence detector (ref.21).

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