DYNAMIC POLARIZABILITY: TDHF THEORY AND RPA SUM RULES

S. Stringari, E. Lipparini, G. Orlandini, M. Traini and R. Leonardi

Dipartimento di Matematica e Fisica Libera Università Trento

- Italy -

ABSTRACT

The linear response of a nucleus to an external oscillating field (dynamic polarizability) is studied in TDHF theory and it is related to RPA energies and matrix elements. An explicit way to evaluate the energy weighted and the cubic energy weighted sum rules with RPA accuracy is given for isoscalar as well as for isovector modes

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We solve time dependent Hartree-Fock equations when an external oscillating field couples to the nucleus through the interaction hamitonian $H_{int} = -\lambda F \cos\omega t$ (λ is the strength of the external field, F is the most general 1-body operator):

$$\delta \langle \Psi(t) | H_0 - \lambda F \cos\omega t - i \frac{\partial}{\partial t} | \Psi(t) \rangle \tag{1}$$

$|\Psi(t)\rangle$ is a Slater determinant and H_0 is the nuclear hamiltonian. One now defines the linear response to the external field as:

$$\alpha^{TDHF}(\omega) = \lim_{\lambda \to 0} \left(\langle \Psi(t) | F | \Psi(t) \rangle - \langle HF | F | HF \rangle \right) / \lambda \tag{2}$$

where $|HF\rangle$ is the static umperturbed HF ground state. By expressing the solution of eq. (1) in terms of RPA solutions, one finds[1] the following remarkable result:

$$\alpha^{TDHF}(\omega) = \left(2 \sum_K \omega_K \frac{|\langle 0 | F | K \rangle|^2}{\omega_K^2 - \omega^2} \right)_{RPA} \tag{3}$$

The static limit ($\omega = 0$) of eq.(3) has been already discussed in ref.(2).
From a suitable ω-expansion [1,3] of the right hand side of eq. (3), it follows that $\alpha^{TDHF}(\omega)$ is related to RPA sum rules; for example when $\omega \to \infty$ one has:

$$\alpha^{TDHF}_{\omega \to \infty}(\omega) = -\frac{2}{\omega^2} \left(S_1^{RPA} + \frac{1}{\omega^2} S_3^{RPA} + \cdots \right) \tag{4}$$

while when $\omega \to 0$:

$$\alpha^{TDHF}_{\omega \to 0}(\omega) = 2 \left(S_{-1}^{RPA} + \omega^2 S_{-3}^{RPA} + \cdots \right)$$

The limit $\omega \to \infty$ is particularly advantageous because eq.(1) can be analitically handled[1]. An explicit expression for the state is obtained in this limit

$$|\Psi(t)\rangle_{\omega\to\infty} = \exp\left(i \frac{\lambda}{\omega} F \sin\omega t - \frac{\lambda}{\omega^2} G \cos\omega t\right) |HF\rangle \qquad (5)$$

G is a one-body operator which depends on the nuclear hamiltonian used; for the Skyrme force one has:

isoscalar modes: $\left(F = \frac{1}{2} \sum_i f(\vec{r}_i)\right)$

$$G = \frac{-i}{4m} \sum_i \left(\vec{\nabla}_i f(\vec{r}_i) \cdot \vec{p}_i + h.c.\right) = [K, F]$$

isovector modes: $\left(F = \frac{1}{2} \sum_i f(\vec{r}_i) \tau_i^3\right)$

$$G = -\frac{i}{4m} \sum_i \left[\vec{\nabla}_i f(\vec{r}_i) \left(1 + \frac{m}{2}(t_1 + t_2)\rho(\vec{r}_i)\right) \cdot \vec{p}_i + h.c.\right] \tau_i^3$$

The explicit knowledge of the state $|\Psi(t)\rangle$ (eq.(5)) permits a direct evaluation[1] of the sum rules S_1^{RPA} and S_3^{RPA} (eqs.(2),(4)) both for isoscalar and isovector modes. Finally we show that eq.(5) allows for the definition of a collective hamiltonian, through the evaluation of H_0 on the state $|\Psi(t)\rangle$:

$$E_{coll} = \frac{1}{2} \frac{1}{2 S_1^{RPA}} \left(\frac{d}{dt} \mathcal{F}\right)^2 + \frac{1}{2} \frac{S_3^{RPA}}{2 (S_1^{RPA})^2} \mathcal{F}^2 \qquad (6)$$

where $\mathcal{F} = \langle\Psi(t)| F |\Psi(t)\rangle - \langle HF| F |HF\rangle$ is the collective variable; from eq. (6) one can extract the mass and the restoring force constant of the collective mode excited by the operator F. Numerical values of the mean excitation energy $\bar{E} = \sqrt{S_3^{RPA} / S_1^{RPA}}$ are reported in table for different isovector modes. The interaction used is Skyrme III.

	Monopole	Dipole	Quadrupole
^{16}O	45	28	41
^{40}Ca	42	24	37

REFERENCES

1) S. Stringari, E. Lipparini, G. Orlandini, M. Traini and R. Leonardi to be published.

2) R. Marshalek and J. Da Providencia, Phys. Rev. C7, 2281 (1973)

3) S. Stringari, Nucl. Phys., A279, 454 (1977).