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ON THE TEMPERATURE DEPENDENCE OF THE IONIZATION RATES IN SEMICONDUCTORS

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The influence of temperature on the ionization rates, α and β, is studied by means of kinetic equation. The analysis incorporates a proposal for the $\alpha(E,T)$ determination which takes into account acoustic phonons. The results obtained agree satisfactorily with those of other authors.

While the field (E) dependence on the ionization rates of electrons (α) and holes (β) in semiconductors has been thoroughly studied in many papers, i.e.[1-8]etc., both theoretically and experimentally, it was not the case with temperature dependences, $\alpha(T)$ and $\beta(T)$, from the point of view of an explicit treatment. An explicit form of these dependences based on universal Baraff's curves[4] is given in [10]:

$$\alpha\lambda_0 = \exp[ax^2 + bx + c] \qquad (1a)$$

$$a = 11.5r^2 - 1.17r + 3.9\cdot 10^{-4};\ b = 46r^2 - 11.9r + 1.75\cdot 10^{-2};\ c = -757r^2 + 75.5r - 1.92, \qquad (1b,c,d)$$

where

$$r = \frac{\langle W_0\rangle}{W_I} \cong \frac{\hbar\omega_0}{C_T\cdot W_I};\quad C_T = \mathrm{cth}\frac{\hbar\omega_0}{2kT};\quad x = \frac{W_I}{qE\lambda_0};\quad \lambda_0 = \frac{\lambda_{00}}{C_T}, \qquad (1e,f)$$

$\hbar\omega_0$ being optical phonon energy, λ_0 the optical phonon mean free path and W_I - the ionization threshold energy, the three essential parameters of the theory.

Another approach consists of solving the kinetic equation of Davydov's type as in[3]), which leads to the relation of the form pointed out in [6,8,9]:

$$\alpha(E, q_i) = \frac{\Psi(E/E_0)}{\langle v\rangle} = A(q_i)\cdot\sqrt{C_T}\,\Psi\left(\frac{E}{E_0}\right),\quad E = E_{00}\sqrt{C_T} \sim \sqrt{C_T}, \qquad (2a,b)$$

where q_i stands for the temperature independent parameters $\hbar\omega_0$, λ_{00} and W_I, and the function $\Psi(E/E_0)$ depends only on the ratio (E/E_0). Thus, the temperature dependent quantity in eqn. (2a) (as well as in (1a)) is C_T and the construction of eqn. (2..) enables us to propose[11]) the procedure for the $\alpha(E,T)$ determination, knowing the parameter $\hbar\omega_0$ and $\alpha(E,T)$ dependence (which could be experimental one) for <u>only one</u> (ambient) temperature (T_1). Indeed, if the form (2a,b) is valid, knowing $\alpha(E,T_1)$ and $\hbar\omega_0$ the "universal" dependence $A\cdot\Psi(x_0) \neq F(T)$ can be determined:

$$A\cdot\Psi(x_0) = \frac{\alpha(E,T_1)}{\sqrt{C_{T1}}}, \quad \left(C_{T1} = C_T(T_1) = cth\,\frac{\hbar\omega_0}{2kT_1}; \quad x_0 = \frac{E_0}{E} \right) \tag{2c}$$

and at $T=T_2$, $\Psi(x_0)$ will have the same value as at $T=T_1$ for the field E_2 from

$$\frac{E_{01}}{E_1} = x_0 = \frac{E_{02}}{E_2} \quad \therefore \quad E_2 = \frac{E_{02}}{E_{01}}\cdot E_1 \equiv E_1\cdot\sqrt{\frac{C_{T1}}{C_{T2}}} \tag{2d}$$

or finally,

$$\alpha(E_2,T_2) = A\cdot\sqrt{C_{T2}}\cdot\Psi(x_0) = \alpha(E_1,T_1)\cdot\sqrt{\frac{C_{T2}}{C_{T1}}}\,. \tag{2e}$$

Both Baraff's and Chuenkov's[3]) treatments leading to eqns. 1a and 2a neglect, among other simplifying assumptions, the influence of acoustic phonon and all other scattering processes. Let us first examine this influence considering the kinetic equation and putting it in the following form (for the symmetric part, f_0, of the distribution function):

$$-\frac{d}{d\eta}\left[\frac{df_0}{d\eta}\,a(\eta) + f_0\cdot b(\eta)\right] = \sqrt{\eta}\cdot\left[g(\eta) - w_I(\eta)\cdot f_0 - w_r\cdot f_0\right], \tag{3a}$$

where the electron energy is related to W_I, $\eta = W/W_I$ and where

$$\frac{a(\eta)}{v_0} = \frac{1}{3}\left(\frac{qE}{W_I}\right)^2 \frac{\lambda(\eta)\cdot\eta}{1+(w_I+w_r)\cdot\tau(\eta)} + \frac{2mv_\ell^2}{W_I\cdot\lambda_{ac}}\cdot\eta^2 + \frac{C_T}{2\lambda_{00}}\left(\frac{\hbar\omega_0}{W_I}\right)^2\cdot\eta, \tag{3b}$$

$$\lambda_{00} = \lambda_0\cdot C_T = \frac{2\pi\hbar^4\rho v_\ell^2}{m^2\omega_0\hbar E_{10}^2} = \lambda_{ac}\,\frac{2kT}{\hbar\omega_0}\left(\frac{E_1}{E_{10}}\right)^2; \quad m^2 = m_\ell^{1/2}\cdot m_t^{3/2}, \tag{3c}$$

$$b(\eta) = v_0\,\frac{2mv_\ell^2}{kT\lambda_{ac}}\cdot\eta^2 + v_0\,\frac{\hbar\omega_0}{W_I\lambda_{00}}\cdot\eta; \quad \left(v_0 = \sqrt{\frac{2W_I}{m}}\right), \tag{3d}$$

$\tau(\eta) = \frac{\lambda(\eta)}{v}$ being the total relaxation time, while g, $w_I f_0$ and $w_r f_0$ are the electron generation, ionization and recombination rates, respectively.

The qualitative analysis of different terms in eqns. (3) must be done for $\eta \approx 1$, because for $\eta \gtrsim 2$ the function f_0 decreases rapidly and for $\eta < 1$ there is no ionization at all. Thus, putting approximately, (for Si)

$v_\ell \approx 10^4$ m/s, $C_T \approx 1$, $\hbar\omega_0 = 2kT_{amb} \approx 0.06$ eV, $W_I \approx 1$ eV and[4]) $\frac{E_1^2}{E_{10}^2} \approx 2 \div 0.25$
we can see that the second and third members in (3b) are

$10^{-6}/\lambda_{ac}$ and $10^{-3}/\lambda_o$, and since

$$\frac{\lambda_o}{\lambda_{ac}} = \frac{2kT}{C_T \cdot \hbar\omega_o} \cdot \left(\frac{E_1}{E_{10}}\right)^2 \equiv \varkappa \frac{kT}{C_T} \approx 2 \div \frac{1}{4} \quad \text{at } T_{amb} = 300\ K, \qquad (3e)$$

the second member can always be neglected, while the first one is preponderant for the field $E \geqslant 10^5$/V/cm, just in the range of interest for silicon. Assuming further that all other processes, except optical and acoustic phonon scattering are of little importance and that (for $\eta \geqslant 1$) $w_r \ll w_I$ but $1 \gg w_I \tau$ eqn. (3b), using (3e) and (1f) can be rewritten as

$$a(\eta) \approx \frac{\vartheta_o}{3}\left(\frac{2E}{W_I}\right)^2 \cdot \frac{\lambda_{oo}}{C_T + \varkappa(kT)} \cdot \eta, \qquad \varkappa = \frac{1}{kT_{amb}} \cdot \left(2.5 \div \frac{1}{3}\right). \qquad (3f)$$

If only optical phonons scattering is actual ($\varkappa = 0$), the characteristic field E_o is as in (2b): $E_o \sim \sqrt{C_T}$, if not -eqn. (3f) gives us

$$E = E_{oo}\sqrt{C_T + \varkappa \cdot (kT)}. \qquad (3g)$$

There is no more temperature dependent terms in (3a) because $\lambda_{ac} \sim 1/kT$ (except eventually in the relation for ionization probability w_I), and eqns (2d) and (2e) still hold with the transition

$$C_T \longrightarrow (C_T + \varkappa kT), \qquad (3h)$$

since α is inversely proportional to the drift velocity $\langle v \rangle \sim (C_T + \varkappa kT)^{-1}$.

Starting from the numerical values of the parameters listed in Table 1 for silicon, taking the dependence (1a) as "exact" one for $T = T_1 = 300$K, the family $\alpha(E, T_2)$ for $T_2 = 100$, 213 and 400 K is displayed using (2d,e) (with $\varkappa = 0$). As one may see from Fig. 1 the agreement with the experimental and theoretical results after [10]) is very good in the entire field range (200 to 500 kV/cm), and especially for higher fields, proving once again that our assumption (3f) is reasonable.

In order to take into account the influence of acoustic phonon scattering, using the transition (3h) in eqns. (2d, e) and considering again the dependence (1a) as exact for 300K, the dependence $\alpha(T)$ with the field as the parameter is calculated for $\varkappa kT_{amb} = 3$, 1/3, 1/19, and 1/38. The changes in α are negative (see Table 2 and Fig. 2) - quite in accordance with the increase of the characteristic field E_o after (2b) ($C_T + \aleph kT > C_T$). Judging from the original data in[10]) given in Fig. 1, the experimental points for T=loo and 213K are always about 4 to 5.7% below of the theoretical curves ($\aleph$=o), which may be explained just with the influence of

TABLE 1

Parameter	n-Si	p-Si	Units
λ_{oo} optical phonon mean free path	76	55	Å
W_I, ionization energy	1.2		eV
$\hbar\omega_R$, optical phonon energy	0.063		eV
v_ℓ, longitudinal velocity of sound	9.00		km/s
p_o, ionization parameter after 3a)	200		-

acoustic phonon scattering with the parameter $\aleph kT_{amb}$ laying between 1/19 and 1/3 (see Table 2; similar results are obtained for p-Si).

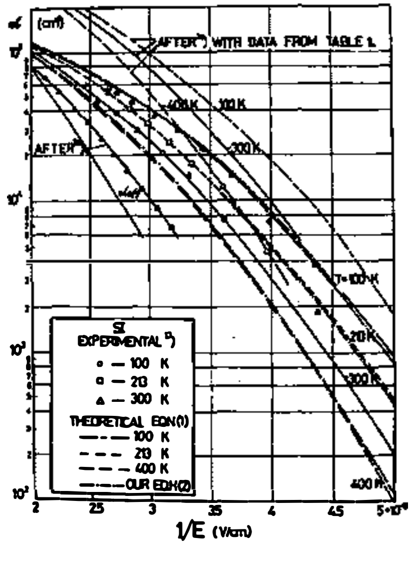

Fig. 1. The $\alpha(1/E)$ dependence with temperature as the parameter, after different approaches.

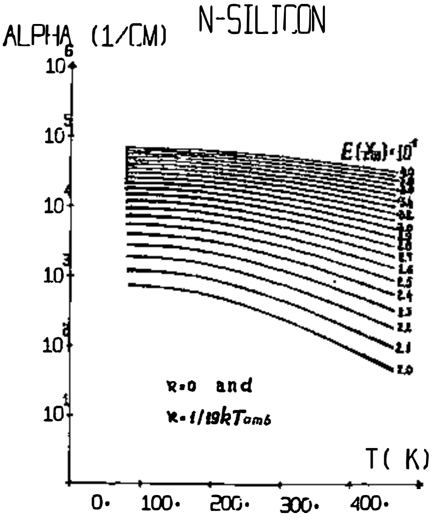

Fig.2. α versus temperature for the electric field as a parameter: $\aleph kT_{amb}$=0 and 1/19.

Table 2 $\Delta\alpha/\alpha_{0}$ (%) for n - Si

		$E(10^5 V/cm)$	3	3.5	4	5	6
T=100 K	$\varkappa kT$	0	1.15	0.1	-0.36	-0.30	-0.18
		1/38	-1.9	-2.2	-2.1	-	-
		1/19	-1.3	-1.4	-1.3	-	-
		1/3	-36	-26	-20	-	-
		3	-178	-113	-78	-	-
T=293 K	$\varkappa kT$	0	0.85	-0.21	-0.04	-0.08	0.07
		1/38	-0.15	-0.53	-0.62	-	-
		1/19	-0.13	-1.4	-1.3	-	-
		1/3	-1o	-7.8	-6.1	-	-
		3	-39	-28	-21	-	-

In conclusion we remark that our proposed method for the $\alpha(E,T)$ evaluation wiht only one parameter known ($\hbar\omega_0$) according to eqns. (2) is eepecially convenient for the new material research. From Figs 1 and 2 and quantitative data in Table 2 it is evident that the role of acoustic phonon in ionization processes is important only in the precise analyses.

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