F I Z I K A V O L U M E 10 S U P P L E M E N T 2, 1978
Proceedings of the VI Yugoslav Symposium of the Physics of Condensed Matter, Kruševac, September, 18-22, 1978

ONE-DIMENSIONAL MAGNETS

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Abstract. A review of theoretical results for the one-dimensional magnetic model systems, with a particular emphasis on effects of the lattice compressibility on the thermodynamic behaviour of the spin system, is presented.

"When a theoretical physicist is asked, let us say, to calculate the stability of an ordinary four-legged table he rapidly enough arrives at preliminary results which pertain to a one-legged table or a table with an infinite number of legs. He will spent the rest of his life unsuccessfully solving the ordinary problem of the table with an arbitrary, finite, number of legs".
(From "Physicists continue to laugh" MIR Publishing House, Moscow 1968)

One-dimensional magnetic model systems have been studied by theorists for more than fifty years [1]. However, there is no more than a decade since experimentalists have found systems which satisfactory mimic theoretical models, and suddenly almost all models turned out to be relevant. These systems are composed of one-dimensional (1d) chains of magnetic ions, so that the exchange interactions whithin a chain is much stronger than the interaction between chains. In some systems this discrimination of the intra- and inter--interaction occurs naturally, but in most of them it has been "engineered". For instance, $CoCl_2 \cdot 2H_2O$ is not a satisfactory 1d magnet, but replacement of the H_2O molecules by the large pyridine molecules pushes the Co^{2+} ion chains further apart and brings about [2] a good 1d magnet $CoCl_2 \cdot 2NC_5H_5$.

There are several comprehensive reviews [3,4,5] of experimental and theoretical results obtained so far for the 1d magnetic model systems, and we will not try to complement them. There is a very nice review [6] of the theoretical results exclusively. Here we shall try to supplement the last review [6] by a summary of results obtained for the compressible 1d magnetic model systems. It means that we are concerned with effects of the thermall fluctuations of distances between neighbouring spins on the magnetic behaviour of the spin system, assuming that the exchange interaction is a function of the distance. We shall restrict our discussion to the Ising model case, as results for the spin 1/2 Heisenberg models are either pertinent to the low-temperature phenomena of the magnon--phonon interaction or they are of a approximate character. In other words, there are no exact and complete statistical mechanical solution of the problem. This is understandable on the grounds of lacking such a solution for the 1d Heisenberg system on a rigid lattice. However, we should mention two papers [7,8] by Barma, who solved the 1d compressible classical (spin $=\infty$) Heisenberg model and investigated the phase transitions at the absolute zero (T=0).

Mattis and Schultz were the first to treat the 1d compressible Ising chain. In the appendix of their paper [9] on "magneto-thermomechanics" they found a transformation which decouples the lattice and spin variables in the Hamiltonian

$$\mathcal{H} = \mathcal{H}_L - \sum_{\langle m,n \rangle} J(X_m - X_n)\, \delta_m \delta_n \qquad (1)$$

Here $\mathcal{H}_L$ contains kinetic and elestic energy of the lattice, the sum goes over the nearest neighbours, X_m is a instantaneous position of the m-th particle of the lattice, and δ_m is the Ising spin variable ($\delta_m = \pm 1$), whereas the exchange coupling is assumed to be the linear function of the distance

$$J(X_m - X_n) = J_0 - J_1(X_m - X_n - a_0) \, , \qquad (2)$$

with a_0 being the distance between the equilibrium positions. It turned out [9] that the effective spin Hamiltonian, apart from a constant term, is equivalent to an Ising chain Hamiltonian on a rigid lattice, with exchange coupling equal to J_0. Therefore, no effect of the spin-lattice coupling was found.

While Mattis and Schultz assumed the free-ends boundary conditions and a phonon spectrum corresponding to nearest-neighbour harmonic interactions, Enting [10] adopted the Einstein phonon spectrum and the periodic boundary conditions. He demonstrated that the effective spin Hamiltonian is equivalent to a rigid Ising chain Hamiltonian with nearest and next-nearest-neighbour interactions. In addition, he investigated "disorder points", i.e. temperatures at which the character of the spin-spin correlation function changes [11].

A notable contribution in this field is a work by Salinas[12]. He studied the 1d compressible Ising chain for two different cases of boundary conditions - the fixed forces case and the fixed volume case. Salinas proved that the resulting free energies are related by a standard Legendre transformation. In particular, he clarified an error in the paper by Bolton and Lee [13], who claimed that the result of Mattis and Schultz [9] was a by-product of their general treatment of the problem. However, it proved to be an incorrect conclusion on account of different boundary conditions adopted in the two papers [9,13].

Since it was experimentally verified [3,4] that 1d magnets, resembling Ising chains, exist embedded within 3d lattices and with very weak inter-chain interaction, Mijatović and Milošević [14] were motivated to study a linear Ising chain (and stacks of non-interacting chains) in the 3d compressible simple cubic lattice. Assuming the harmonic potential energy with the Debye phonon spectrum, separation of the lattice and spin degrees of freedom was accomplished and the following effective spin Hamiltonian was obtained

$$\mathcal{H}_{eff} = -J_0 \sum_{\langle m,n\rangle} \sigma_m \sigma_n - \frac{J_1^2}{2\Phi_2} \sum_{\substack{\langle i,j\rangle \\ \langle m,n\rangle}} W(i,m)\,\sigma_i \sigma_j \sigma_m \sigma_n \,, \qquad (3)$$

where Φ_2 is the second derivative of the potential energy and W(i,m) is an oscillating and decreasing function of the distance $(m-i)\cdot a_0$ between the spin pairs. As $J_1^2/2\Phi_2$ is in reality much smaller than Jo, Mijatović and Milošević[14] treated the four-spin interaction as a perturbation. Thus they found that the effect of the lattice compressibility is to increase the specific heat peak and to shift it to lower temperatures. Afterwards Djordjević and Milošević [15] found an almost exact way to tackle the statistical mechanics of the Hamiltonian (3), and thereby they confirmed that the magnetic specific heat peak is shifted to lower temperatures, but it is decreased rather than increased. Hence it was demonstrated that the counter-effect, which was obtained [16] by regarding the function W(i,m) as a positive constant, was a wrong prediction (cf. Fig. 1).

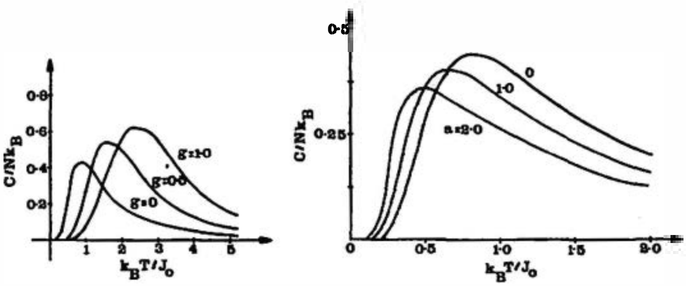

Figure 1. (a) The specific heat of the Hamiltonian (3) according to Ref. 16, where W(i,m) was supposed to be a constant $J_1^2 W(i,m)/2\Phi_2 \equiv J_4/N$. The parameter g is the ratio $J_4/^1 J_0$ and N is the number of spins in the chain. (b) The specific heat of the same Hamiltonian according to Ref. 15 $(a = J_1^2/2\Phi_2)$.

At the end we would like to mention that Knežević and Milošević [17] have recently solved the 1d compressible Ising model with spin S = 1 and S = 3/2. We should also mention two papers which treat models that are similar to 1d compressible Ising chains. Jasnow and Wagner [18] studied two planar Ising models on a compressible lattice whose elastic forces were one-dimensional (like the lattice of rods in the table-hockey game). Figueiredo et al.[19] have solved a one-dimensional version of one of the Jasnow-Wagner models, i.e. they-solved the thermodynamic problem of a system of two compressible Ising chains that are coupled with rigid vertical rods and with exchange interactions along the rods and diagonally between every two rods.

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