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SPHERICAL ISING MODEL WITH REGULARLY SPACED DEFECTS

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1. The critical behaviour of magnetic systems with different type of defects has been the subject of many investigations. The defects are normally randomly distributed in the crystal. The difficulties in treating random systems are such that exact results are very few. For finite densities McCoy and Wu [1] have solved the two-dimensional Ising problem with random distribution of infinite line defects and Rauh [2] has solved the spherical model of randomly distributed layers of spins. Some limiting results are available [3] at very small densities. In several publications, Fisher and his co-workers have solved exactly the two-dimensional Ising model with line [4] or point [5] defects of finite density which are periodically distributed throughout the lattice.

2. In this paper we consider an infinite simple cubic lattice (with lattice constant a) with finite density of defects which are regularly distributed through the lattice. Let the spacing between the defects be ma, na, pa along the x, y, z -axes respectively, where m, n, p are integers. Thus the defects themselves form a regular lattice. The density of defects is $c=(mnp)^{-1}$.

We adopt a Hamiltonian of the Ising type

$$\mathcal{H}=\mathcal{H}_o-\sum_{\{d.p.\}} J'_{\vec{l}\vec{l}'}\sigma_{\vec{l}}\sigma_{\vec{l}'} \qquad (1)$$

where $\sigma_{\vec{l}}$ is the spin at site $\vec{l}$, $\mathcal{H}_o$ is the Ising Hamiltonian of the system without defects and

$$J'_{\vec{l}\vec{l}'}=J_{1\vec{l},\vec{l}'}-J_{o\vec{l},\vec{l}'}$$

where $J_{o\vec{l},\vec{l}'}$ is the exchange interaction between host particles at sites $\vec{l}$, $\vec{l}'$ and $J_{1\vec{l}\vec{l}'}$ is the exchange interaction between an impurity at $\vec{l}$ and a host particle at $\vec{l}'$. The summation in (1) proceeds over all distinct pairs $\{d.p.\}$ of particles. We assume that J_o and J_1 are even functions dependent on $\vec{l}-\vec{l}'$ only.

An analytical treatment of the above problem is possible if we allow a continuous variation of the spins subject to the spherical constraint [6].

$$\sum_{\vec{\ell}} \sigma_{\vec{\ell}}^2 = N$$

where the summation is over all the lattice sites.

The partition function can be evaluated after diagonalizing the Hamiltonian with a Fourier transform [7] and applying the method of steepest descent [6, 7]. The long-range order sets in below the critical temperature T_c which is determined from the saddle-point equation

$$\frac{\hat{J}(0) + c\hat{J}'(0)}{kT_c} = \frac{a^3}{(2\pi)^3} \iiint \frac{d\vec{k}}{1 - \dfrac{\hat{J}_0(\vec{k}) + c\hat{J}'(\vec{k})}{\hat{J}_0(0) + c\hat{J}'(0)}} \tag{2}$$

where $\hat{J}(\vec{k}) = \sum_{\vec{\ell}} J(\vec{\ell}) \cos \vec{k}\vec{\ell}$

respectively for $\hat{J}_0(\vec{k})$ and $\hat{J}'(\vec{k})$. The integration in (2) is taken over the first Brillouin zone and k is the Boltzmann constant.

3. The critical properties of the system remain unaltered by the presence of defects. This has been observed in [5] and is probably due to the translational symmetry of the model which is still present when the defects are regularly distributed. The critical temperature, as can be seen from (2), is a function of c only and does not depend on m, n, p separately, as in [5].

If the exchange interaction is with nearest neighbours only and is isotropic for both J_{0i} and J_{1i} where i denotes the x, y, z -axes, the integral in (2) is independent of c and we have a linear relation between the critical temperature and the concentration of defects

$$T_c = \mathrm{Const}\left[J_0 + c(J_1 - J_0)\right]$$

The linear relationship remains even if we include interactions with further neighbours but keep $J'(\vec{\ell}) = J_1(\vec{\ell}) - J_0(\vec{\ell})$ proportional to $J_0(\vec{\ell})$ i.e.

$$J'(\vec{\ell}) = \alpha J_0(\vec{\ell}), \quad \alpha = \mathrm{Const}.$$

In the case of anisotropic interactions $T_c(c)$ is a non-linear function. For some particular values of the interaction constants, integrals of the type (2) have been evaluated [7].

Non-analytic dependence on c can occur in so-called Lifshitz points [8]. Such behaviour can be achieved if we include interaction with the second nearest neighbours along one of the axes. For example, let

$J_{0i} = J_0;\ J_{1x} = J_{1y} = J_0;\ J_{1z}(a\vec{n}_z) = J_1;\ J_{2z}(2a\vec{n}_z) = J_2.$

The Fourier transform in (2) becomes

$$\hat{J}_0(\vec{k}) + c\hat{J}'(\vec{k}) = 2J_0(\cos k_x a + \cos k_y a + \cos k_z a) + 2c(J'\cos k_z a + J''\cos 2k_z a)$$

where $J' = J_1 - J_0$, $J'' = J_2 - J_0$. Using limiting results [8] for the behaviour of (2) we obtain

$$T_c = T_L + \begin{cases} \text{Const}\,\sqrt{c_0 - c} & c < c_0 \\ \text{Const}\,(c - c_0) & c_0 < c \end{cases}$$

where the critical concentration c_0 is found from

$$J_0 + c_0(J' + 4J'') = 0.$$

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