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A SIMILARITY BETWEEN AN ISING CHAIN IN THE THREE-DIMENSIONAL PHONON FIELD AND SOME MODELS OF SPIN GLASSES

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Abstract. Certain similarities between an Ising chain embedded in a three-dimensional compressible lattice and spin glasses are observed. In particular, it is demonstrated that an Ising antiferromagnetic chain with nearest and next-nearest neighbour interactions, imitating the effective spin model of the compressible Ising chain, has spikes in the zero-temperature entropy similarly to some one-dimensional models of spin glasses.

Systems with dilute magnetic impurities in a non-magnetic host metal have been termed spin-glasses. Such a system has, at the so-called freezing temperature T_f, a sharp cusp in the zero-field magnetic susceptibility, whereas its specific heat shows a broad maximum at a temperature $T_m \gtrsim T_f$ (see, e.g., the review by K. Fisher[1]). In a finite magnetic field the susceptibility peak is rounded as well. Below T_f each spin, associated with the magnetic impurity, is expected to be locked in a particular direction, but these directions are randomly distributed so that there is no net total magnetization. The properties of spin glasses may be ascribed to the magnetic interactions between impurities. It is considered that the most important should be the Rudermann - Kittel - Yosida interaction, whose exchange function is of a long-range oscillatory form[1)].

Without any pretension to study spin glasses Mijatović and Milošević[2)] have investigated the influence of the spin-lattice coupling on the thermodynamic behaviour of the magnetic subsystem in the case of a three-dimensional lattice and one-dimensional Ising chain. It was found that the effective spin Hamiltonian corresponds to an Ising Hamiltonian with a pseudo-field, proportional to the original excahange interaction, and with an effective exchange interaction of a long-range oscillatory form. According to the perturbative treatment[2)] of the effective Hamiltonian, the corresponding specific heat peak, compared with the specific heat of the rigid Ising chain, is shifted to lower temperatures, whereas magnetic susceptibility may have a sharp maximum at low temperatures as well. A more rigorous treatment[3)] of the same Hamiltonian proved that the specific heat peak is reduced and shifted to lower temperatures. Thus, the two firmly established facts, the long-range oscilatory interaction and a broadened specific heat, resemble the properties of spin glasses.

On the other hand there have appeared[4-6)] theoretical models of spin glasses which assume an effective Hamiltonian with spins distributed regularly on the lattice sites but with the exchange interaction equals $\pm$ J randomly for nearest neighbours and zero otherwise. Such a system situated on a one-dimensional lattice has peculiar behaviour of entropy at low temperatures. Its entropy has a series of spikes for various values of the magnetic field at the zero temperature. These spikes turn into sharp maxima for non-zero temperatures. The largest spike appears at H=2J. As the existence of the entropy maximum was crucial in explaining[3)] the specific heat behaviour of an Ising chain embedded in a three-dimensional compressible lattice, it is of interest to see whether the corresponding entropy in fact has more than one maximum. If it proves to be so it will be one additional similarity with the spin glasses.

Since the effective exchange interaction of the Hamiltonian describing the compressible Ising chain was found to have a predominant antiferromagnetic part[2,3)], we will study here an Ising antiferromagnetic chain with both nearest and next-nearest exchange interactions and in the field H

$$\mathcal{H} = J_1 \sum_i \sigma_i \sigma_{i+1} + J_2 \sum_i \sigma_i \sigma_{i+2} - H \sum_i \sigma_i \ , \tag{1}$$

where J_1 and J_2 are assumed to be positive. Thermodynamics of this system can be determined by using the Montroll-Dobson method[7)] for the many-neighboured Ising 'hain. According to this method system is divided into blocks so that each block contains two spins (n spins in the case of n-neighboured interaction), and the transfer matrix is formed of elements

$$V_{c_k c_l} = \exp\left\{-\beta\left(\tfrac{1}{2} X_{c_k} + Y_{c_k c_l} + \tfrac{1}{2} X_{c_l}\right)\right\} \equiv \exp\left\{-\beta \tilde{\mathcal{H}}_{c_k c_l}\right\} , \tag{2}$$

where sufficies c_k and c_1 denote configurations of the k-th and 1-th block in the chain, X_{c_k} and X_{c_1} are the corresponding energies of these blocks, while $Y_{c_k c_1}$ is the coresponding mutual energy; β is the reciprocal of the product of temperature T and the Boltzmann constant k_B. The partition function of the system is determined by the largest eigenvalue λ_{mV} of $\hat{V}$, i.e. $Z \sim \lambda_{mV}^N$ where N is the number of blocks in the chain. In our case V is a 4x4 matrix.

The elements $\tilde{\mathcal{H}}_{c_k c_1}$ are linear function of H, and thereby it is not difficult to determine the smallest one in a particular region of H, for given values of J_1 and J_2. When T goes to zero we will multiply and divide each matrix element of $\hat{V}$ by the largest of them, which is obviously related to the smallest $\tilde{\mathcal{H}}_{c_k c_1}$. Hence, when $T \to 0$ the matrix $\hat{V}$ can be written in the form

$$\hat{V} = \max \{ V_{c_k c_\ell} \} \hat{A} , \tag{3}$$

where A is a "sperse" matrix, whose elements are either zero or one. It can be verified that the entropy is then given by formula

$$S = \frac{1}{2} k_B \ln \lambda_{mA} \tag{4}$$

where λ_{mA} is the largest eigenvalue of the sparse matrix $\hat{A}$. Therefore, in order to see whether entropy has spikes at particular values of H it is necessary to look for eigenvalues of $\hat{A}$ that are larger than one.

Taking care that only a cyclical sequence of the block configurations, which corresponds to a cyclical sequence of the matrix elements of A different from zero, may build up a state of all spins in the chain, it can be shown that depending on the value of H there are five possible forms of the sparse matrix to be studied for each case $2J_2 > J_1$ and $2J_2 < J_1$. Three of them in each group, have the largest eigenvalue equal to 1 and thus according to (4) the corresponding entropy is zero.

For $2J_2 > J_1$ there are two particular values $H = 2J_2 - J_1$, and $H = 2(J_1 + J_2)$ for which entropy can be different from zero at $T = 0$. The respective equations for determining eigenvalues of the sparse matrix are

$$\lambda^4 - 2\lambda^2 - \lambda + 1 = 0 \tag{5}$$

and

$$\lambda^3 \quad \lambda^2 - 2\lambda - 1 = 0 . \tag{6}$$

A simple analysis of these equations can demonstrate that their largest solutions are larger than 1, and thereto equation (6) has a larger solution than equation (5).

For $2J_2 < J_1$ the two critical values of field are $2J_1-4J_2$ and $2(J_1+J_2)$. The respective eigenvalue equations are

$$\lambda^3 - 2\lambda^2 + \lambda + 1 = 0 \tag{7}$$

and

$$\lambda^3 - \lambda^2 - 2\lambda - 1 = 0 . \tag{8}$$

Again, a simple analysis of these equations demonstrates that their largest solutions are larger than one, and in addition the entropy spike is larger for $H = 2(J_1+J_2)$ than for $H = 2J_1 -4J_2$.

We may conclude that an additional similarity between the spin glasses and the model of an Ising chain embedded in a three--dimensional compressible lattice has been established. Of course, it would be interesting to demonstrate the existence of the entropy spikes for an Ising antiferromagnet with a longer exchange interaction. However, such a demonstration could not be analytical likewise the presented one, as it would require to tackle the eigenvalue problem of 8x8 and larger matrices. A computer calculation[8)] indicates existence of spikes in these cases as well.

One might speculate that the above-mentioned similarities are due to the fact that the three-dimensional phonons cause similar disordering effects on the local ordering of spins as the chaotic arrngement of spin bonds in the case of an absolutely rigid lattice. However, only further investigations of both systems should show whether there is a deeper significance of these similarities or they will turn out to be fortuitous. As for the compressible Ising chain the present work represents an analytical support for the clarification of the entropy and specific heat behaviour. Besides, it appears to be an extension of previous treatments of the zero-temperature entropy for Ising antiferromagnets which take into account only the nearest-neighbour interaction.

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