

A correlation of low temperature resistivity of normal
 metals with the anisotropy of electron-phonon scattering

Rudolf Krsnik and Emil Babić

Institute of Physics of the University, Zagreb

The behaviour of the low temperature resistivity ($\rho = \rho(\rho_0, T/\theta_D)$, where θ_D is Debye characteristic temperature and ρ_0 is residual resistivity) is different both for the different pure metals and for their dilute alloys^{1,2}. In the "pure limit", i.e. for $\rho_0 < \rho'_0$ (ρ'_0 is "breakdown resistivity") the relation $\rho_T - \rho_0 = B_1 T^5$ holds for all the systems which have been investigated. With addition of nonmagnetic impurities an "impure region" ($\rho_0 > \rho'_0$) is reached, where for a constant temperature $\rho_T - \rho_0$ is increasing with ρ_0 . There is no universal law for different metals. In the empirical relation^{3,2}

$$\rho_T - \rho_0 \propto T^n \cdot \log \frac{\rho_0}{\rho'_0(T)} \quad (1)$$

n varies from 3 (Al³, Ga) to 5 (Zn⁴). The logarithmic dependence on ρ_0 is universal up to the highest values of ρ_0 achieved in dilute region, but with different values of $\rho'_0(T/\theta_D)$ and different slope ($\rho_T - \rho_0$) per decade of ρ_0 for different metal systems.

So far no theory explain in detail all these differences among the various systems. But the approach, based on the differences in the anisotropy of the Fermi surfaces and consequently on different contribution of the "umklapp"(U) electron-phonon scattering in different metals, seems promising^{5,6,7}. Particularly important seems to be the result⁵ that strong U processes contribute to the temperature variation of resistivity with $n=5$ for $T/\theta_D < 0.05$ followed by a somewhat slower variation ($n=2$ and 4) at higher T.

With this general picture in mind we will compare the existing experimental results for the resistivity of different metals (most of them appeared in ref. 2) to see whether they can be related to the strength of the low temperature U processes in electron-phonon scattering.

In Table 1 we compare for different systems the calculated coefficient (B_N) of the Bloch-Grüneisen resistivity term⁸ (which includes only normal (N) processes and does not include U processes) with the coefficients B_i of the measured T^5 resistivity term² in "pure limit". In the systems with a strong intrinsic anisotropy of el.-ph. scattering the contribution of U processes in B_i will be large and therefore we expect $B_i \gg B_N$. In spite of a considerable uncertainty in the intrinsic resistivities ($\rho_{T-\rho_0}$)_i and θ_D values, the ratio B_i/B_N is bigger for Zn, Pt and Sn i.e. for the systems where a pure T^5 dependence is shown (or at least indicated) and which also have the most anisotropic Fermi surfaces. For Al and the noble metals B_i/B_N is much smaller although still greater than 1 which indicates that also in these metals the contribution of the U processes does not vanish even at the lowest temperature.

Furthermore these results imply that the intrinsic resistivity $\rho_i(T/\theta_D)$ should be relatively higher for those metals which have the more anisotropic Fermi surface (i.e. where low temperature U processes are relatively stronger). This is really so, as one can see in Fig.1 where ρ_i is plotted vs T/θ_D for all the systems where it was determined with reasonable accuracy².

So far we have considered only the parameters (B_N , B_i , ρ_i) which deal with intrinsic resistivity, i.e. without any influence of impurities. The role of impurities (for $\rho_0 > \rho_0'$) should be to isotropize (gradually remove) the intrinsic anisotropy of el.-ph. scattering,

Table 1: Data relevant to the investigated systems.

θ_D is the Debye temperature at $T=0K$, B_N is the coefficient of the Bloch-Gruneisen resistivity term, B_i is the coefficient of a T^5 term in the resistivity (in pure limit) and T is the temperature at which B_i is taken.

	$\theta_D(K)$	$B_N(10^{-18}\Omega_m/K^5)$	$T(K)$	$B_i(10^{-18}\Omega_m/K^5)$
Cu	340	2.2	15.7 23.4	2.6 2.4
Ag	225	9.6	10	11
Au	162	51	7.2	77
Zn	310	11	7 10 20	150 130 100
Al	428	1.5	20	2.2
In	111	800	4.2	2300
Sn	195	120	4.2	570
Pt	240	52	20.3	120

causing the appearance of an additional term in the temperature dependent part of the resistivity (or causing it to change). This change is rather different for the different systems. In Fig.2 we plotted the increase in the temperature dependent part ($\rho_T - \rho_0$) of the resistivity per a decade of ρ_0 in impure region vs T/θ_D for various metals. The result is rather obvious: The stronger the intrinsic anisotropy of Fermi surface, the larger is the increase of the resistivity in the impure region. The differences among various systems are pretty large: up to more than a factor 100.

The order of different systems in Fig.1 and Fig.2 as well as in Table 1 is essentially the same. This reflects the fact that the dominant mechanism of temperature dependent resistivity is the same both in the

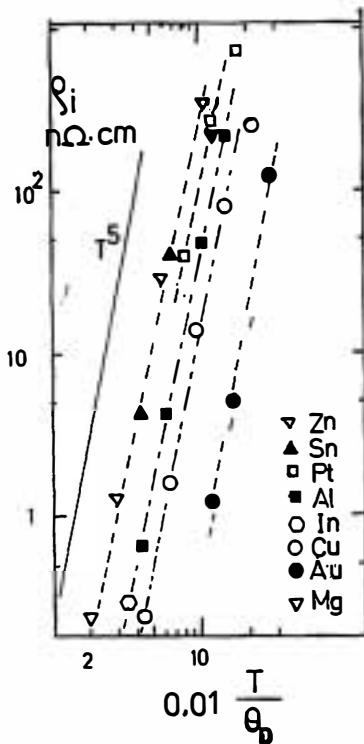


Fig. 1: Low temperature intrinsic resistivity deduced from "pure limit" for different metals.

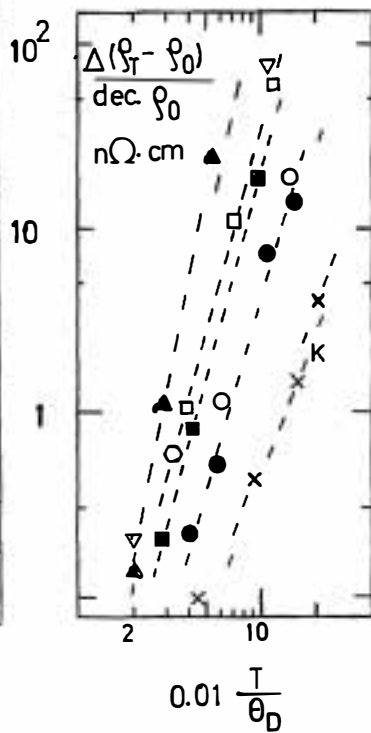


Fig. 2: Increase of temperature dependent resistivity $(\rho_T - \rho_0)$ per decade of ρ_0 vs T/θ_D for different systems.

pure limit and in the impure region. This could be better seen from Fig. 3 where we plotted the ratio of $\Delta(\rho_T - \rho_0)$ per decade of ρ_0 and ρ_i vs. T/θ_D for various metals. In spite of the fact that this ratio depends on T/θ_D in a somewhat peculiar way, there is a rather universal behaviour for different systems. It follows that impurities themselves make no essential change in the el.-ph. scattering. The influence of impurities is mainly to cause the appearance of an interference term between inelastic el-ph. scattering and elastic electron-impurity scattering. This interference is expected (i.e. becomes significant) when the impurity resistivity ($\rho_{\sigma} \rightarrow \rho'_0$) is of about the

same magnitude as the intrinsic resistivity. So, ρ'_0 should depend on T . Once that interference appeared it is stronger for larger ρ_i .

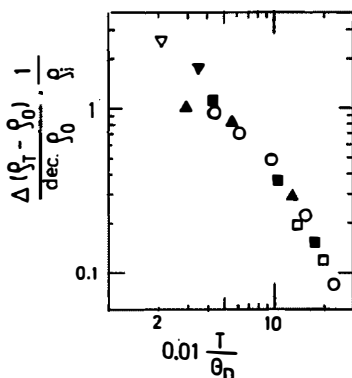


Fig.3: Ratio $\frac{\Delta(\rho_T - \rho_0)}{(\text{decade of } \rho_0) \rho_0} \frac{1}{\rho_i}$ vs. $\frac{T}{\theta_D}$ for different metals exhibits nearly universal behaviour.

As it was shown earlier² ρ'_0 increases approximately with the square of T/θ_D and it is higher for the more anisotropic systems. But because it is implicitly related to $\rho_i(T)$, there is a need for a physically more explicit picture. We hope that more relevant information could be given by the ratio ρ_i/ρ'_0 i.e. the ratio between effective fractions of el.-ph. and el.-imp. scattering in the situation when the interference scattering is just appearing. This ratio was plotted for various systems in Fig.4. We can see two important facts: a) For a given system ρ_i/ρ'_0 is increasing with T/θ_D . I.e. at lower temperature there should be relatively more scattering on impurities present (than on the phonons) for

the interference process to appear. The reason should be the smaller total amount of U processes in el.-ph. scattering. b) There are remarkable differences among different systems. $\rho_i/\rho'_0(T/\theta_D)$ is higher for the systems with more anisotropic el.-ph. scattering. I.e. there should be relatively less impurities present to influence the beginning of the isotropization process than in less anisotropic systems. Probably the reason is again higher total amount of intrinsic el.-ph. scattering, which allows earlier appearance of the interference term.

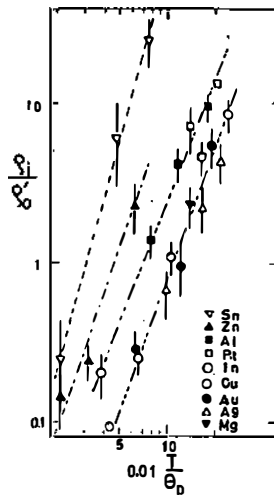


Fig.4: Ratio ρ_i/ρ'_0 of the intrinsic resistivity and "breakdown" resistivity for different metals vs. (T/θ_D) . Note that both the dependence on temperature and on anisotropy of intrinsic el.-ph. scattering support the same physical picture.

This plot could provide a slight correction to the previous ones. Namely, we do not know exactly whether large $\rho_i(T)$ always means large ρ_U . Consequently Fig. 1, 2 and 3 could give a less precise description of real situation. Fig. 4 contains physically more relevant data which possibly could give us a better idea about the concurrence of different scattering mechanisms in metals.

In conclusion we believe that all discussed parameters are in agreement with general theoretical approach based on anisotropy of Fermi surfaces.

References:

- 1) Bass J, (1972) Adv. Phys. 21, 431
- 2) Cimberle M.R., Bobel G., Rizzuto G., (1974) Adv. Phys. 23, 639
- 3) Caplin A.D. and Rizzuto G., (1970) J.Phys.C 3, L117
- 4) Salvadori E., Babić E., Krsnik R. and Rizzuto G., (1973) J. Phys. F 3, L195
- 5) Lawrence W.E. and Wilkins J.W., (1972) Phys.Rev.B 6 4466
- 6) Ekin J.W. and Bringer A., (1973) Phys.Rev.B 7, 4468
- 7) Kagan Ju. and Fleurov V.N., (1974) JETP 66, 1374
- 8) Ziman J.M., Electron and Phonons, Clarendon Press, Oxford (1960)