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PAIR CORRELATIONS IN ^{3}He

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It is by now generally belived that the low temperature phases of superfluid ^{3}He, i.e. ^{3}He-A and ^{3}He-B, correspond to the so-called Anderson - Morel (AM) [1] and Balian - Werthamer (BW) [2] states, respectively. Both AM and BW states are formed on the basis of the spin triplet pairing.

In the present work we analyze this type of pairing with the help of the Hartree - Bogoliubov (HB) theory formulated in terms of one-particle operators. This method, originally developped by Herbut and Vujičić, [3] introduces two basic trial operators, the density operator $\hat{\rho}$, and the pairing operator $\hat{P}_a$ defined by the properties

$$\hat{P}_a^{+} = \hat{P}_a^{-1} = -\hat{P}_a \tag{1}$$

In the simple case of BCS singlet spin pairing $\hat{P}_a$ is the time-reversal operator $\hat{T}_a$.

In the AM model the total spin of a pair is $S = 1$, the orbital angular momentum $L = 1$, and $S_z = \pm 1$. This is so-called equal spin pairing (ESP). The "up" and "down" spin particles form completely independent and non - interacting systems (at first sight at least). A possible choice for the pairing operator in the ESP state is $\hat{P}a = i\hat{C}$, where $\hat{C}$ is the conjugation operator. $\hat{P}a = i\hat{C}$ is an antiunitary involution, i.e. satisfies the conditions (1) and has the canonical form in the one - particle basis $\{|k\rangle\}$ formed of plane waves (PW). The Bogoliubov transformation for the present case is the special one

$$d^+_{\kappa z} = u_{\kappa z} a^+_{\kappa z} + v_{\kappa z} a_{-\kappa z} \qquad (2)$$

The density operator $\hat{\rho}$ is diagonal in the (PW) representation, and commutes with the pairing operator $\hat{P}a = i\hat{C}$ and with the correlation operator $\hat{t}_a = (\hat{\rho}-\hat{\rho}^2)^{1/2}\hat{P}a$, i.e. the Bogoliubov conditions[3] are satisfied when $\hat{P}a = i\hat{C}$. On the other hand, starting from the Hamiltonian

$$H = \sum_{\kappa} \varepsilon_{\kappa} a^+_{\kappa z} a_{\kappa z} - \sum_{\kappa,\kappa'} V_{\kappa\kappa'} a^+_{\kappa' z} a^+_{-\kappa' z'} a_{-\kappa z'} a_{\kappa z} \qquad (3)$$

it is easy to see that the set of operators $\hat{h}$, $\hat{\rho}$, $\hat{P}a$, $\hat{\Delta}_a$ (where $\hat{h}$ is the HB one-particle Hamiltonian and $\hat{\Delta}_a$ pairing potential operator) is commutative. The solution for the density operator matrix elements is of the BCS-like form[3].

However, the spin triplet pairing with $S_z = 0$ has been neglected in the AM model. The general triplet pairing, which includes this possibility was studied by Balian and Werthamer[2]. They introduced a canonical transformation coupling the electrons of opposite momenta and of both same and opposite spins

$$d^+_{\kappa z} = \sum_{z'} (u^{\kappa}_{zz'} a^+_{\kappa z'} + v^{\kappa}_{zz'} a_{-\kappa z'}) \qquad (4)$$

This transformation mixes four states, and thus is more general than the usual one. However, as Bloch and Messiah pointed out[4], each canonical transformation U can be regarded as pairing of the BCS-Bogoliubov type between two single particle states in an appropriate representation. In the present case

$$U_{gen} = U_{spec} \cdot U_1 \qquad (5)$$

where U_1 is the transformation leading to the representation in which $\hat{\rho}$ and $\hat{t}_a$ are in the canonical form, and U_{spec} is a special transformation one has to performe after U_1 in order to diagonalize the Hamiltonian. U_{spec} is given by

$$d_{k\lambda} = u_{k\lambda}\, b_{k\lambda}^{+} + v_{k\lambda}\, b_{-k\lambda} \tag{6}$$

where $b_{\pm k\,\lambda}$ are linear combinations of the spin-up and the spin-down annihilation operators, i.e. U_1 is given by

$$b_{\pm k\lambda} = A_{\pm k\lambda}\, a_{\pm k\uparrow} + B_{\pm k\lambda}\, a_{\pm k\downarrow} \tag{7}$$

and λ is a new quantum number to be defined below. The coefficients in (7) can be determined if one knows the pairing operator $\hat{P}a$, and vice versa. Taking $\hat{P}a = \hat{T}a$ we determine $A\pm k$, $B\pm k$ from the condition that $b_{k\lambda}^{+}$ and $b_{-k\lambda}$ create mutually time-reversed states. This gives the relations

$$\begin{aligned} B_{k\lambda}^{+} &= A_{-k\lambda} \\ B_{-k\lambda} &= -A_{k\lambda}^{+} \end{aligned} \tag{8}$$

which do not determine A, B in a unique way. A possible choice, corresponding to the helicity representation, is

$$\begin{aligned} b_{k+} &= \cos\frac{\theta_k}{2}\, e^{i\varphi_k} a_{k\uparrow} + \sin\frac{\theta_k}{2}\, e^{-i\varphi_k} a_{k\downarrow} \\ b_{k-} &= -\sin\frac{\theta_k}{2}\, e^{i\varphi_k} a_{k\uparrow} + \cos\frac{\theta_k}{2}\, e^{-i\varphi_k} a_{k\downarrow} \end{aligned} \tag{9}$$

where $\lambda = \pm$ is the helicity quantum number.

In fact, U_1 amounts then to the rotation

$$e^{i\theta_k \frac{1}{2}\tau_y}\; e^{-i\varphi_k \frac{1}{2}\tau_z} \tag{10}$$

in the spin space, specific for each k. θ_k and φ_k are the polar angles of the momentum k with the z - axis. The matrix elements of the pairing potential operator $\hat{\Delta}_a$ and of the correlation operator $\hat{t}_a$ are non-zero between the states of the same helicity only, i.e.

$$\begin{aligned} (\hat{\Delta}_a)_{k\lambda,k'\lambda'} &= (\hat{\Delta}_a)^{\lambda}_{k,-k}\, \delta_{\lambda,\lambda'} \\ (\hat{t}_a)_{k\lambda,k'\lambda'} &= (\hat{t}_a)^{\lambda}_{k,-k}\, \delta_{\lambda,\lambda'} \end{aligned} \tag{11}$$

In the special case of $\theta_k = 0$

$$(\hat{\Delta}_a)^{\lambda}_{k,-k} = \Delta^{BCS}_{k,-k} = \lambda\,\Delta_k$$

while for $\theta_k =$

$$(\hat{\Delta}_a)^{\lambda}_{k,-k} = \Delta^{ESP}_{k,-k} = \Delta_k$$

i.e. the general triplet pairing obtained with $\hat{P}_a = \hat{T}_a$ includes the simpler cases of the BCS and of the ESP pairing. Another possible choice of the pairing operator is $\hat{P}a = i\hat{C}$, but in this case the matrix elements of $\hat{\Delta}_a$ never reduce to the simpler form of the BCS or of the ESP pairing.

We conclude that the most general triplet pairing in the superfluid ^{3}He is on the basis of the time reversal operator $\hat{P}_a = \hat{T}_a$, in contrast to the ESP case, where $\hat{P}a = i\hat{C}$. Usually, the time-reversal based pairing is associated with the singlet pairing only; however we have shown explicitly that for the triplet pairing $\hat{P}_a = \hat{T}_a$, but the canonical basis is no more the (PW) basis. Finally, one should notice that the thermodynamically stable state (BW) state) is a non - ESP state for $L = 1$, and probably this holds for $L > 1$ also[(5)].

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