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DISPERSION IN A CLASSICAL ELECTRONIC PLASMA

Ž. Crljen
"Rudjer Bošković" Institute, 41001 Zagreb, Croatia, Yugoslavia
V. Šips
Faculty of Science and "Rudjer Bošković" Institute, 41001 Zagreb, Croatia, Yugoslavia

The random phase approximation (RPA) has been widely used to find an approximate solution of the many-body problem. Suhl and Werthamer[1,2] have shown how the most general linear solution for the ground-state energy and the elementary-excitation spectrum can be achieved starting from the RPA. In Ref. 3, we derived an expression for the electron dielectric function valid in the second-order RPA (SRPA), using the generalized RPA procedure of Suhl and Werthamer. In the present paper we have found a classical limit for long wavelength plasma oscillations. It is well known[4] that the damping of such oscillations is negligible at high frequency, and we do not consider it here. An important result is that collisions between particles (which is a classical equivalent to many-body excitations) included via SRPA, give rise to negative dispersion for a sufficiently dense plasma.

Let us start with the quantum expression for the dielectric function in SRPA, eq. (10) in Ref. 3:

$$\varepsilon(\vec{q},\omega) = 1 + V_{\vec{q}} \sum_{s\vec{k}} F^{(2)}_{\vec{k}\vec{q}} (N_{s\vec{k}+\vec{q}} - N_{s\vec{k}}) \ . \qquad (1)$$

$F^{(2)}_{\vec{k}\vec{q}}$ is the solution of the integral equation (8) in Ref. 3, obtained by second iteration, where $V_{\vec{q}}$ is the Coulomb interaction. To find a classical limit of the expressions under consideration, we make the following approximations:

i) We omit the exchange terms.

ii) We keep only the linear terms in the distribution function.

iii) We expand the quantum expressions $G(\vec{p}+\vec{q})$ as

$$G(\vec{p}+\vec{q}) = G(\vec{p}) + \vec{q}\,\frac{dG}{d\vec{p}} + \ldots \rightarrow G(\vec{v}) + \frac{\hbar\vec{k}}{m}\,\frac{dG}{d\vec{v}} + \ldots .$$

and in the final results we use $\hbar \to 0$.

By using the above-mentioned approximations and introducing the function $f(\vec{v}) = \hbar\, F^{(2)}_{\vec{k}\vec{q}}$, we find the equation for

$f(\vec{v})$ as

$$f(\vec{v}) = \frac{1}{\omega - \vec{q}\,\vec{v}} \{1 - \frac{1}{m^2} \sum_{\vec{q}'\vec{v}'} \frac{V_{\vec{q}'}\, n(\vec{v}')}{\omega + \vec{v}(\vec{q}' - \vec{q}) - \vec{v}'\vec{q}'}$$

$$\times \Big[V_{\vec{q}'}\, \vec{q}' \cdot \frac{d}{d\vec{v}}\, \vec{q} \cdot \frac{df(\vec{v})}{d\vec{v}} + V_{\vec{q}'-\vec{q}}\, \vec{q} \cdot \frac{d}{d\vec{v}'}\, (\vec{q}' - \vec{q}) \cdot \frac{df(\vec{v}')}{d\vec{v}'}$$

$$+ \frac{\vec{q}'(\vec{q} - 2\vec{q}')}{\omega + \vec{v}(\vec{q}' - \vec{q}) - \vec{v}'\vec{q}'} \, (V_{\vec{q}}\, \vec{q}\, \frac{df(\vec{v})}{d\vec{v}} + V_{\vec{q}'-\vec{q}} (\vec{q} - \vec{q}') \, \frac{df(\vec{v}')}{d\vec{v}'}\Big]\} \; . \tag{2}$$

Hence, the dielectric function in the classical limit is

$$\varepsilon(\vec{q},\omega) = 1 - V_{\vec{q}} \frac{1}{k_B T} \sum_{\vec{v}} f(\vec{v})\, \vec{q}\cdot\vec{v}\; n(\vec{v}) \;, \tag{3}$$

where $n(\vec{v})$ is the Maxwell-Boltzmann distribution function and k_B is the Boltzmann constant. Taking $f(\vec{v})$ to be $f(\vec{v}) = 1/\omega - \vec{q}\vec{v}$, we obtain the RPA expression for the dielectric function:

$$\varepsilon(\vec{q},\omega) = 1 - V_{\vec{q}} \sum_{\vec{v}} \frac{1}{\omega - \vec{q}\,\vec{v}} \frac{\vec{q}\vec{v}}{k_B T}\, n(\vec{v})' \; . \tag{4}$$

Expression (4) leads to the long wavelength plasma dispersion relation

$$\frac{\omega^2}{\omega_p^2} = 1 + \frac{q^2 \overline{v^2}}{\omega_p^2} \; . \tag{5}$$

To avoid unphysical divergences in eq. (2), we employ the screened Coulomb interaction $V_{\vec{q}} = 4\pi e^2/q^2 + q_s^2$, with the screened wave vector q_s equal to the Debye-Hückel screened wave vector $q_s = (4\pi N e^2/k_B T)^{1/2}$. A somewhat lengthy calculation for $f(\vec{v})$ from eq. (2) by second iteration gives the dispersion relation in SRPA in the small q limit:

$$\frac{\omega^2}{\omega_p^2} = 1 + \left[1 - 0.935 (N (\frac{e^2}{k_B T})^3)^{1/2}\right] \frac{q^2 \overline{v^2}}{\omega_p^2} + O(q^4) \, . \tag{6}$$

A qualitatively new result with respect to RPA is the appearance of a negative dispersion term in eq. (6). RPA is essentially a mean field theory, and SRPA is an extension of this, taking into account collisions between particles. To characterize the importance of collisions, we estimate a mean time τ between two collisions as follows:

Taking $e^2/k_B T$ as a classical "distance of closest approach", we obtain a crude estimate for the mean free path, $\lambda = 1/N\sigma$, with the cross section $\sigma \sim (e^2/k_B T)^2$. The relaxation time $\tau = 1/(\overline{v^2})^{1/2}$ gives for the product $\omega_p \tau$

$$\omega_p \tau \simeq \gamma^{-1} ,$$

with $\gamma = (4\pi N (e^2/k_B T)^3)^{1/2}$ as a plasma parameter. For $\gamma \ll 1$, the "kinetic" regime is established, where the relaxation time is much longer than the period of plasma oscillations, and collisions between particles do not change the RPA assumption markedly. However, for $\gamma > 1$, τ is shorter than ω_p^{-1} and we cannot expect any mean field theory to be valid.

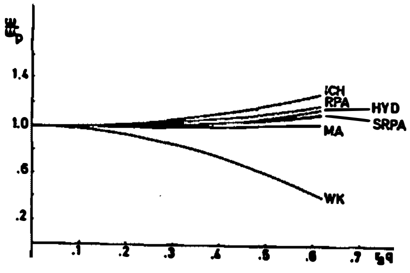

Fig.1

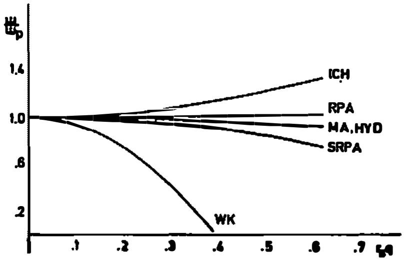

Fig. 2

Fig.1. The long wavelengths plasma dispersion relation. ICH stands for Ichikawa, HYD for hydrodynamic limit, MA for momentum analysis and WK for Wu and Klevans. Γ=0.993.
Fig. 2. Same as Fig. 1 for Γ=9.7.

In Figs. 1 and 2 we compare our results with the results of some other approaches. For clarity, let us introduce a new plasma parameter, $\Gamma = e^2/k_B T r_s = 3^{-1/3}\gamma^{2/3}$, where $r_s = (3/4\pi N)^{1/3}$ is the electron or ion sphere radius. Eq. (6) now reads:

$$\frac{\omega}{\omega_p} = 1 + \frac{1}{2\Gamma} q^2 - 0.234 \left(\frac{3}{\pi}\right)^{1/2} \Gamma^{1/2} q^2 + O(q^4)$$

with q as a wave vector in r_s units. This shows a change of dispersion for $\Gamma = \Gamma_o \sim 1.68$. To the best of the authors' knowledge, experiments on plasmas are performed with Γ several orders of magnitude smaller than Γ_o, which falls in the "kinetic" regime.

We quote some other results for comparison. Starting from BBGKY equations, Ichikawa[5] obtained the following expression in the limit of small q, including also binary correlations:

$$\frac{\omega}{\omega_p} = 1 + \frac{1}{2\Gamma} q^2 + 0.263 \left(\frac{3}{\pi}\right)^{1/2} \Gamma^{1/2} q^2 .$$

Willis[6], Wu and Klevans[7], performing plasma-parameter expansions, found

$$\frac{\omega}{\omega_p} = 1 + \frac{1}{2\Gamma} q^2 + 2.25 \left(\frac{3}{\pi}\right)^{1/2} \Gamma^{1/2} q^2 \qquad \text{(Ref. 6)},$$

$$\frac{\omega}{\omega_p} = 1 + \frac{1}{2\Gamma} q^2 - 0.659 \left(\frac{3}{\pi}\right)^{1/2} \Gamma^{1/2} q^2 \qquad \text{(Ref.7)} .$$

A positive contribution to the last dispersion term for all possible values of Γ would mean no difference in the plasma behaviour in passing from the "kinetic" to the "hydrodynamic" limit. A negative dispersion is known also from some other approaches. Momentum analysis of "molecular dynamics experiments" (i.e. computer simulation on a classical one-component plasma) and the hydrodynamic plasma limit[8-10] based on dynamical structure factor calculations gave for the dispersion relation, respectively,

$$\frac{\omega}{\omega_p} = 1 + \frac{q^2}{6\Gamma} - \frac{3^{1/2}}{24} \Gamma^{1/2} q^2 ,$$

$$\frac{\omega}{\omega_p} = 1 + \frac{5}{18\Gamma} q^2 - \frac{3^{1/2}}{24} \Gamma^{1/2} q^2 .$$

It should be noted that"molecular dynamics experiments" also show that a sufficiently dense plasma does not exhibit an important damping of plasma oscillations. The above expressions have different factors even in the first dispersion term. It is not possible to make a term-to-term comparison with the previous results, because these approaches[8-10] are based on macroscopic considerations.

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