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MICROWAVE HALL MOBILITY MEASUREMENT

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1. Introduction

The charge carrier mobility, defined as the ratio between the drift speed and the electric field causing it, is practically the same at high frequencies (lo GHz) as under direct current conditions. This fact makes it possible to measure charge carrier mobility by means of microwave measuring methods. The main advantages are:

- Ohmic contacts are not required on the sample
- measurements are made on small crystals
- measurements are performed within a narrow frequency range and the noise effect is reduced
- microwave devices are sensitive and precise

2. Measuring method

A sample is placed at the center of a bimodal cavity (e.g. cylindrical cavity). The TE_{111} mode in the cavity is excited by the TE_{10} mode which is propagated through the rectangular wave guide "1" (Fig. 1).

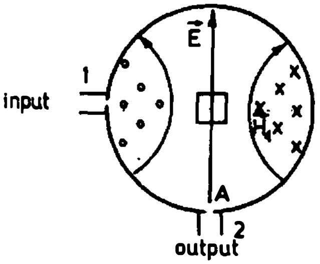

Fig.1 Configuration of the TE_{111} mode in cylindrical cavity

$\vec{E}_1$ - vector of electric field
$\vec{H}_1$ - vector of magnetic field

In the neighbourhood of point A the amplitude of magnetic field is zero. Due to the boundary condition $(\vec{n}\times\vec{H}=\vec{K})$ on the metal wall of the cavity surface, a current will not flow in this area. Consequently, it is not possible to excite a wave in the output wave guide by this configuration of the electromagnetic field.

Let us put the whole devices in a homogenous magnetic field, so that magnetic vector is in the direction of the cylinder axis. Then electrons suffer a magnetic force.

$$F_b = Bev \qquad u = v/E_1 \qquad F_b = BeuE_1 \qquad (1)$$

The electron drift speed falls to zero when the magnetic force is counterbalanced with an electron force (Fig.2). This electric force results from a accumulated charge.

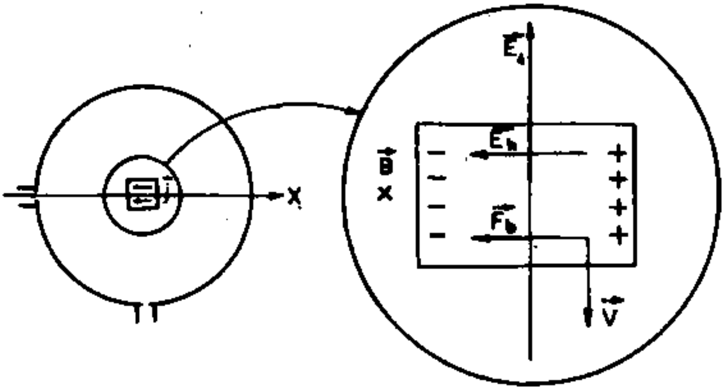

Fig.2 Hall effect

A cosequence of the electric field E_h is the current perpendicular to the original electric field E_1. It oscillates with the same frequency as the electric field E_1. For this reason the sample behaves like a short antenna and excites a new TE_{111} mode in the cavity. If the asymmetry, caused by apertures, is neglected, it is easy to see that the electric field of the new TE_{111} mode is symmetric with respect to "x" axis. This mode will excite only wave guide "2". The amplitude of the electric field in the wave guide is proportional to the amplitude of the electric field in the cavity. By means of foregoing relations (1) it can be readily shown that:

$$u = k \, \frac{1}{B} \left[\frac{P_2}{P_1} \right]^{1/2}$$

B- magnetic induction

P_1- wave power propagated into the cavity through wave guide "1"

P_2- wave power propagated out of the cavity through wave guide "2"

It is necessary to determinate constant k, which depends on a form of the cavity and sample, and on the material of which they are made. A detailed mathematical analysis of the problem must be made to determinate this constant. It is necessary to solve Maxwell's equations for this structure. Due to the magnetic field the sample conductivity has tensorial character.

One starts from the equation describing resonant cavity with apertures and a conducting medium(1.).

$$\mu\frac{d^2}{dt^2}\int\varepsilon\vec{E}\vec{E}_i^{*}dv+k_i^2\int\vec{E}\vec{E}_i^{*}dv=-\mu\frac{d}{dt}\left[\int\vec{J}\vec{E}_i^{*}dv-\int_{S'}(\vec{n}\times\vec{H})\vec{E}_i^{*}da\right]-k_i\int_{S}(\vec{n}\times\vec{E})\vec{H}_i^{*}da$$

$\vec{E}_i$ - orthonormal TE_{111} mode
k_i - eigenvalue of mode
S - conducting surface of the cavity wall
S' - surface of the apertures

Using the fact that the amplitude of the electromagnetic field in the cavity is directly proportional to the amplitude of the electromagnetic fieldin the wave guide it is possible to relate the elements of the scattering matrix with mobility (2., 3.). The element s_{21}, defined as the ratio of the outgoing wave amplitude in wave guide "2" and the ingoing wave amplitude in wave guide "1" is especially interesting. Namely, as we have already seen s_{21} gives an information about mobility.

$$u=\frac{1}{B}\,\frac{\left[(1+\Gamma_{10})(1+\Gamma_{20})\right]^{1/2}}{\left[(1+\Gamma_{11})(1+\Gamma_{21})(\Gamma_{10}-\Gamma_{11})(\Gamma_{20}-\Gamma_{21})\right]^{1/2}}\left[\frac{P_2}{P_1}\right]^{1/2} \qquad (2)$$

Γ_{i0} -- reflection coefficient of the input or output for an empty cavity
Γ_{i1} - reflection coefficient of the input or output with a sample in the cavity

In the case of symmetric cavity with respect to the input and output ($\Gamma_{10}=\Gamma_{20}=\Gamma_0$, $\Gamma_{11}=\Gamma_{21}=\Gamma_1$) relation (2) becomes:

$$u=\frac{1}{B}\,\frac{1+\Gamma_0}{(1+\Gamma_1)(\Gamma_0-\Gamma_1)}\left[\frac{P_2}{P_1}\right]^{1/2}$$

3. Measurement

The measurement is performed according to Fig.3.

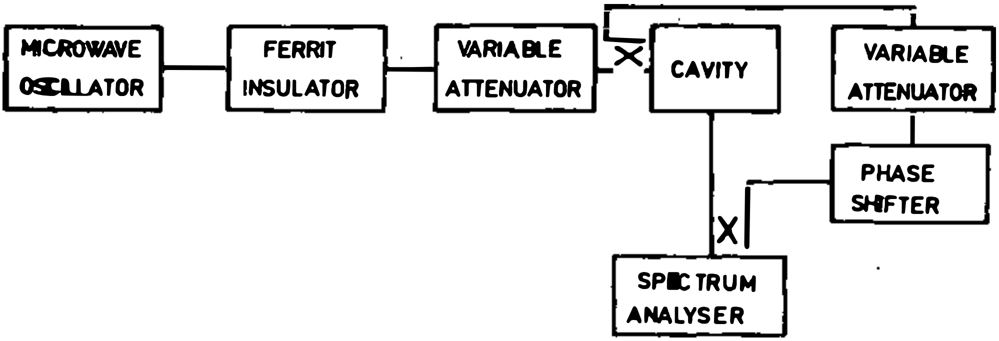

Fig.3 Simplified block diagram of the microwave measurement system

The measurement starts when the cavity is tuned (4.).
First the ratio P_2/P_1 is measured. It is determinated by recording the attenuation necessary to reduce the level of the input signal to that of the output signal.
Next step is Γ_1 measurement. The reflection signal is led into the spectrum analyser. The attenuation required to reduce the level of the input signal to that of the reflected signal is Γ_1^2.
The same procedure is repeated with an empty cavity for Γ_0 determination.
This method has been applied to a sample of sintered aluminium oxide with metal excess in the ratio 3:1. The sample resistivity is $10\,\Omega$cm. The measured mobility is 192 $\mathrm{cm}^2/\mathrm{Vs}$.
Since reflecting coefficient can be measured more precisely, our measurement gives only the order of magnitude.

References

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