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APPLICATION OF COHERENT STATES IN THE THEORY OF MAGNETIC SOLITONS

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ABSTRACT

Nonlinear effects in ferromagnetic systems can be studied in classical limit, but it is necessary to develop the methods for a strict quantum - classical transition, because the exchange interaction is of the purely quantum origin. This realized by the application of coherent states (CS). Two types of states are studied: spin (generalized) CS and Glauber boson CS, combined with boson representation for spin operators. An example of anisontropic Heisenberg ferromagnet is discussed in detail.

1. Introduction

Since the discovery that "solitary waves" -solitons, which are well localized excitations created in the physical systems as the consequence of the competetion of the nonlinearity and dispersivity of the medium, the magnetic systems were considered to be a suitable medium for their appearance. Theoretical calculations for ideal one-dimensional (1d) structures indicated that such a possibility exists but, for the sake of experimental verification, it was necesarry to find (synthesize) materials which can be described by the idealized systems. First such experiment was neutron scattering on $CsNiF_3$, which simulates S=1 xy ferromagnet [1]. It was followed, for example by the measurements of soliton contribution to the heat capacity, heat conductivity and (nuclear spin)-lattice (NSLR) relaxation time in the materials like (TMMC) $(CH_3)_4NMnCl_3$ (S=5/2 antiferromagnetic) and CHAB $(C_6H_{11}NH_3)CuBr_3$ (S=1/2 ferromagnetic) [2,3], to quote only some earlier experiments.

As the first theories were scrictly classical, let us treat the example of 1d ferromagnetic with "easy-plane" anisotropy, described by the Hamiltonian

$$\hat{H} = -J\sum_n \hat{\vec{S}}_n\cdot\hat{\vec{S}}_{n+1} + A\sum_n (\hat{S}_n^z)^2 - \mu\mathcal{H}_{ex}\sum_n \hat{S}_n^x \qquad (1.1)$$

We shall suppose that all spins are *classical variables*

$$\hat{\vec{S}}_n \to \vec{S}_n \equiv (S\sin\theta_n\cos\phi_n,\ S\sin\theta_n\sin\phi_n,\ S\cos\theta_n) \qquad (1.2)$$

Next step is so called *continuum approximation*, which means the transition from the lattice sites na to continual variable x. We shall expand all expressions up to the second order terms in lattice parameter a:

$$A_n(t)\to A(x,t)\ ;\ A_{n\pm1}(t)\to A(x\pm a,t)\approx A(x,t)\pm a\left(\frac{\partial A}{\partial x}\right)_t + \frac{a^2}{2}\left(\frac{\partial^2 A}{\partial x^2}\right)_t$$

$$\sum_n \to \frac{1}{a}\int dx$$

In order to simplify the calculations, we shall set a = 1 (dimensionless variable). The quantity (1.1) is then treated as the classical Hamilton's function of the system and we treat ϕ and $S\cos\theta$ as canonical variables. In the approximation of small out-of-plane excitations ($\sin\theta \approx \theta$), we obtain:

$$H = E_0\int dx\left[\frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - \frac{1}{2c^2}\left(\frac{\partial\phi}{\partial t}\right)^2 + m^2(1-\cos\phi)\right] \qquad (1.2)$$

$$E_0 = JS^2 \qquad c^2 = 2JAS^2 \qquad m^2 = \frac{\mu\mathcal{H}_{ex}}{JS^2}$$

The equation of motion is the well known sine-Gordon nonlinear equation

$$\frac{\partial^2\phi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = m^2\sin\theta \qquad (1.3)$$

with the soliton solution of the following form

$$\phi_s(x,t) = 4\ \mathrm{arctg}\left\{\exp\left[\pm\frac{m\,(x-vt)}{\sqrt{1-\frac{v^2}{c^2}}}\right]\right\} \qquad (4.1)$$

where v is the soliton velocity.

Let us not that it was obtained by treating the spins as classical vari·ables from the very begining. As we are interesed in the influence of quantum effects, we must apply a different approach.

In the next sections we shall explain the way in which one can perform the transition from quantum to classical quantities through the application of spin and boson (Glauber) coherent states (CS).

2. SPIN (GENERALIZED) COHERENT STATES

Spin (generalized) coherent states (SCS) were introduced in a pioneer work of Radcliffe [4], although the fundamental reference nowadays is an excellent review by Perelomov [5].

SCS can be defined for the case of spin $S = 1/2$ in the following way

$$|\Omega_m\rangle = e^{-i\phi_m \hat{S}^z_m} e^{-i\Theta_m \hat{S}^y} |0\rangle_m \qquad (2.1a)$$

We adopt the convention that $|0\rangle_m$ represent the eigenstate of the operator $\hat{S}^z_m$ with $S^z = +1/2$ ($|0\rangle_m = |\uparrow\rangle_m$), so that $|1\rangle_m = |\downarrow\rangle_m$ is the other possible eigenstate. Using this notation, we have

$$(\Omega_m\rangle = e^{-i\phi_m/2} (\cos\tfrac{\Theta_m}{2} |0\rangle_m + e^{i\phi_m} \sin\tfrac{\Theta_m}{2} |1\rangle_m) \qquad (2.1b)$$

The fundamental property of SCS's is:

$$\langle\Omega_m(\hat{S}^x_m |\Omega_m) = S \sin\Theta_m \cos\phi_m \qquad (2.2a)$$

$$\langle\Omega_m(\hat{S}^y_m |\Omega_m) = S \sin\Theta_m \sin\phi_m \qquad (2.2b)$$

$$\langle\Omega_m(\hat{S}^z_m |\Omega_m) = S \cos\Theta_m \qquad (2.2c)$$

enabling the transition to the classical variables.

Let us treat xxz anisotropic Heisenberg ferromagnet as an example.

$$\hat{H} = -\mu \mathscr{H}_{ext} + \sum_j (\hat{S}^z_j - S) - J \sum_j (\hat{\vec{S}}_j \hat{\vec{S}}_{j+1} - S^2) - \sigma J \sum_j (\hat{S}^z_j \hat{S}^z_{j+1} - S^2) \qquad (2.3)$$

where J is the exchange integral, σ - anisotropy paramerer, μ - - magnetic moment and $\mathscr{H}_{ext}$ - the external magnetic field along z-axis. The constans are added in order to avoid divergencies of the ground state energy.

We shall suppose that the function $|\Omega\rangle = \prod_m |\Omega_m\rangle$ satisfies Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\Omega\rangle = \hat{H} |\Omega\rangle \tag{2.4}$$

giving

$$|\Omega(t)\rangle = e^{-i/\hbar \hat{H}t} |\Omega(0)\rangle \tag{2.5}$$

Using the following definition of Lagrange function which is suitable for the field theory [6]

$$L = \frac{i\hbar}{2} \langle\Omega| \overleftrightarrow{\frac{\partial}{\partial t}} |\Omega\rangle - \langle\Omega |\hat{H}| \Omega\rangle \tag{2.6}$$

we obtain that

$$L = S \sum_m \dot{\phi}_m(\cos\Theta_m - 1) - \bar{\mathcal{H}} (\Theta_m, \phi_m) \tag{2.7}$$

Equations of motion follow from Lagrange's equations for (2.7)

$$\dot{\phi}_m = \frac{\partial \bar{\mathcal{H}}}{\partial (S \cos\Theta_m)} \tag{2.8a}$$

$$S \frac{\partial}{\partial t} (\cos\Theta_m) = - \frac{\partial \bar{\mathcal{H}}}{\partial \phi_m} \tag{2.9a}$$

Now we can apply the continuum approximation up to terms of order a^2 :

$$\bar{\mathcal{H}} = \frac{1}{a} \int \mathcal{H}\, dx \tag{2.9}$$

where we have defined the Hamiltonian density $\mathcal{H}$:

$$\mathcal{H} = \frac{1}{2} JS^2a^2(\Theta_x{}^2 + \sin^2\Theta\, \phi_x{}^2) - J\sigma S^2(\cos^2\Theta - 1) + \frac{1}{2} J\sigma S^2 a^2 \left(\frac{\partial \,(\cos\Theta)}{\partial x}\right)^2 -$$

$$- \mu\, \mathcal{H}_{ext}\, S (\cos\Theta - 1) \tag{2.10}$$

Subscripts denote partial derivatives over given variables. Classical equations of motion for the variables $\Theta(x,t)$ and $\phi(x,t)$ are

$$(\cos\Theta)_t = JS (\phi_x \sin\Theta)_x \tag{2.11a}$$

$$\phi_t \sin\Theta = -\mu\, \mathcal{H}_{ext} \sin\Theta - 2J\sigma S \sin\Theta \cos\Theta + JS\sigma \sin\Theta\, (\Theta_x \sin\Theta)_x +$$

$$+ JS\Theta_{xx} - JS \sin\Theta \cos\Theta\, \phi_x{}^2 \tag{2.11b}$$

Now we introduce a new variable $\xi = x - vt$, where v is the soliton velocity, and the assimptions:

$$\phi(x,t) = \Omega t + \tilde{\phi}(\xi)$$

$$\Theta(x,t) = \Theta(\xi)$$

$$S^z(\pm\infty) = S$$

First integration gives

$$\tilde{\phi}_\xi = \frac{v}{JS}\frac{1}{1+\cos\Theta} \tag{2.12}$$

It is suitable to transform (2.11b) by introducing a new variable $\beta = \frac{\Theta}{2}$:

$$\cos^2\beta\, \beta_\xi(1 + 4\sigma \sin^2\beta \cos^2\beta) = \Gamma \sin^2\beta(\cos^2\beta + \frac{2\sigma}{\Gamma}\cos^4\beta - \frac{v^2}{4J^2S^2\Gamma}) \tag{2.13}$$

with the notation

$$\Gamma = \frac{\mu\, \mathscr{H}_{ext} + \Omega}{JS}$$

Let us look for the soliton solution in the case of small anisotropy $\sigma = \frac{J - J_z}{J} << 1$. We shall express it in terms of the parameters of the system: magnetization

$$M_z = S\int_{-\infty}^{\infty}(1 - \cos\Theta)\, dx \tag{2.14a}$$

and momentum

$$P = S\int_{-\infty}^{\infty}(1 - \cos\Theta)\frac{\partial\phi}{\partial x}\, dx \tag{2.14b}$$

The energy is then given by:

$$E = \frac{1}{2}\int \mathscr{H}\, dx = 4JS^2(2\sigma)^{1/2}\frac{\mathrm{ch}\left[\frac{M_z(2\sigma)^{1/2}}{2S}\right] - \cos\left[\frac{P}{2S}\right]}{\mathrm{sh}\left[\frac{M_z(2\sigma)^{1/2}}{2S}\right]} \tag{2.15}$$

The solution is then (S=1/2)

$$\cos\Theta = 1 - \frac{2A_0}{1 + \frac{2}{y+1}\mathrm{sh}^2\sigma^{1/2}A_0\sqrt[2]{\frac{yH}{y-1}}(x - vt)} \tag{2.16}$$

where

$$y = \mathrm{ch}\ \frac{M_z(2\sigma)^{1/2}}{2S} = \mathrm{ch}\ M_z(2\sigma)^{1/2} \qquad (2.17a)$$

and soliton amplitude

$$A_0^2 = \frac{y - \cos\frac{P}{2S}}{y + 1} = \frac{y - \cos P}{y + 1} \qquad (2.17b)$$

In order to perform the semi-classical quantization [7], we shall let the total magnetization have only the integer multiple of $\hbar$:

$$M_{z'} = m\hbar \qquad (m=0,1,2...) \qquad (2.18)$$

So the energy is given by

$$E_m = J\ \mathrm{sh}\ \alpha\ \frac{\mathrm{ch}\ (m\alpha) - \cos P}{\mathrm{sh}\ m\alpha} \qquad (2.19)$$

where: $\mathrm{ch}\ \alpha = \sigma + 1$ $\quad\mathrm{sh}\ \alpha \approx \sqrt{2}\ \sigma$

3. GLAUBER'S COHERENT STATES

Glauber [8] introduced coherent state as the eigenstate of the boson annihilation operator:

$$\hat{B}_j\,|\alpha_j\rangle = \alpha_j\,|\alpha_j\rangle \qquad (3.1)$$

The state satisfying (3.1) has the form

$$|\alpha_j\rangle = \exp(\alpha_j \hat{B}_j^{\dagger} - \alpha_j \hat{B}_j)|0\rangle \qquad (3.2)$$

The coefficient α_j is the coherent amplitude and $|0\rangle$ boson vacuum. Since we are dealing with the system of particles, we need the direct product state:

$$|\alpha\rangle = \prod_j\ |\alpha_j\rangle \qquad (3.3)$$

In order to use Glauber's CS, it is necessary to write down the spin Hamiltonian (2.3) in terms of Bose operators. It turned out that Holstein-Primakoff representation [9] is the most suitable for that purpose:

$$\hat{S}_j^{+} = \left[2S\left(1 - \frac{\hat{B}_j^{\dagger}\hat{B}_j}{2S}\right)\right]^{1/2}\hat{B}_j \qquad (3.4a)$$

$$\hat{S}_j^{-} = (\hat{S}_j^{+})^{+} \qquad (3.4b)$$

$$\hat{S}_j^z = S - \hat{B}_j^\dagger \hat{B}_j \qquad (3.4c)$$

The sqare root is formally expanded in operator series which is then put in the normal product form to give:

$$\left[1 - \frac{\hat{B}_j^\dagger \hat{B}_i}{2S}\right]^{1/2} = \sum_{\nu=0}^{\infty} \binom{1/2}{\nu} \left(-\frac{1}{2}\right)^\nu \frac{\hat{B}_i^{\dagger\,\nu} \hat{B}_i^{\,\nu}}{S^\nu} \left[1 + 0\left(\frac{1}{S}\right)\right] \qquad (3.5)$$

We shall show that this degree of accuracy is sufficient for the calculation in the classical limit [10].

Using (3.4), we obtain

$$\langle \hat{S}_j^z \rangle = \langle \alpha | \hat{S}_j^z | \alpha \rangle = S - \langle \alpha | \hat{B}_j^\dagger \hat{B}_j | \alpha \rangle = S - |\alpha_j|^2$$

Having in mind the classical parametrization

$$\langle \hat{S}_j^z \rangle = S \cos\theta_j \longrightarrow |\alpha_j|^2 = S(1 - \cos\theta_j)$$

We conclude that $|\alpha_j|^2$ defined in this way diverges in the limit $S \to \infty$. For this reason we introduce a new amplitude $\tilde{\alpha}_j$

$$\tilde{\alpha}_j = \frac{\alpha_i}{\sqrt{S}}$$

which is expressed as

$$|\tilde{\alpha}_j|^2 = 1 - \cos\theta_j$$

So it does not depend on S.

$$\langle \hat{S}_j^z \rangle = S(1 - |\tilde{\alpha}_j|^2) \qquad (3.6)$$

In the same way

$$\langle \alpha | \hat{S}_j^\dagger | \alpha \rangle = \sqrt{2S} \langle \alpha_j | \sum_k \binom{1/2}{k} \left(-\frac{1}{2}\right)^k \left(\frac{\hat{B}_i^\dagger \hat{B}_i}{S}\right)^k \hat{B}_j | \alpha_j \rangle$$

Using [3,5] in the limit $S \to \infty$, we neglect the terms of order $\frac{1}{S}$ and obtain:

$$\langle \hat{S}_j^\dagger \rangle = \sqrt{2}\, S \sqrt{1 - \frac{|\tilde{\alpha}_i|^2}{2}}\; \tilde{\alpha}_j \qquad (3.7a)$$

$$\langle \hat{S}_j^- \rangle = \sqrt{2}\, S \sqrt{1 - \frac{|\tilde{\alpha}_i|^2}{2}}\; \tilde{\alpha}_j \qquad (3.7b)$$

The classical limit implies that $S \to \infty$, $\hbar \to 0$, with the quantity $\hbar S$ remaining finite

$$S_c = \lim_{\substack{\hbar \to 0 \\ S \to \infty}} \hbar S \qquad (3.8)$$

S_c is classical angular momentum.

In order to evaluate the classical limit, we must firstly "restore" the dimension of spins in (2.3) $S \to \hbar S$ and only after that to take the limit.

Using the continuum approximation, we obtain in the classical limit

$$\mathcal{H} = \langle \alpha \,|\hat{H}\,(\hat{B}^{+},\hat{B})|\,\alpha\rangle = \frac{1}{a}\int \varepsilon\,(x,t)\,dx \qquad (3.9)$$

$$\varepsilon\,(x,t) = h|\tilde{\alpha}|^2 - \sigma\,JS_c^2|\tilde{\alpha}|^4 + JS_c^2a^2|\tilde{\alpha}_x|^2 + \frac{1}{4}\,JS_c^2a^2(\tilde{\alpha}^2\tilde{\alpha}_x^{*2} + \tilde{\alpha}^{*2}\alpha_x^{\,2}) +$$

$$+ \frac{1}{2}\,\sigma\,JS_c^2a^2(|\tilde{\alpha}|_x^2)^2 + \frac{1}{16}\,JS_c^2a^2\,\frac{|\tilde{\alpha}|^2}{1-\frac{|\tilde{\alpha}|^2}{2}}\,(|\tilde{\alpha}|_x^2)^2 \qquad (3.10)$$

$$h = \mu\,\mathcal{H}_{ex}S_c + 2\sigma JS_c^2$$

It is important to notice that the relevant quantities are in fact μS_c and JS_c^2 having the dimension of magnetic moment and energy, respectively.

Equations of motion are of the form

$$iS_c\,\frac{\partial\tilde{\alpha}}{\partial t} = \frac{\partial\varepsilon}{\partial\tilde{\alpha}^*} = \frac{\partial\varepsilon}{\partial\tilde{\alpha}^*} - \frac{\partial}{\partial x}\left(\frac{\partial\varepsilon}{\partial\tilde{\alpha}_x}\right) \qquad (3.11)$$

We shall look for the soliton solution of the form

$$\tilde{\alpha}\,(x,t) = A(x-vt)e^{i[\phi(x-vt)+\Omega t]} \qquad (3.12)$$

giving

$$\phi_x = \frac{V}{2}\,\frac{1}{1-\frac{A^2}{2}} \qquad V = \frac{v}{JS_c^2a^2} \qquad (3.13)$$

and

$$A_x^2\left[1 + 2\sigma A^2\left(1-\frac{A^2}{2}\right)\right] + \Gamma A^2\left[1 - \frac{V^2}{4\Gamma} + \frac{2\sigma}{\Gamma a^2} - \frac{A^2}{2}\left(1+\frac{4\sigma}{\Gamma a^2}\right) + \frac{\sigma}{2\Gamma a^2}\,A^4\right]$$

$$\Gamma = \frac{\mu\,\mathcal{H}_{ex}+\Omega}{JS_c a^2} \qquad (3.14)$$

For $\sigma = 0$, this is the result for the isotropic Heisenberg chain. It was first obtained by de Azevedo et al [11], who has considered the Holstein-Primakoff approximation up to four Bose-operators. On the other hand, the result for $\sigma \neq 0$, but small was first derived by us [12] using the expansion up to six Bose-operators. All these results agree with the classical results obtained by Tjon and Wright [13].

It is very important to understand the meaning of the above results. We have shown that in the classical limit the calculation with the complete HP - series leads to the results obtained by the classical treatment. Furthermore, we have shown that it is sufficient to truncate the series at fourth of sith order in Bose-operators when treating isotropic or anisotropic case respectively. Thus, we justified the usual approximate calculations.

4. CONNECTION BETWEEN SPIN CS AND GLAUBER CS AND CONCLUSION

Let us perform the following parametrization:

$$\tilde{\alpha}_j = A_j \; e^{i\phi_j} \qquad A_j^2 = 1 - \cos\Theta_j \tag{4.1}$$

In this case (3.6) and (3.7) lead to:

$$\langle\alpha \; |\hat{S}_j^z| \; \alpha\rangle = S\,(1 - |\tilde{\alpha}_j|^2) = S\cos\Theta_j \tag{4.2}$$

$$\langle\alpha \; |\hat{S}_j^+| \; \alpha\rangle = \sqrt{2S}\sqrt{1 - \sin^2\frac{\Theta_i}{2}}\;\sqrt{2}\,\sin\frac{\Theta_i}{2}\,e^{i\phi_j} = S\sin\Theta_j \; e^{i\phi_j} \tag{4.3}$$

In other words, in the classical limit ($S \to \infty$, $S\hbar \to S_c$), Glauber and spin CS are equivalent and the choice, which ones to be applied depends on the nature of particular problem.

The approach using Glauber CS and HP representation enables us a systematic calculation of the quantum corrections, which appear themselves as the corrections of order $\frac{1}{S}$. The work on this problem is currently under progress.

On the other hand, the approach using spin CS permits the study of a more subtle problem, concerning the time evalution of CS. Treating the definition of SCS as the equation of time evolution

$$|\Omega(t)\rangle\rangle = \hat{U}(t) \; |0\rangle \tag{4.4}$$

with $\quad \hat{U}(t) = e^{-i\sum_m \phi_m \hat{S}_m^z} \; e^{-i\sum_m \Theta_m \hat{S}_m^x}$

the "classical" dynamics is given by Lagrangian equations for $\dot{\theta}_m$ and φ_m

Quantum evolution is determined by the Schrödinger equation

$$i\hbar \frac{\partial|\psi\rangle}{\partial t} = \hat{H}\ |\psi\rangle \qquad (4.5)$$

Let us suppose that the state $|\psi\rangle$ satisfies the same initial conditions, as the function $|\Omega\rangle$

$$|\psi(0)\rangle = |\Omega(0)\rangle \qquad (4.6)$$

The question that arises is wheather this implies

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t}\ |\Omega(0)\rangle (=)\ |\Omega(t)\rangle \qquad (4.7)$$

or, equivalenty, for some physical quantity $\hat{F}$

$$\langle\Omega(t)\ |\hat{F}|\ \Omega(t)\rangle\ (=)\ \langle\Omega(0)\ |\hat{F}(t)|\ \Omega(0)\rangle \qquad (4.8)$$

Strictly speaking, for $\hat{F}(t) = e^{-\frac{i}{\hbar}\hat{H}t}\ \hat{F}\ e^{-\frac{i}{\hbar}\hat{H}t}$, this last equality is valid only in the classical limit. The question arises about the error introduced by substituting these two quantities, for a particular Hamiltonian. Our preliminary results [14] indicate that no error is introduced for the energy and an error of order $\sim t^2$ is introduced in the propagator.

$$\langle\Psi(t)|\Omega(t)\rangle = 1 + O(t^2)$$

One can conclude that spin CS and Glauber's CS in the classical limit lead to the identical results which agree with the classical ones.

KOHERENTNA STANJA U TEORIJI MAGNETNIH SOLITONA

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Nelinearne fenomene u magnetnim sistemima moguće je analizirati u klasičnoj aproksimaciji, ali je pri tome potrebno razviti odgovarajuće metode kvantno-klasičnog prelaza s obzirom da je interakcija izmene čisto kvantne prirode. Jednu takvu mogućnost svođenja kvantnih jednačina na klasične daje nam primena koherentnih stanja (CS). U radu se analizira primena dva tipa koherentnih stanja: spinskih (generalizíranih) CS i Gauber-ovih bozonskih CS u kombinaciji sa Holstein-Primakoff (HP). reprezentacijom spinskih operatora. Detaljnija diskusija rezultata data je za slučaj anizotropnog Heisenber-ovog feromagneta.

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