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Mixed Feelings Modeled with the Fuzzy Logical Hexagon

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ABSTRACT: In this paper, I propose a logical model of mixed feelings. I claim that contrary emotions of desire, fear and indifference can be felt towards the same state of affairs and that this phenomenon can be modeled with the fuzzy logical hexagon. In the fuzzy logical hexagon, contrary notions are allowed to be true simultaneously, but with a special proviso: the sum of their values must be 100%. The same ratio, I claim, appears among the emotions of desire, fear and indifference when they are felt towards one and the same state of affairs. To calculate the percentage of an emotion in an 'emotional ratio', I propose an analysis of states in terms of their aspects, understood as relevant implications. For instance, a state can be both desired and feared if some of its relevant implications are desired and some feared. The present approach is in opposition to a non-realist stance in philosophy of affectivity about the possibility of simultaneous contrary emotions, where differentiation between aspects of a state is often used to explain *away* the possibility of mixed feelings. The proposed model also differs from some other graded formal approaches to desire and fear, which treat these two emotions separately, each according to its own logic. I claim that logic of desire and logic of fear is one and the same – that of the fuzzy logical hexagon.

KEY WORDS: Logical hexagon, fuzzy logic, mixed feelings, desire, fear, indifference.

1. Introduction

In this paper, I speak about the relation between the intentional states of desire, fear and indifference. I claim that these contrary emotions can be felt at the same time towards the same object, and that this phenomenon can be modeled with the fuzzy logical hexagon.

The classical logical hexagon is an extension of the Aristotelian square of oppositions. This extended structure has proven to be a useful tool for classifying and analyzing concepts: it can show in a concise and expressive way all the logical relations between three mutually exclusive and jointly exhaustive terms and their negations. The *fuzzy* logical hexagon takes into account the fact that properties can be a matter of degree. Using non-classical semantics, it allows for opposing notions to be true about the same (propositional) object at the same time. However, there is an important semantic proviso in the fuzzy logical hexagon: although contrary notions can be true simultaneously, they can be so only *partly*, and the truth value of a notion can grow only *at the expense* of its contraries. More precisely, the sum of truth values of contrary notions has to be equal to 1 or 100%, yielding a ratio of contrary notions.[1]

I claim that this ratio can also be present among feelings of desire, fear and indifference. To calculate the amounts of the three emotions, I propose a fine-grained analysis of their objects in terms of aspects, understood as (relevant) implications. For instance, as I will argue by means of an example (travelling to Antarctica), a state of affairs can be both desired and feared if some of its aspects are desired and some feared. In philosophy of affectivity, however, aspects are sometimes used to explain *away* the possibility of experiencing contrary emotions at once.

The plan of the paper is the following. In Section 2, I give preliminaries about the classical and the fuzzy logical hexagon and their conceptual usage. In Section 3, I provide an example of a situation in which one can be said to feel desire, fear and indifference towards the same state of affairs at the same time. I propose to model this by a fuzzy logical hexagon, exploring the implications and potential difficulties of the approach. In Section 4, I put the proposed model in context, comparing it with two alternative logical treatments of desire and fear, as well as with different philosophical views on the (im)possibility of simultaneous presence of conflicting emotions, i.e., of mixed feelings.

2. The Classical and the Fuzzy Logical Hexagon

2.1 The Square of Oppositions

The square of oppositions is perhaps the most famous logical figure. It is a diagrammatic representation of the four semantic relations between

[1] For fuzzy truth values I also, interchangeably, use percentages. This is because I find this more in line with common parlance. The percentages are meant to express amounts (not probabilities).

the four prototypical quantified propositions (or sentences) used by Aristotle in his syllogistic.

Namely, the four propositions are: the universal affirmative, the particular affirmative, the universal negative, and the particular negative. After Aristotle, these four kinds of sentences got shortened names. Affirmative sentences got their names form the Latin word *affirmo*, meaning "I affirm". The first two vowels of the word are A and I, so the universal affirmative got to be named A (or A-sentence), and the particular affirmative got the name I. Regarding negative sentences, they are named according to the Latin word *nego*, meaning "I negate". The two vowels are E and O, corresponding to, respectively, the universal negative and the particular negative.

Regarding the semantic relations between the sentences, the four of them are, in modern parlance: contrariety, subcontrariety, contradiction, and implication. Here are the definitions. Two propositions are *contrary* iff they cannot both be true at the same time, but they can both be false at the same time. Two propositions are *subcontrary* iff they cannot both be false at the same time, but they can both be true at the same time. Two propositions are *contradictory* iff they cannot both be true at the same, nor can they both be false at the same time. Lastly, there is implication, the only relation in the square of oppositions that is not an opposition. Nevertheless, it is needed to make the diagram complete. For any two propositions, we say that the first *implies* the second iff the second proposition is true if the first proposition is true.

The square of oppositions with example sentences is shown in Figure 1. There, we can see, e.g., that the universal affirmative (or the A-sentence) "Every human is mortal" implies the particular affirmative (or the I-sentence): "Some human is mortal". The A-sentence is, moreover, contradictory to the O-sentence "Some human is not mortal". Finally, the A-sentence is contrary to the E-sentence "No human is mortal". Sentences A and O cannot be true at the same time, and the same applies to A and E. However, sentences A and E can both be false, which is not the case for A and O.

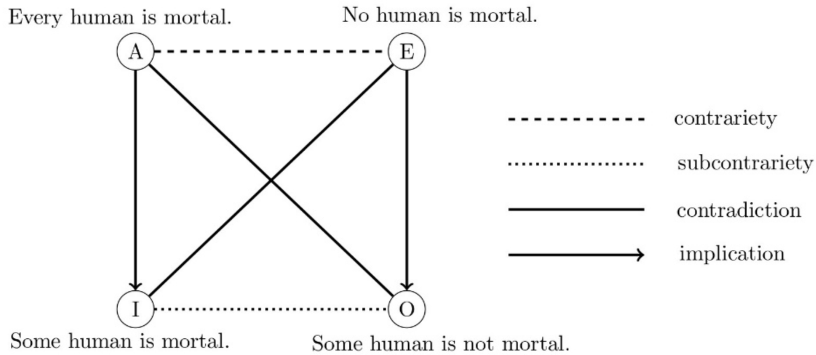

Figure 1: The square of oppositions

Now, there is an important convention in drawing the square and connected diagrams, which I adopt in this paper. The vowels that represent the names of sentences are also taken to be the names of *corners* (or points) of the square. So, the A-sentence is in the A-corner, the B-sentence is in the B-corner, etc. This works perfectly for the square, in the visual sense. Each corner has a sentence that literally corresponds to it. However, sentences come and go. That is, we can put other things in the square. The four prototypical Aristotelian sentences (or, more precisely, the examples thereof) presented in Figure 1 are, in logic, called "a decoration" of the square. But we can consider different decorations, some of which have nothing to do with the four Aristotelian sentences. (I will do this for the next figure, the logical hexagon.) Also, we can consider the square of oppositions without any decorations, in which case we would only have the shape. But in that shape, the lettered corners remain. Corners are named after Latin names for Aristotelian sentence – even when we consider more complex structures of oppositions, where in many cases the names of the corners lose the association with the four prototypical propositions.

2.2 The Classical Logical Hexagon, in the Abstract

Let me now turn to the logical hexagon, the star of this paper. We get the classical[2] logical hexagon if we add two more corners to the square and connect them by some of the four sematic relations with other

[2] I call the figure "classical" because it is proposed in the setting of classical logic. More commonly, it is just called "the logical hexagon".

corners and with each other. The classical logical hexagon was discovered independently by Jacoby (1950), Sesmat (1951), and Blanché (1953). The two new corners added to the square are conventionally named U and Y. This time, the names do not come from any Latin expressions, nor do they specifically refer to any (forms of) sentences. Instead, their names come from the works by the two later co-discoverers of the hexagon, who were French. In the French alphabet, U and Y are the remaining vowels, so the notation was perhaps an intuitive choice for the two authors and later became standard in the literature.

The classical logical hexagon is a highly symmetrical structure, having three distinct symmetry axes, while the square has only one (Moretti 2009). It is shown in Figure 2. The figure represents the logical hexagon, along with three decorations of the corners, each in its own row. Let me start from the shape itself. We see how adding two more corners makes the hexagon quite more "busy" than the square. That is, there are now many more relations between the corners. The square of oppositions itself is already somewhat busy, in the sense that each corner is connected to *every other* corner by some type of relation (i.e., line). In the square, there are *three* lines emanating from every point. In the hexagon, the number jumps to *five*. Moreover, shape-wise, there are two important triangles in the classical logical hexagon: *the triangle or contraries* and the *triangle of subcontraries*. The former is formed by the dashed lines (representing contrariety) in the figure, i.e., it is the triangle A-E-Y. The latter triangle, drawn in dotted lines, is U-I-O.

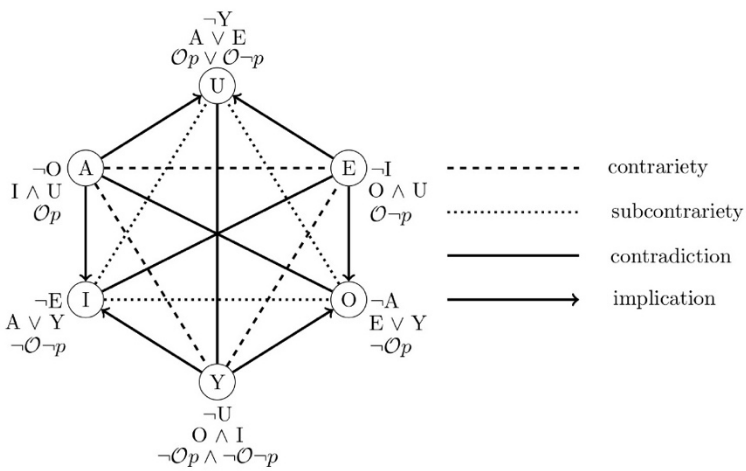

Figure 2: The classical logical hexagon

Let me now turn to the three decorations of the corners in Figure 2. They do not say anything particularly concrete, save for the last one, which is included to foreshadow the conceptual usage of the diagram, discussed at the end of this section. This is not to say the provided decorations are not informative, even in this abstract form.

The first two decorations show how each corner can be defined in terms of other corners. The first one shows that what is said in each corner can be expressed as the *negation* of what is said in another corner, namely, the one which is connected to it by the opposition of *contradiction*. This also works in the *square* of oppositions, shown in Figure 1. The sentence in the O-corner, "Some human is not mortal", can be defined as the negation of the sentence in the A-corner, which states that "Every human is mortal". The second decoration in Figure 2, however, brings novelties. It shows that each corner can be defined in terms of its two neighboring corners. This is where the symmetry of the hexagon (as opposed to the square) really becomes visible. What is said in a given corner of the hexagon can be defined as either a *disjunction* or as a *conjunction* of what is said in its two neighboring corners. Whether it is a disjunction or a conjunction depends on the two triangles mentioned above. Corners A, E, and Y, which form the triangle of contraries, are defined as conjunctions. E.g., A is equivalent to I ∧ U. Corners U, I and O, which form the triangle of subcontraries, are defined as disjunctions. E.g., O is equivalent to E ∨ Y.

Lastly, the third decoration intends to show how logically elegant and expressive the hexagon can be. I introduce the symbol $\mathcal{O}$ (not to be confused with O, which signifies the corner). The symbol $\mathcal{O}$ is a variable, and it stands for "operator". (The lowercase letter, p, stands for "proposition", like in propositional logic). To get actual logical formulas, we need to replace $\mathcal{O}$ with a propositional operator, i.e., an operator that is put in front of sentences. A strong candidate is K, used in epistemic logic, where "Kp" is read as "the subject knows that p". If, in Figure 2, we take $\mathcal{O}$ to represent K, we get a myriad of logical relations between subject's knowledge states. Let me mention only a couple. We see that, if the subject knows that p, this implies that they do not know that $\neg p$ (since A implies I). We also see that a subject cannot know both p and $\neg p$ at the same time, but they can fail to know either p or $\neg p$ (since corners A and O are contrary, so they cannot be true at the same time but can be false at the same time). We can also replace $\mathcal{O}$ with $\Box$, the necessity operator in modal logic, in which case the hexagon shows the relations between metaphysical notions. Or, as I will argue, we can replace O with an operator that stands for *desire*.

2.3 The Fuzzy Logical Hexagon, in the Abstract

There recently appeared the *graded* or *fuzzy* version of the logical hexagon, first proposed by Dubois and Prade (2012), who soon also offered more complex fuzzy structures of oppositions (2015).[3] The fuzzy logical hexagon is built on fuzzy logic, where truth can be *partial*, i.e., propositions can take truth values from the interval [0,1], as opposed to the set {0,1}, like in classical logic (cf. Łukasiewicz 1970). In this paper, I also use *percentages* to express fuzzy truth values. This is because I find this more in line with common parlance. The percentages are meant to express amounts (not probabilities).

Here are the truth values for the connectives in fuzzy logic, which correspond to Łukasiewicz' (1970) continuum-valued semantics. Let p stand for any proposition and v for truth value:

- $v(p) \in [0, 1]$ (where 1 is the designated value)
- $v(\neg p) = 1 - v(p)$
- $v(p \rightarrow q) = min(1, 1 - v(p) + v(q))$
- $v(p \vee q) = max(v(p), v(q))$
- $v(p \wedge q) = min(v(p), v(q))$
- $v(p \oplus q) = min(v(p) + v(q), 1)$
- $v(p \otimes q) = max(v(p) + v(q) - 1, 0)$.

For example, if proposition p is 30% true and proposition q is 50% true, their conjunction takes the lesser value and is 30% true. Further, if p is 30% true, its negation is true to the (rest) 70%. This has an interesting consequence – sometimes considered counterintuitive (cf. Smith 2008: ch. 5) – that both a proposition and its negation can be simultaneously true. (But at most a 50%; the truth value of $p \wedge \neg p$ in the above example is 30%.)

The last two connectives on the above list – not distinguishable in classical, two-valued, logic – are the strong disjunction and the strong

[3] Unlike Dubois and Prade, who use the term "graded", I call the logical hexagon "fuzzy", because it corresponds to the popular fuzzy logic paradigm, championed by Zadeh (1975) in AI. Fuzzy logic is also endorsed in philosophy, particularly concerning the problem of vagueness (cf. Smith 2008).

conjunction respectively, which take different values from their weak counterparts. They are defined in this way (Łukasiewicz 1970):

- $p \oplus q =_{df} \neg p \rightarrow q$ (strong disjunction)
- $p \otimes q =_{df} \neg(p \rightarrow \neg q)$ (strong conjunction).

The different account of negation in fuzzy logic has a vast influence on the meaning of oppositions. For instance, and importantly for the present purpose, contradictory and contrary propositions now *can* be true at the same time. Two contradictory propositions are each other's negation, which is discussed above. Regarding the contrary propositions, in the fuzzy hexagon, we get a more interesting picture. It has to do with the triangle of contraries, i.e., the corners A, E, and Y. The truth is split between the three propositions found in these corners, according to the specific *ratio*. Namely, they all have to add up to 1 or 100% (Dubois and Prade 2015). Said formally:

$$v(\text{A}) + v(\text{E}) + v(\text{Y}) = 1.$$

The fact that contraries can be simultaneously true may sound odd at first glance, but the relation between contrary notions in the fuzzy hexagon is not that different from the classical version. In the two-valued picture, we can say that one contrariety is true *at the expense* of the others: in Figure 2, only one among the corners A, E, and Y can say something true. The same dynamic is present in the fuzzy hexagon: the *more* true one contrariety, the *less* true the other two.

The fuzzy logical hexagon is shown in Figure 3. The figure also shows four decorations of the corners. The diagram (shape) itself looks the same as its classical counterpart, shown in Figure 2. However, everything is now interpreted in accordance with the above-given continuum-valued or fuzzy semantics. The first three decorations of the corners shown in Figure 3 are analogous to the decorations in Figure 2. The first shows that each corner can be defined as (fuzzy!) negation of its contradictory corner. The second shows that each corner can be defined in terms of its two neighboring corners – this time by means of operators of *strong* conjunction or disjunction. The third decoration shows how we can define each corner just by means of one sentential operator (denoted by $\mathcal{O}$), along with operators of negation, strong conjunction, and strong disjunction.

The fourth decoration is new. It shows how truth values behave between the corners in the fuzzy setting. The fourth row represents one possible logical model. Let us see how we get the fuzzy numbers. We start with a given. We are given the truth values of contrary corners[4] A, E and Y. These values, as stated above, behave according to the ratio: $v(\mathrm{A}) + v(\mathrm{E}) + v(\mathrm{Y}) = 1$. Then it is easy to get the values of the remaining, subcontrary corners, since each of them is the negation (contradiction) of a particular contrary corner, and $v(\neg p) = 1 - v(p)$. We can also see that each corner can as well be defined either as a strong conjunction or a strong disjunction of the two neighboring corners, depending on whether it is a contrary or a subcontrary corner. Take the contrary corner A. We get its value if we add the values of its two neighboring corners, I and U, and subtract 1, i.e. $v(\mathrm{A}) = v(\mathrm{I} \otimes \mathrm{U}) = max(v(\mathrm{I}) + v(\mathrm{U}) - 1, 0) = max(0.9 + 0.7 - 1, 0) = 0.6$. Take now its negation, the subcontrary corner O. Its value is the sum of its two neighboring corners, i.e. $v(\mathrm{O}) = v(\mathrm{E} \oplus \mathrm{Y}) = min(v(\mathrm{E}) + v(\mathrm{Y}), 1) = min(0.1 + 0.3, 1) = 0.4$.

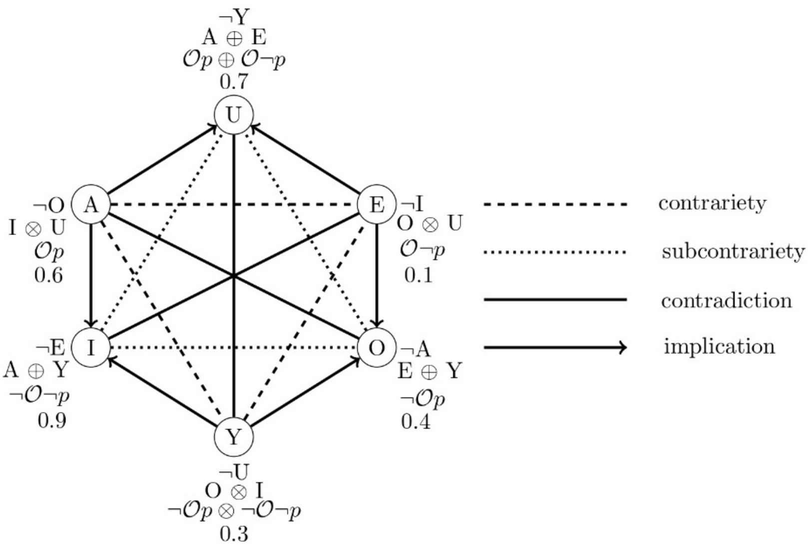

Figure 3: The fuzzy logical hexagon

An axiomatization of the classical logical hexagon can be found in Kalinowski (1967). Axiomatizations of the classical and fuzzy logical hexagon, accounting also for internal negation, can be found in Restović (2022).

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[4] By '(sub)contrary corner', I mean a corner which is in the triangle of (sub)contraries.

2.4 Using the Hexagons in Conceptual Analysis

The logical hexagon – both in its classical and fuzzy version – can also be used to model *concepts*, alongside propositions. This means that the four semantic relations found in the hexagon can also be applied to concepts, *mutatis mutandis.* We do not say that concepts are "true", so we cannot define the four relations in terms of truth and falsity, like we do when we talk about propositions. But we can adapt the definitions in an intuitive way. For instance, we can say that two concepts (or notions) are contrary iff they cannot both *apply* at the same time, but they can both *fail to apply* at the same time. Consider, for example, the notions *impossible* and *necessary.* They cannot both apply at the same time. But they can both fail to apply at the same time. Also, regarding the concept/proposition distinction, sometimes we can define concepts, for logical purposes, in terms of propositions. For instance, instead of "necessity", we can speak about a proposition $\Box p$.

The classical logical hexagon is much more popular than the fuzzy one when it comes to conceptual analysis. This is probably because the fuzzy version is used to model concepts that can come in *degrees*, which arguably makes them less straightforward. Plus, the classical logic is the go-to logic, and so is the classical logical hexagon.

And indeed, many concepts can fit into the classical hexagon. The most common way is to start with a tripartition or a trichotomy. In other words, we take any three concepts that are *jointly exhaustive* and *mutually exclusive*. For example, statements can be either *necessary*, *impossible*, or *contingent*. Or, more mundanely, money can be either *spent*, *saved*, or *invested*. For every such tripartition, there is a classical logical hexagon, in which the three trichotomous concepts are assigned to the three corners in the *triangle of contraries*, A, E, and Y – in no particular order. Then, the rest of the corners are assigned their own concepts, according to the opposition of contradiction indicated in the hexagon. They are defined as negations of their contradictory corners. For modal concepts, we get: *non-necessary, possible* and *non-contingent*. We can see this modal decoration in Figure 2, if we take O to mean the necessity operator, $\Box$.

But let me now turn to the "emotional decoration" of the hexagon. I the following section, I will argue that its fuzzy version can be used as a model of mixed feelings – simultaneous contrary emotions felt towards one and the same state of affairs – given that emotions are one of those things which can come in degrees.

3. A Fuzzy Logical Hexagon of Emotions

3.1 A Case for the Model

Blanché (1966) was the first to arrange the emotions considered in this paper in the classical logical hexagon. He labeled the U-corner 'pathy', the Y-corner 'apathy', A 'philia', I 'aphilia', E 'phobia' and O 'aphobia'. I present both the classical and the fuzzy version of the 'emotional hexagon', in English terms, in Figure 4. The structure is based on the trichotomy desire-fear-indifference. Unlike in Figures 2 and 3, here I present different amount of information attached to each corner (different amount of rows). Assigned to each corner, I show all the different ways to say the thing said in the very first row, i.e., all the ways to logically express desire, fear, indifference, non-desire, non-fear, and non-indifference. These concepts are logically expressed as propositions (in accordance with the use of symbol O in the previous two figures). Also, Figure 4 shows that all the states can be defined only by means of operator D, which stands for "desire" (for corners U and Y this is omitted, but it can be read from other corners). Alternatively, everything can be defined using only operator F ("fear").

I believe that the (trichotomy of) emotions of desire, fear and indifference can be meaningfully modeled with the fuzzy logical hexagon.[5] I make the case with the help of an example.

Before that, an important remark. The three emotions are only considered as they apply to *propositions* which express *states of affairs*. I do not consider the three emotions when they are felt towards *objects*. This choice is made to make sense of the talk of, say, desiring a negation of something: you can wish not to go for a picnic, but you cannot want a 'non-apple'. I do not think, however, this is a serious limitation, since the talk about objects can be plausibly translated into propositional terms, like "I desire not *having* an apple".[6]

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[5] I use the term 'model' for the fuzzy logical hexagon in a less formal, wider sense, to mean a (diagrammatic) representation of *semantic* relations. In the formal, narrow sense of the word, the (fuzzy) logical hexagon is "an abstract structure or *theory* which can be interpreted in many different ways. We can say that it is a *theory with different models*" (Béziau 2012: 5, added emphasis), one of them being the here considered concepts describing emotional states.

[6] Or, more long-windedly: "I desire it not to be the case that I eat an apple right now". Cf. Świętorzecka's (2008) 'situational counterparts of substances' in her logic of change.

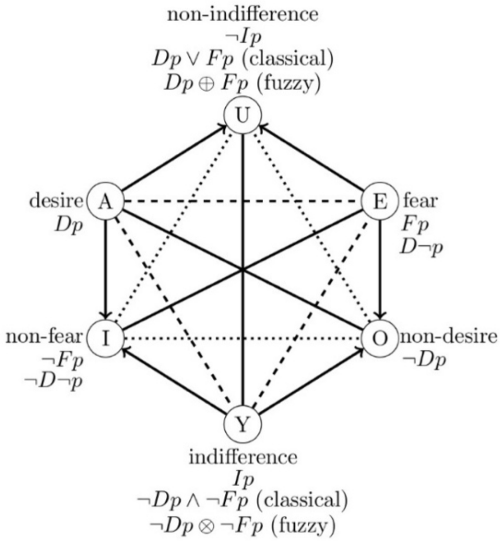

Figure 4: The classical and the fuzzy emotional hexagon

Now, consider a particular state of affairs – travelling to Antarctica. As a matter of fact, personally and *prima facie*, I somewhat desire to go there. Now, I do not only claim that it makes sense to say that I *somewhat* desire it, but that it also, at the same time, makes sense to say that, to a certain degree, I am afraid of the experience, as well as that I am partially indifferent towards it. (And also, that this is applicable to other people, *mutatis mutandis* and possibly taking another destination or example – and not just a feature of my own psychological idiosyncrasies.) How do I make sense of this? It is sensical if we consider the *relevant implications* of the proposed propositional object of emotion.

All propositions have implications. That which follows (by logical implication) from a proposition are, obviously, its logical consequences, among which there are also the *necessary conditions* for the proposition.[7] However, not all implications of a state can be considered (emotionally) relevant. Those that are, I here call 'aspects'. In a nutshell (and, I maintain, quite uncontroversially): towards different aspects of one and the same state of affairs a person may feel different emotions. When aspects

[7] Now, speaking of states of affairs or situations in the world (which propositions are here meant to represent), the consequences of a particular situation come *after* the fact, while the necessary conditions come *before*. However, both are logically implied by (i.e., logically follow from) the considered situation and can thus be, I believe, legitimately called 'implications'.

are sorted according to different emotional categories, I will argue, a pattern emerges.

Returning to the above example, going to Antarctica and spending quality time there (for me) implies a lot of things. For a start, to get there I definitely have to take a plane and/or a boat. This is an aspect of going. Now, I can consider my emotions towards it. For instance, I may be nervous about my tendency to get seasick, and also be afraid of flying. Taking that into account, these unpleasant aspects add to the fear of the initial experience. I may also be afraid that, when I get there, I will constantly be cold, which would make me want to go even less. But there are, obviously, implications that add to the desirability of going to Antarctica. Firstly, going means you can (truthfully) say you have been there, which is something I would surely want to brag about. Not to mention all the beautiful scenery, and the peace and quiet to clear my thoughts (or so I imagine). Finally, once there, I am sure the food would be different, but I am usually indifferent about what I eat. So, there are also aspects which I neither desire nor fear.

Of course, people cannot be expected to list *all* the implications of a particular state of affairs. This would make us logically omniscient. We do not have to, however, since not all implications are relevant – some of them would not be considered 'aspects'. But still, we probably can neither list all the *aspects* of a particular state. Psychologically, there will always be some aspect of our (potential) desire that we had not predicted or noticed. But I suppose that there will always be some list of aspects we *did* manage to produce.

Let us say we have a (probably incomplete) list of aspects of some state of affairs, expressed by proposition *s*. For simplicity, let there be 100 of them. Let us also say that a person feels desire towards 53 of them, indifference towards 27, and fear towards the remaining 20. Then, the degree of desire towards *s* is 53%, and accordingly for fear and indifference. What we get is a *ratio* of contrary feelings, call it the 'emotional ratio', resulting from an 'emotional calculation' (and in accordance with the fuzzy logical hexagon). For the state of affairs *s*, desire prevails. Then we can say that the person 'overall desires' that state.

In my opinion, this may be what we sometimes mean when we say that we desire something – that we *mostly* (or 'overall') desire it. We mostly do not absolutely desire anything. There are almost always downsides (fears) and 'middlesides' (indifferences) to our desires.

Consider the old expression "Be careful what you wish for (for it may come true)!". It may be interpreted as a warning against unexamined desires, i.e., against their unexplored undesirable consequences – a suggestion that there may be a 'high measure of fear' in a given desire.

Generally, then, overall desire/fear/indifference is a result of a particular ratio of the three contraries, where one element, desire/fear/indifference respectively, is predominant. But in any case, the sum of all contrary concepts gives 100% or, in the language of fuzzy truth values, 1.

Of course, there may be cases where, after an emotional calculation, no element prevails. What is the overall value then? Is it indifference or not even that? It may be tempting to assert that, when a subject feels equal degrees of desire and fear towards *s*, they are overall *indifferent* towards *s*. However, in the proposed model of emotions, this need not be the case. Indifference is not just a point of separation between desire and fear, it is an entire area in its own right (cf. Massin 2014). It occupies a part of the emotional ratio, just like desire and fear do. Let $d(s)$, $f(s)$ and $i(s)$ respectively stand for the degree of desire, fear and indifference towards the state *s*. Say that $f(s) = 40\%$, $d(s) = 40\%$, and $i(s) = 20\%$. Even though the strength of desire is equal to that of fear, the subject cannot be said to be overall indifferent towards *s*, since the value of indifference does not prevail. Rather, what prevails is its exact opposite (i.e., its contradiction), *non-indifference*, the value of which corresponds to the sum of values of desire and fear (see the U-corner in Figures 3 and 4). Analogously, if we had $f(s) = 40$, $d(s) = 20$ and $i(s) = 40$, the prevailing emotion would be *non-desire*, and so on. In the case where the values of all three contraries are equal, no emotion prevails.

Let me now address some possible concerns regarding the (so far) proposed model.

3.2 Some Concerns about the Proposed Model

The above example of an emotional calculation with 100 states of affairs is grossly oversimplified. Our emotions towards different consequences of (and requirements for) some state of affairs – an easily observable fact upon a minute in(tro)spection – each carry their own weight. In other words, aspects are emotionally fuzzy, too. This is expressible in the proposed model by recognizing that each aspect has its own aspects (i.e., relevant implications), which have their own aspects and so on, not all of which can fall under the same emotion. But then, any emotional calculation seems hopeless – since we go deeper and deeper in listing the (aspects of) aspects and assigning to them different feelings, it looks like we will never come back to the surface to give a ratio for the initial state of affairs we want to analyze. There lurks an *infinite regress* among the aspects.

For instance, I do not like flying, but I like an aspect of it – the airplane food. Further, good food for me is more desirable provided I can have a cigarette after the meal, but this desire will surely be frustrated by the airline policy. Then again, not smoking when I want to brings me closer to quitting, something I overall desire. However, nervousness is a probable negative side effect of quitting cigarettes. What are the pros of being nervous? Following all the implications of an aspect deeper and deeper could make me lose track of what I was initially considering – it was actually a trip to Antarctica, not only the experience of being on a flight, that I started thinking about.

Well, I admit that weighing aspects is psychologically demanding and that it may make us lose ourselves in particularities – and forget what we were initially trying to calculate. However, from the logical point of view, these particularities are still relevant to the thing we started from. Psychologically, we may lose the thread, but logically, we are still on point. This is because aspects are (relevant) implications, and implications are *transitive*. All the aspects of aspects (of aspects...) of *s* are themselves aspects of *s*. There is still only one list.

However, if we hope to find any emotional ratio, listing relevant implications has to stop at some point. At that point, we have to assign emotional ratios to propositions on the list *without* further listing their aspects by means of which we could perform an emotional calculation. How do we do that?

I provide only a sketch. I assume that there will be some subjective *ordering of preferences* among the aspects on the list. I simply take that as a given. So, at some point, one must decide not to list aspects (of aspects) of the initial state *s* any further. Then, they can look at the list in search of aspects that are emotionally clear-cut. For instance, the subject can identify an aspect of *s* that they find absolutely unbearable, which would get a degree of fear of 100%. Let us call these types of aspects 'prototypes'. Or, alternatively, the prototypes can be the aspects that the subject is the most sure about, which would not all have to be 100% on the side of one emotion. In any case, prototypes are good starting points for comparative analysis. The subject then proceeds to comparatively assign ratios to remaining aspects, having prototypes in mind and following the pre-existing preference ordering. For instance, there may be an aspect which they find less desirable than a prototype (which itself has a desire-value of 100%), so they assign to it the measure of desirability of 90% and decide that the rest 10% is occupied by fear, comparing it with another prototype towards which they feel there is tenfold more fear.

Now, having in mind that all the aspects we list are the aspects of the initial state *s*, there is a way to calculate the percentage of each of the three contrary emotions towards *s*. The percentage of desire assigned to the state of affairs *s* would correspond to the *arithmetic mean* of desire values of its aspects. Analogous descriptions apply to the percentage of fear and the percentage of indifference towards *s*. In the end, the values still behave according to the emotional ratio, which states that values of desire, fear, and indifference towards a state of affairs add up to 100%. The underling logic is the same as in the simplified example provided in the previous subsection, even though aspects can be assigned fuzzy (i.e., decimal) numbers.

There is another thing to hold in mind, though. When we decide to call some aspects prototypes, we must ensure that these states of affairs do not imply one another. (I admit, this solution presupposes a kind of logical omniscience.) Otherwise, the entire calculation of emotional values may collapse. It is intended that we assign values to prototypes *independently*. And if one of them logically implies the other (or they both imply each other), their values cannot be independent. If we later discover that some prototypes are mutually logically dependent, all the comparative analysis is rendered contradictory – and useless. Not counting on logical omniscience, another solution may be to delete some aspects from the list, if we are not sure if they imply (or are implied by) some other aspects.

All that being said, to fully describe the phenomenon of assigning emotional ratios, we would presumably need more tools. For instance, in a possible calculation we may have to take into account that some relevant implications are only *probable* and/or *possible*. E.g., it is not the same to feel fear towards an aspect which will *certainly* happen and towards an aspect which only *can* happen. So, there might be additional weights to the aspects on the list. These considerations are beyond the scope of this paper. Consequently, the way in which we get ratios remains, at least in part, a black box. However, I claim that, whatever comes out of it, will be divided among the logico-conceptual space in accordance with the triangle of contrary notions we find in the fuzzy logical hexagon.

Having hopefully made the case for a fuzzy interpretation of simultaneous contrary emotions towards the same state of affairs, in the following section I weigh the proposed model against some opposing logical and philosophical approaches.

4. A Fuzzy Logical Hexagon of Emotions in Context

4.1 Graded Formal Accounts of Desire and Fear

A nuanced formal theory of desire, coming in part from the authors who gave us the fuzzy logical hexagon, can be found in Dubois, Lorini and Prade (2017). The authors also acknowledge that desire comes in percentages, however, some of their underlying philosophical assumptions are at odds with the approach proposed here.

Most importantly, although they recognize the logico-conceptual space inhabited by desire – by providing attention also to fear and indifference – their resulting logic nevertheless explicitly excludes the possibility of simultaneous desire and fear (where fear is also understood as desire for negation or 'negative desire') towards a particular state of affairs. *A fortiori*, their whole enterprise can be seen as built on that assumption, generating a need for further logical means of avoiding this phenomenon:

> [D]esiring φ and $\neg\varphi$ at the same time is not usually regarded as rational, since it does not make much sense to desire one thing and its contrary at the same time. Thus when a new desire is added to the set of desires of an agent, a revision process may be necessary. (Dubois, Lorini and Prade 2017: 199)

The authors provide a desire revision process counterpart to the AGM theory of belief change (Gärdenfors 1988). On the account proposed here, however, there is nothing problematic in desiring both p and $\neg p$, so this is not a reason for desire revision. But this does not mean that there is never any need for a revision of any kind. It may be necessary during the assignment of comparative emotional ratios, if it turns out that there are aspects of some state of affairs which are not mutually independent.

Moreover, to avoid conflict of emotions, Dubois, Lorini and Prade (2017) consider only *positive* desires, where states which are not in the set of desires are understood as the ones towards which the agent is *indifferent*. Nevertheless, they are not oblivious to fears, acknowledging that "modelling both desires and the fear of unbearable situations would require a bipolar setting [...] leaving room for both positive and negative desires" (Dubois, Lorini and Prade 2017: 200). This is done elsewhere, which I consider shortly. In the uni-polar setting, they develop a crucial insight that desires behave in a way *reverse* to beliefs. They claim that, if one desires q and $p \rightarrow q$, then they should also desire p (and if they do not, they should revise their desires according to that norm). On the other hand, for *beliefs*, the implication goes in the regular, non-reverse, way – if one believes p and $p \rightarrow q$, then they should also believe q.

The authors acknowledge, however, that this only works if fears are excluded:

> There is an obvious objection to the reverse entailment for desires. Namely, desiring coffee does not imply desiring coffee while being in a very unpleasant situation, like having a close friend killed. It is clear that you would find this situation unbearable, even with a coffee. However, we have to recall that we do exclude unbearable situations. (Dubois, Lorini and Prade 2017: 206)

They probably have in mind situations like a wake for a deceased friend, where one can expect to be served coffee. Wake implies coffee, we desire coffee, but we surely do not desire wake. So, new desires can come from the stock of situations that are either already desirable or potentially desirable.

On the other hand, the fuzzy logical hexagon model of emotions does not exclude fears, modeling them *simultaneously* with desires and indifferences. How does it deal with the proposed unbearable situation? So, if wake, then coffee. In the proposed model, coffee is an aspect of a wake, along with a myriad of other things. Controversially, getting coffee turns out to be a desirable aspect of the tragedy. And if this is so, having a friend die turns out not to be an absolutely undesirable event. Of course, the tragedy will be (overall) feared, but finding *any* 'silver lining' in it may sound inappropriate and/or just false. Some may find it bordering on the macabre. I have no clear and convincing answer to this. Examples like these *do* pose a problem for the here given model, call it 'the silver lining problem'.

Now, there are alternative accounts that take both desires *and* fears into account. Dubois, Lorini and Prade (2017) themselves point to the formalisms of Benferhat et al. (2006) and Dubois and Prade (2009). These approaches also differ from the one taken here. They presuppose a *bipolar setting*, having in mind a specific type of bipolarity, called 'asymmetric' or 'heterogeneous' bipolarity (Dubois and Prade 2009), treating the negative and the positive preferences *separately*: the agent first eliminates the intolerable states, and only then establishes the desirable ones among those which remain.[8] To represent this, the said formalisms provide two different functions, π and δ, defined on a set of mutually exclusive interpretations (also called 'situations' or 'solutions') and ranging over the interval [0,1]. The higher the value of $\pi(s)$, the more tolerated (i.e., less rejected, more feasible) the situation in question. The higher the value of $\delta(s)$, the more satisfactory (i.e., desirable) the situation s. In their asymmetric approach, e.g., if some s is fully satisfactory, that means both that $\pi(s) = 1$ and that $\delta(s) = 1$. The two measures are independent and not mutually definable, as is the case with the fuzzy hexagon model.

From the tolerability and satisfiability measures of *interpretations*, the two models arrive at the tolerability and satisfiability measures of *propositions*. This is done in the framework of *possibility theory*, the standard arsenal of which includes possibility and necessity measures. Let me here outline these measures as they appear in possibility theory not (yet) applied to the notions of desire and fear. The central notion of possibility theory is that of a *possibility distribution*, denoted by π (not to be confused with the above mentioned π as opposed to δ). It is a mapping from the set of situations (interpretations) S to the unit interval [0,1], expressing the extent to which a given situation $s \in S$ is considered possible (plausible) by an agent, ranging from fully impossible to fully possible. Given an ordering of possibility of situations, we can define two possibility measures and two necessity measures for propositions (cf. Dubois, Hájek and Prade 2000). *Potential* (or *weak*) possibility of a proposition p, denoted $\Pi(p)$, is defined in the following way:

$$\Pi(p) = max\{\pi(s) : s \in S \text{ and } s \vDash p\}.$$

In other words, it is the value of the most plausible situation in which p is the case. The dual operator of potential possibility is *strong* necessity, denoted $N(p)$ and defined as:

[8] Thus avoiding the 'silver lining problem'.

$$N(p) = 1 - \Pi(\neg p).$$

Additionally, there is the notion of the *guaranteed* (*strong* or *actual*) possibility of p, denoted $\triangle(p)$, which takes the value of the least plausible situation in which p is the case:

$$\triangle(p) = min\{\pi(s) : s \in S \text{ and } s \vDash p\}.$$

Its dual is *weak* (or *potential*) necessity of p, denoted $\nabla(p)$, the measure of which corresponds to:

$$\nabla(p) = 1 - \triangle(\neg p).$$

The logic of potential possibility differs from that of guaranteed possibility. Significantly, in the former:

- $\Pi(p_n \vee p_m) = max(\Pi(p_n), \Pi(p_m))$
- $\mathrm{N}(p_n \wedge p_m) = min(N(p_n), N(p_m))$.

On the other hand, in the logic of guaranteed possibility:

- $\triangle(p_n \vee p_m) = min(\triangle(p_n), \triangle(p_m))$
- $\nabla(p_n \wedge p_m) = max(\nabla(p_n), \nabla(p_m))$.

As mentioned earlier, Benferhat et al. (2006) and Dubois and Prade (2009) use two distinct functions for modeling negative and positive preferences, more precisely, they use *two possibility distributions*, π and δ respectively. Now, on that approach, negative and positive desires follow different logics: negative preferences are modeled in the logic of *potential* possibility, while positive preferences are modeled in the logic of *guaranteed* possibility.[9] Even though positive and negative desires are treated separately, they do not live in two disparate niches – they are both possessed by the same agent. In the two formalisms, desires and fears are ultimately *merged*. When this is done, coherence has to be established. In other words, just like in the unipolar setting, some revision may be necessary. Coherence between positive and negative preferences is established by the following condition:

$$\forall s, \delta(s) \leq \pi(s).$$

[9] Dubois, Lorini and Prade (2017) use a logic of guaranteed possibility in their (unipolar) model of positive preferences, so their approach there can be viewed as a (more nuanced) fragment of the bipolar setting.

On the contrary, in the proposed view, the logic of desires (positive preferences) and the logic of fears (negative preferences) is one and the same, since these emotions are mutually definable and can be simultaneously meaningfully represented by the fuzzy logical hexagon.

Moreover, as discovered by Dubois and Prade (2012), the measures of potential possibility and strong necessity can *themselves* fit into the graded logical hexagon, resulting in a structure analogous to the classical (i.e., non-fuzzy) hexagon of modalities $\Box$ and $\Diamond$, where $\Box p \leftrightarrow \neg\Diamond\neg p$. In my opinion, the fact that potential possibility and strong necessity fit the same structure as operators of desire and fear may suggest that the appropriate logic for both these emotions is that of *potential* possibility. However, I will not go any further into strictly logical considerations, turning now rather to philosophical ones.

4.2 Philosophy of Desire vs. Fear

The view that desires and fears can be present simultaneously is not universally accepted also in the field of philosophy of affectivity, where there is an ongoing discussion on the possibility and meaning of 'mixed feelings'. It is exactly the (classical) notion of *contrariety* appearing in the description of this phenomenon that makes it philosophically problematic. How can one have contrary emotions towards the same state of affairs at once if contrary notions cannot all be true? There are, as I see it, two general approaches to answering this question.

The first approach is to argue that in mixed feelings there are actually no contrary emotions present since they do not have the same object (cf. Tappolet 2005; Zaborowski 2020). A popular strategy (and term) in dispelling the possibility of two contrary emotions being true about exactly the same thing is to talk, as I do here, about *aspects*. Different aspects of the (propositional) object are usually understood as its different features, different modes of presentation or different properties. In Tappolet's (2005: 231) words:

> [G]iven that something can well be both attractive in one respect [...] while unattractive in another respect [...], there is no contradiction. This is not different from the situation in which something, like most good things in life, is desirable *qua* being pleasurable and undesirable *qua* being treat to your health.

Similarly, Zaborowski (2020: 204) argues:

> [I]f Paul likes Peter because of one feature and he dislikes him because of another feature, it may be said that Paul likes Peter's first feature while he dislikes Peter's

> other feature. More precisely, it is manifest that Paul likes Peter *qua* x (= Px), while he dislikes him *qua* y (= Py). This is to say that liking and disliking relate to two differently described or constructed persons (Px, Py), that is, to two different objects.

The other approach is to claim that the simultaneous presence of contrary emotions towards the same state of affairs is not (as) problematic as it is within, say, beliefs, since emotions have a logic of their own. This approach is taken, e.g., by Greenspan (1980) and Marino (2009).

The former argues that emotions cannot be identified with 'judgments' (i.e., beliefs). For one, these two sorts of states follow different adequacy criteria: "appropriateness is the value for emotions which comes closest to truth for judgments" (Greenspan 1980: 236). While contrariety between judgments has to be settled on pain of irrationality, contrary emotions *may* be appropriate, for different *reasons*: "an emotion is appropriate as long as there are reasons *for* it, whatever the reasons against it" (Greenspan 1980: 237). However, contrary emotions cannot be appropriate for the *same* reason. It is in this sense, Greenspan maintains, that we can say some emotions are (classically) *contrary*, i.e., that they cannot both be true at the same time, but can both be false at the same time.

Although they feature in adequacy criteria for emotions, reasons need not be taken as defining the *object* of an emotion: contrary emotions are still felt towards *the same content*. Using the example of a rivalry with a friend, Greenspan (1980: 233) argues:

> [E]ven if I do feel happy for my rival, or happy about his winning *in that* it satisfies a desire of someone I identify with, I would normally still feel happy about his winning – *simpliciter* – so that my emotion cannot be said to be truly qualified. Hence emotions should not be identified with qualified evaluative judgments.

Reasons do not define the objects of emotions, she maintains, also because we can have mixed feelings *before* we know all the reasons for them. This remark may propose a problem for the fuzzy logical hexagon, which I consider shortly. However, the hexagonal model of mixed feelings is similar to Greenspan's account in that she recognizes different (fine-grained) reasons for mixed feelings, but does not thereby analyze away contrariety, considering such a move "fiddling with the object of contrary emotions" (1980: 229).

Marino (2009), speaking specifically about desires, does allow that sometimes we can speak about different, independent aspects (*x* and *y*) of a desired object (*a*), but that there are some cases (like Greenspan's 'friendly rivalry' example) where, ultimately, contrary emotions are felt towards *one and the same* propositional content: "may I not still feel that I want my rival to win (*a*) because he is my friend, and want him not to win (not-*a*), because he is my rival? It is not clear what we would take to be *x* and *y* in this case" (Marino 2009: 278). Another case that can bring us to desire both a proposition and its negation is, she maintains, *instrumentality*. If I want seemingly non-contrary *x* and *y*, but *a* is the only means of getting *x*, while not-*a* is the only means of getting *y*, contrary emotions towards the same content come right back.

However, Marino sees nothing irrational or unusual with wanting both *a* and not-*a*: "[t]o be for and against something is not, in itself, to make any sort of error" (2009: 288). This only becomes problematic when one starts planning (f)or acting based on the underlying conflicting desires. A plan needs to be consistent, desires do not. She concludes that "'consistency' for desires is indeed importantly different from consistency for beliefs" (2009: 291).

The fuzzy hexagon model of emotions has similarities with both general approaches to the "problem" of mixed feelings. In line with the second approach, it claims that mixed feelings are ultimately felt towards the same proposition and that the logic of emotion is not classical. In line with the first approach, it claims that a more fine-grained analysis (in terms of aspects) is necessary for explaining mixed feelings.

However, in contrast to the first approach, it does not go as far as to claim that mixed feelings are thus explained away. While the usual 'aspect-strategy' claims that the emotional conflict is *not* about the same (propositional) object but about different aspects of that (propositional) object, according to the fuzzy hexagon, the conflict *is* about the same propositional object exactly *in virtue of* being about different aspects of that propositional object. (Also, aspects are understood somewhat differently, as relevant implications.)

Moreover, proponents of the aspect-strategy (plus Greenspan 1980) seem not to allow any mixture of contrary emotions among aspects themselves: feelings towards aspects are unambiguous, wholehearted, discrete or crisp (as opposed to fuzzy). This is where the here given proposal differs: the 'emotional ratio' desire/fear/indifference goes through and through – there can be a ratio of contrary emotions even for the aspects on the 'final list', acquired based on a given (subjective) preference ordering.

Let me now return to Greenspan's remark mentioned earlier. It seems plausible, as she notices, that one could have mixed feelings towards a (propositional) object without or prior to knowing all the reasons for the emotions. In this scenario, the mixture of feelings cannot be the result of any background 'calculation', since there is nothing to calculate. This phenomenon seems to go against the fuzzy hexagon (plus the corresponding method of emotional calculation) as a model of mixed feelings.

Now, on the one hand, the proposed model ultimately depends on a given list of (ordered) preferences as a means of avoiding an infinite regress brought by listing the relevant implications of (relevant implications of...) a given state of affairs. In doing so, as sketched earlier, it *does* allow for mixed feelings about aspects even without (an underlying) calculation. (I.e., there can be mixed feeling even for the 'prototypes'.)

But, on the other hand, I claimed that there has to be *some list* of aspects of the *initial* state of affairs which we want to analyze and ascribe to them a ratio of contrary emotions. If there are no aspects (reasons), there is no support for mixed feelings. This problem may be alleviated if we take the same approach to the initial state of affairs as we do towards the 'final' aspects. If the initial situation s (e.g., travelling to Antarctica) is itself already contained in the given set of (ordered) preferences, we can ascribe to it an emotional ratio without listing *any* further aspects of it.

I do admit, however, that this maneuver may just relocate the problem, without really addressing it. The question remains of what the elements (and structure) of the presupposed ordering of preferences are and whether the considered state *appears in* it, or just *emerges from* it. In other words, I may instantly develop a strong desire to go to Antarctica upon merely entertaining that thought and without really considering the idea or knowing why I feel this way – and the answer that going to Antarctica is somehow among my 'atomic', deepest or ultimate desires may not be convincing. In any case, the ordering of preferences, here only presupposed, may be further developed, perhaps modeled with a 'desirability distribution' function akin to the possibility distribution (π) one finds in possibility theory. I leave this for another occasion.

Let me here mention another critique of the possibility of mixed feelings, which I did not consider in detail. Instead of denying that contrary emotions are felt towards the same object, one can also deny that contrary emotions are felt at exactly the same time. Mixed feeling may mean that one *oscillates* between contraries (cf. Zaborowski 2020). I do not think that the fuzzy hexagon forbids emotions towards aspects of a state of affairs to be felt at the same time. They may not all be at the forefront at a given moment, e.g., I may be dwelling on a particular undesirable consequence of going to Antarctica. But the list of aspects is considered and kept in mind in its entirety at the same time – only by acknowledging and weighing the aspects *simultaneously*, i.e., synchronously, do we get an emotional ratio.

The proposed model of contrary emotions leaves a number of concerns not fully addressed. There remain questions concerning the phenomenology of mixed feelings (e.g., the conflict felt when experiencing this phenomenon), the ethical implications of mixed feelings, as well as contrary desires in the framework of action theory, where one can act according only to one among the simultaneously present contrary emotions.

5. Conclusion

The possible (and not uncommon) experience of simultaneous emotions of desire, fear and indifference towards one and the same state of affairs *s* (i.e., a case of 'mixed feelings') can be modeled – I argue in this paper – by the fuzzy logical hexagon, a non-classical structure of oppositions in which truth comes in degrees, allowing for contrary notions to be true at the same time, but with a special proviso that the truth values of contraries always add up to 100%.

According to the fuzzy hexagon, for instance, towards *s* we may feel desire to the degree of 50%, fear to the degree of 30%, and be indifferent the rest 20%. To arrive at this 'emotional ratio', I propose to consider the relevant implications of *s* (which I call 'aspects') and classify them under the three mutually contrary notions. Some implications of *s* may be desirable, some feared, and towards some there may be indifference. Then, in accordance with the classification of its aspects, we can say that the said state (partially) satisfies all three contrary notions at once. There, however, appears an infinite regress in this process, since aspects have aspects of their own (and so on, *ad infinitum*). The regress is here avoided by taking as a given the existence of some underlying ordering of preferences, on the basis of which any calculation of emotions is performed.

The present approach is in opposition to a non-realist stance in philosophy of affectivity about the possibility of simultaneous contrary emotions, where 'aspects' are used to explain *away* the possibility of mixed feelings. There, it is claimed that a fine-grained analysis of a state shows that contrary emotions are ultimately felt towards *different* (propositional) objects, so genuine mixed feelings do not exist. I argue for a different, non-classical interpretation, in which the mixture of emotions is about one and the same state of affairs exactly *in virtue of* conflicting emotions being felt towards different implications of that one and the same state.

I consider also some fuzzy formal theories of emotion coming from the opposing philosophical camp, in which an underlying anti-realist stance towards mixed feelings generates a need for a desire revision mechanism and separate logical treatment of desires and fears. Contrary to them, I argue that logic of desires and fears is one and the same – that of the fuzzy logical hexagon, and that it can perhaps be further developed using the logic of potential possibility as opposed to (also) the logic of guaranteed possibility.

The proposed model naturally faces problems, some possibly endemic. One is here called 'the silver lining problem', where some unbearable situations necessarily include desirable dimensions. Another is the possibility of *emotions without reasons*, mixed feelings that "just appear" and cannot be explained in terms of calculation of aspects, as the proposed model suggests. Lastly, some important facets of the proposed model are treated only in outline, like the phenomenon of the (here presupposed) ordering of preferences, which is necessary to perform the 'emotional calculation'.[10] All this (not) being said, I hope this paper helps the realists' case for mixed feelings and contributes to the logical analysis of the phenomenon.[11]

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[10] As an anonymous reviewer suggests, the presupposed preference ordering is transitive, which need not be the case in real life. Moreover, as they note, the fact that the aspects of aspects (of aspects of aspects…) of state *s* are still all aspects of the initial state *s* – via transitivity – can also be challenged, if we want to model away the logical omniscience.

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