Exploring Social Interactions on the Adriatic Network

Žiga Velkavrh

This paper explores social interactions on the Adriatic network consisting of six countries surrounding the Adriatic Sea. Using game theory, we analyze how three well-known classes of 2x2 strategic games, namely Prisoner's dilemma, anti-coordination and coordination games, would be played on the Adriatic network. We determine all Nash equilibria, i.e., steady states, and obtain two main results. First, anti-coordination games on the Adriatic network always induce multiple (4, 5, 7 or 12) Nash equilibria that vary with payoffs and may differ in efficiency. Second, coordination games on the Adriatic network have only trivial equilibria, unless a specific condition on payoffs is met, in which case two new equilibria emerge. Our findings may be of great interest for policy makers and other scholars interested in maritime pollution control and other water-related problems, as well as biodiversity conservation, as they indicate at which maritime borders (anti)coordination issues and resulting inefficiencies may arise. Knowing that, one may give special attention to the critical maritime borders and take extra care there, thus helping to prevent potential catastrophic events. Finally, our study can also be used for academic purposes, e.g., in classroom, to demonstrate how to perform a complete Nash equilibrium analysis on some real-world network which has a relatively simple structure.

KEY WORDS

~ Adriatic
~ Anti-coordination games
~ Coordination games
~ Game theory
~ Nash equilibrium
~ Networks
~ Prisoner's Dilemma
~ Spatial games

University of Primorska, Faculty of Mathematics, Natural Sciences and Information Technologies, Koper, Slovenia

e-mail: ziga.velkavrh@upr.si
doi: 10.7225/toms.v14.n03.024
Received on: 9 Sep 2024; Revised on: 8 Oct 2025; Accepted on: 8 Oct 2025; Published online: 21 Oct 2025

1. INTRODUCTION

The motivation for the present paper comes from one of the major environmental problems, maritime pollution. This problem is especially acute for countries surrounding semi-enclosed seas, such as the Adriatic Sea, which represents the core of the paper. The Adriatic Sea is vital for Adriatic economy (e.g., tourism, fishing, maritime transport) but due to the high number of vessels vulnerable to maritime pollution, especially because of the slow exchange of water with the Mediterranean Sea (Vidas, 2009). As an illustration, consider the coastal countries that must simultaneously decide whether or not to engage in pollution control and mitigation along their maritime borders. Will they engage in these activities or will they free-ride on neighboring countries? Will their decisions depend on their geographic location or perhaps on the magnitude of potential negative consequences of pollution? These sorts of questions can be explored and answered by applying game theory to networks. Game theory is now a standard tool used to analyze steady states of interactive situations, involving rational entities such as national governments and authorities. In game-theoretic models like those presented in this paper, in which individuals make decisions simultaneously and autonomously, steady states are called Nash equilibria and represent the situations in which no one has an incentive to deviate from their current action, given the fixed actions of others (Nash, 1950; 1951).

The main goal of this study is to identify all Nash equilibria for certain classes of games played on network of Adriatic countries, or, shortly, the Adriatic network. In general, this can be a very difficult computational task (Bramoullé, 2007). On the Adriatic network, however, this is possible since it contains only six countries surrounding the Adriatic Sea: Albania, Bosnia and Herzegovina, Croatia, Italy, Montenegro and Slovenia.

Nowadays, game theory is widely used in many academic fields, including those that investigate transboundary pollution problems, water resources management and pollution control (Fernandez, 2002; Madani, 2010; Grdović Gnip and Velkavrh, 2022, among others). Given the spatial nature of the problems considered in the present paper, the most closely related literature is that which considers games played on networks. A nice introduction to these game-theoretic models can be found in various books such as Young (1998, subchapter 6.1) and Goyal (2007). This literature together with the below-mentioned Bramoullé (2007) and Muñoz and Mc Gettrick (2021) considers local effects, i.e., the impact of direct neighbors, and shows, among other things, that Nash equilibria generally depend on particular network structure – topology. The present paper addresses three well-known classes of 2x2 strategic games – Prisoner’s Dilemma, coordination and anti-coordination games, and examines how they would be played on the Adriatic network. All these three classes have gained attention not only among economists and other social scientists, but also among scholars interested in game-theoretic modelling of water-related problems. Madani (2010), for example, promoted game theory among water scholars by demonstrating how groundwater exploitation by farmers can be modelled as a Prisoner’s Dilemma game; past Iran–Afghanistan water dispute on Helmand (Hirmand) River as a chicken game (i.e., an example of an anti-coordination game); and lake ecosystem maintenance by two countries as a stag-hunt game (i.e., an example of a coordination game).

In this paper the main attention is devoted to anti-coordination games, as they are, at least from a game-theoretic point of view, the most interesting of the three to study on our network of Adriatic countries, because they generate the highest number of Nash equilibria. The literature closest to ours, in the sense that its main focus are anti-coordination games played once, on a fixed network, is Bramoullé (2007). In his seminal paper the author examines refinements of Nash equilibria of these games played on certain classes of networks (complete, bipartite and core-periphery, among others) and proves some general results, for example, that players can anti-coordinate with all their neighbors if and only if a network is bipartite.[1] This and other results of

[1]A bipartite network is one that does not contain an odd cycle (see, e.g., Bondy and Murty, 1976, Theorem 1.2).

his study highlight how important the network structure may be for overall efficiency and differs from the result for coordination games, where coordination is always possible, regardless of the network structure (see, e.g., Goyal, 2007, Theorem 4.1).

Our paper can be seen as a real-world application of Bramoullé (2007). The scope of our study is much narrower – as it considers only one special network – but more detailed. In particular, we do not limit ourselves to some Nash equilibrium refinements, but instead determine all Nash equilibria for (anti-coordination) games played on the Adriatic network.

Apart from Bramoullé (2007), anti-coordination games on networks have recently caught attention of theorists studying asymptotic growth of Nash equilibria. Muñoz and Mc Gettrick (2021), for example, focus on anti-coordination games played on particular networks (graphs), called (circular) ladder graphs and investigate how the number of Nash equilibria grows with the number of players, i.e., with order and size of the graph. They show that the number of Nash equilibria grows exponentially with the number of players, and also illustrate that network topology plays an important role when players engage in anti-coordination games. While the findings of this paper are interesting, they cannot be applied to our network, which has different topology and fixed number of players (i.e., countries).

The present paper contributes to the literature by providing all Nash equilibria for certain classes of games played on the Adriatic network. Our results may be relevant for everyone interested in maritime pollution control and/or other water-related problems, such as water resources management, as it indicates at which maritime borders (anti)coordination problems and resulting inefficiencies may arise. Knowing that, one may give special attention to the critical maritime borders and take extra care there, thus helping to prevent potential damage. Since we use generic parameters (i.e., payoffs), our analysis also shows how the situation and steady states change with different parameter values. Finally, our Adriatic example can also be used for academic purposes, e.g., in classroom, to demonstrate to students how Nash equilibria of some real-world network can be relatively easily found if it has a relatively simple structure.

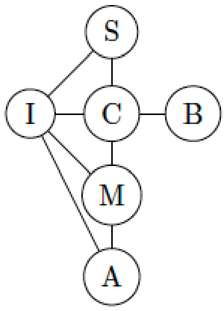
The paper has the following structure: after this brief introductory section, section 2 presents the Adriatic network and its properties. Section 3 describes first the three different classes of strategic games that are to be analyzed on the Adriatic network and then the solution concept, Nash equilibrium, that is used to analyze them. In section 4, the Nash equilibrium analysis on the Adriatic network is carried out, and the main results are given in Propositions 1 and 2. Section 5 is reserved for discussion, while section 6 concludes the paper.

2. THE ADRIATIC NETWORK

The decisions of countries are often influenced by the decisions of neighboring countries, which are in turn influenced by the decisions of their neighboring countries, and so on. Therefore, the decision-making process is in general structure-dependent and more involved than a simple optimization problem where the objective function, e.g., decision maker's (expected) utility, depends only on decision maker's decisions.

The simplified relationship between Adriatic countries can be represented with a simple network or graph. Denote by $\mathcal{N} = \{A, B, C, I, M, S\}$ the set of six Adriatic countries: Albania, Bosnia and Herzegovina, Croatia, Italy, Montenegro and Slovenia, respectively. Some of these countries are neighboring countries in the sense that they have maritime jurisdictional boundaries adjacent to each other while the others are not. We assume that only neighboring countries directly influence each other, that is, we consider local rather than global effects. In particular, let a binary variable $g_{ij} \in \{0, 1\}$ describe a relationship between two countries: $g_{ij} = 1$ if countries i and j directly influence each other, and $g_{ij} = 0$ otherwise. We assume that relationships are symmetric or reciprocal, $g_{ij} = g_{ji}$, meaning that country i directly influences country j if and only if country j directly influences country i. Since the delimitation of maritime boundaries is a rather complex issue

(Klemenčić and Topalović, 2009), we turn to Fortuna et al. (2015) to determine the direct connections between Adriatic countries. Fortuna et al. (2015, Figure 1.5, p.8) depicts the maritime jurisdictional boundaries according to which Albania is adjacent to Italy and Montenegro; Bosnia and Herzegovina to Croatia; Croatia to all but Albania; Montenegro to Albania, Croatia and Italy; Italy to all but Bosnia and Herzegovina; and Slovenia to Croatia and Italy. Let g denote the network (graph) that consists of the set of Adriatic countries, $\mathcal{N}$, and links between them. Taken together, the interactions between Adriatic countries can be summarized by a simple network, depicted in Figure 1, with 6 nodes (vertices) and 8 links (edges).



A-Albania; B-Bosnia and Herzegovina; C-Croatia; I-Italy; M-Montenegro; S-Slovenia.

Figure 1. The Adriatic network.

A closer examination of the Adriatic network reveals that it has a structure of what Bramoullé (2007) calls a *core-periphery* network with core nodes $\mathcal{C} = \{C, M, I\}$ and periphery nodes $\mathcal{P} = \{A, B, S\}$.[2] The main features of this type of network are that the core nodes form a clique (i.e., every pair of distinct core nodes is directly connected) and that no pair of distinct periphery nodes is directly connected – periphery nodes may be connected with core nodes, though. Formally, $g_{ij} = 1$ for each $i \neq j \in \mathcal{C}$, and $g_{ij} = 0$ for each $i \neq j \in \mathcal{P}$. The network structure is depicted in Figure 2.

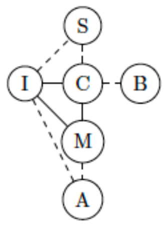


Figure 2. Network type: Core-periphery network with dense core $\mathcal{C} = \{C, M, I\}$ and unconnected periphery $\mathcal{P} = \{A, B, S\}$. Solid links connect core nodes and dashed links connect core nodes with periphery nodes.

In general, the structure of the underlying network is extremely important as it may shape the behavior of players (Goyal, 2007; Bramoullé, 2007). As demonstrated below in Appendix A, *A remark on the proof of Proposition 1*, knowing or recognizing the network structure can also greatly facilitate the Nash equilibrium analysis. Finally, since each network is characterized by certain properties, some basic conclusions can be drawn by only looking at its structure. For example, if there are three adjacent countries forming a clique, each choosing between strict but expensive ship monitoring or lenient but inexpensive ship monitoring, then if they have to choose the same monitoring type at both of their borders, at least two of them will always take the same action. Assuming that it is sufficient for one of the two neighboring countries to choose strict ship monitoring in order to successfully address maritime threats on their border, then (undesirable) coordination will result either

[2]There exist different (but similar) definitions of a core-periphery network, however. We adopt the definition of Bramoullé (2007) which follows Borgatti and Everett (2000) and slightly differs from that of Goyal (2007).

in unnecessarily high total monitoring costs (if both choose strict monitoring) or increased risks from maritime threats (e.g., pollution, smuggling) on that border (if both choose lenient monitoring).[3]

3. GAMES ON THE ADRIATIC NETWORK AND THE NASH EQUILIBRIUM CONCEPT

The present paper investigates the steady states of various *spatial games* played on the Adriatic network. A spatial game – a term used by Young (1998), among others, refers to a collection of all identical 2-player strategic games that are simultaneously played by each pair of neighboring countries. Since the Adriatic network consists of 6 nodes and 8 links, our spatial games are 6-player strategic games consisting of 8 identical 2-player strategic games. We consider three different classes of strategic games, each playing an important role in everyday decision-making. A Prisoner's Dilemma (Figure 3, left payoff matrix) is a model for a social dilemma where the selfish interest of an individual conflicts with the collective interest of a group (pair). In particular, it describes a dyadic situation in which both individuals prefer to choose their strictly dominant (selfish) action *Y* resulting in an outcome (i.e., pair of actions) that is not efficient, in the sense that there exists some other outcome that is better for both of them; hence the dilemma.[4] In the context of the present paper, this game can be used to model water sharing and climate change problems. A second class includes anti-coordination games (Figure 3, middle payoff matrix). An anti-coordination game describes a dyadic situation in which individuals prefer to choose opposite actions, for example, taking a leading/submissive role, or, in our case, prefer to coordinate on the outcome (*X*, *Y*) or (*Y*, *X*). An anti-coordination may be efficient, but not easily agreed upon, since individuals would generally disagree on who should profit (more) from anti-coordination. This game can be used to model congestion and pollution problems and inspection regimes. A third class includes coordination games (Figure 3, right payoff matrix). A coordination game describes a dyadic situation where both individuals prefer to choose the same action and thus coordinate on the outcome (*X*, *X*) or (*Y*, *Y*). While individuals agree that coordination is efficient, it may still not be achieved, because individuals' decisions may depend on various factors such as risk attitudes and/or beliefs about the opponent. This game can be used to model adoption of new technologies or practices where both parties would benefit only if they are "on the same page". In game-theoretic language, considering only situations in which players always choose one of their actions with probability 1, a Prisoner's Dilemma has a unique inefficient Nash equilibrium, (*Y*, *Y*), while the other two games have two Nash equilibria: an anti-coordination game (*X*, *Y*) and (*Y*, *X*); and a coordination game (*X*, *X*) and (*Y*, *Y*).[5]

Player 1 \ Player 2	X	Y
X	r, r	s, t
Y	t, s	p, p

Prisoner's Dilemma
$t > r > p > s$

Player 1 \ Player 2	X	Y
X	r, r	s, t
Y	t, s	p, p

Anti-coordination game
$t > r, s > p$

Player 1 \ Player 2	X	Y
X	a, a	c, b
Y	b, c	d, d

Coordination game
$a > b, d > c$

Figure 3. 2-player games. Prisoner's Dilemma with one Nash equilibrium, (Y, Y); Anti-coordination game with two Nash equilibria, (X, Y) and (Y, X); Coordination game with two Nash equilibria, (X, X) and (Y, Y).

[3]This is an example of application of a well-known result of graph theory, namely that a graph with an odd cycle is not bicolorable (see, e.g., Asratian et al., 1998, or some other standard textbook on graph theory).

[4]Both individuals would be better off by enforcing – via a binding agreement – the socially optimal decision *X*. However, such binding agreements are not permitted in non-cooperative game theory models (see, e.g., Osborne, 2004, or some other standard textbook on game theory).

[5]An outcome, i.e., a pair of actions, one for each player, is a Nash equilibrium if and only if neither player wants to deviate from their action given the fixed action of the other player. Formally, (a_1, a_2) is a Nash equilibrium if and only if $u_1(a_1, a_2) \geq u_1(a_1', a_2), \forall a_1'$, and $u_2(a_1, a_2) \geq u_2(a_1, a_2'), \forall a_2'$ (Nash, 1950; 1951), where a_i denotes player i's action, and u_i is player i's utility or payoff function, describing their preferences over outcomes in the sense that if the player prefers outcome (a_1, a_2) over (b_1, b_2), then $u_i(a_1, a_2) \geq u_i(b_1, b_2)$.

We analyze the network effects under the following two standard assumptions:[6] 1) each country chooses the *same* (fixed) action in all 2-player games it plays, i.e., in all pairwise interactions with its neighbors,[7] 2) its total payoffs are the sum of the payoffs from playing each of its neighbors. In particular, if country i chooses action $a_i \in \{X, Y\}$ and the other countries' action profile is a_{-i},[8] the total (or spatial game) payoffs of country i on the Adriatic network g are:

$$\Pi_i(a_i, a_{-i} \mid g) = \sum_{j \in N_i(g)} \pi(a_i, a_j),$$

where $N_i(g) = \{j \in \mathcal{N} \mid g_{ij} = 1\}$ is the set of country i's direct neighbors. $\pi(a_i, a_j)$ are country i's payoffs in a 2-player strategic game in which it chooses a_i and its neighbor j chooses a_j.

As mentioned above, we are interested in finding steady states of spatial games played on the Adriatic network. Steady states correspond to Nash equilibria on network which are in the literature (e.g., Young, 1998; Goyal, 2007) defined analogously as they were originally defined by Nash (1950; 1951): an action profile $(a_A, a_B, a_C, a_I, a_M, a_S) = (a_i, a_{-i})$ is a Nash equilibrium in network g if $\Pi_i(a_i, a_{-i} \mid g) \geq \Pi_i(a_i', a_{-i} \mid g), \forall a_i' \in \{X, Y\}$ and $\forall i \in \mathcal{N}$. For later purposes, we also define the *efficiency* of outcome (a_i, a_{-i}) played on network g as:

$$\frac{\sum_{i \in \mathcal{N}} \Pi_i(a_i, a_{-i} \mid g)}{6},$$

which is the average payoff per country when the outcome (a_i, a_{-i}) is played on network g.

In the following section we present the results of the Nash equilibrium analysis of spatial games played on the Adriatic network. To avoid confusion, we name each spatial game after the underlying 2-player games that induce it, e.g., if the underlying games are anti-coordination games, then the spatial game is called anti-coordination spatial game.

4. NASH EQUILIBRIUM ANALYSIS ON THE ADRIATIC NETWORK

4.1. Prisoner's Dilemma spatial games

From a game-theoretic perspective, Prisoner's Dilemma spatial games are the easiest and least interesting to analyze, since everyone is governed by a strictly dominant action Y, resulting in the unique Nash equilibrium in which everyone chooses action Y (illustrated in Subfigure 8b). This Nash equilibrium yields payoffs of $p|N_i(g)|$ to country i, where $|N_i(g)|$, or shortly $|N_i|$, denotes the number of neighbors of country i. It is not efficient, though, as the socially desirable outcome in which everyone chooses action X brings higher payoffs of $r|N_i|$. By adding a spatial component, the situation therefore does not change and the dilemma remains unresolved. In fact, this is a known result (see, e.g., Goyal, 2007, section 4.3), but we nevertheless give it as a warm-up example to better understand the analysis of the other two classes of spatial games that have more than one steady state.

---

[6]See, for example, Young (1998, subchapter 6.1), Bramoullé (2007) or Goyal (2007, subsection 4.2).

[7]So, it is not possible to choose one action when playing against one neighbor and another action when playing against different neighbor. This is essentially the same as assuming that each country makes only one decision, despite being involved in multiple interactions.

[8]Let $\boldsymbol{a}$ denote the profile (vector) of actions choosen by each of the six countries, $\boldsymbol{a} = (a_A, a_B, a_C, a_I, a_M, a_S)$. Action profile a_{-i} then denotes actions choosen by all countries other than country *i*, i.e., $a_{-i} = a \setminus a_i, i \in \mathcal{N}$.

4.2. Anti-coordination spatial games

The main focus of the present paper are anti-coordination spatial games that are on the Adriatic network the most interesting and challenging to analyze, because they have the highest number of Nash equilibria. Since the Adriatic network is not bipartite, anti-coordination spatial games also do not have a Nash equilibrium in which all countries anti-coordinate, that is, an equilibrium in which opposite actions are played at each of 8 maritime borders.[9] In particular, since there are only two available actions and core countries of the Adriatic network form a cycle (clique) of size 3, at least two of them always choose the same action in a network equilibrium (for some general results, see Bramoullé, 2007). Recall that this is different than in coordination spatial games where "complete" coordination is always possible in equilibrium (see, e.g., Goyal, 2007, subsection 4.2). Let anti-coordination games be defined as in Figure 3, middle payoff matrix, and let $x = s - p$ and $y = t - r$.[10] Then we have the following Proposition 1 (the proof is in Appendix A).

Proposition 1. *Let n^* denote the number of Nash equilibria. In anti-coordination games on the Adriatic network, the set of Nash equilibria and n^* vary with x and y. In particular, $n^* \in \{4, 5, 7, 12\}$, with the maximum value being attained when $y = x$.*

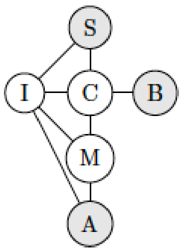
In each Nash equilibrium, both available actions are played by at least two countries.

Table 1 lists all Nash equilibria (right column) for a given condition (left column), whereas Figures 4 and 5 show under which conditions a certain action profile is a Nash equilibrium.[11] For example, if $y > 3x$, an anti-coordination spatial game has four Nash equilibria depicted in Subfigures 4f, 5a, 5d and 5h. All cases (conditions) are also visually represented in Figure A1 (see Appendix A).

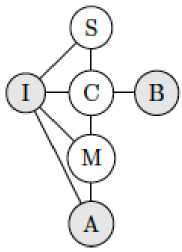
Case	Condition	Nash equilibria
1	$y > 3x$	4f, 5a, 5d, 5h
2	$y = 3x$	4f, 4g, 5a, 5d, 5h
3	$3x > y > 2x$	4f, 4g, 5a, 5d, 5h
4	$y = 2x$	4c, 4f, 4g, 5a, 5d, 5e, 5h
5	$2x > y > x$	4c, 4f, 4g, 5d, 5e
6	$y = x$	4b, 4c, 4d, 4e, 4f, 4g, 5b, 5c, 5d, 5e, 5f, 5g
7	$x > y > \frac{x}{2}$	4d, 4e, 5c, 5f, 5g
8	$y = \frac{x}{2}$	4a, 4d, 4e, 4h, 5c, 5f, 5g
9	$\frac{x}{2} > y > \frac{x}{3}$	4a, 4d, 4h, 5f, 5g
10	$y = \frac{x}{3}$	4a, 4d, 4h, 5f, 5g
11	$y < \frac{x}{3}$	4a, 4d, 4h, 5f

Table 1. All Nash equilibria (right column) for a given condition (left column).

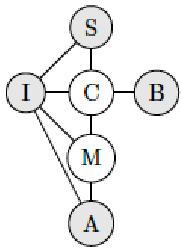
[9]Recall that only opposite actions consititute a Nash equilibrium in anti-coordination games played at each maritime border.
[10]Value x (y) can be thought of as satisfaction of choosing an equilibrium action X (Y) instead of Y (X) when the opponent chooses Y (X).
[11]The figures display only action profiles that are a Nash equilibrium under at least one condition.



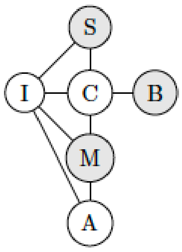
(a) *NE:* (Y,Y,X,X,X,Y).
Requirement: $x \geq 2y$.



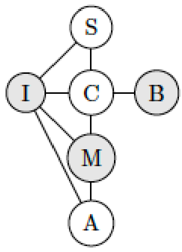
(b) *NE:* (Y,Y,X,Y,X,X).
Requirement: $x = y$.



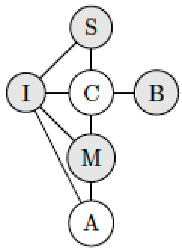
(c) *NE:* (Y,Y,X,Y,X,Y).
Requirement: $x \leq y \leq 2x$.



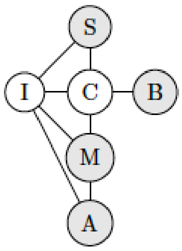
(d) *NE:* (X,Y,X,X,Y,Y).
Requirement: $x \geq y$.



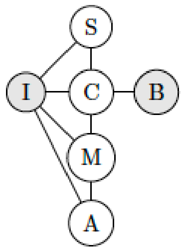
(e) *NE:* (X,Y,X,Y,Y,X).
Requirement: $y \leq x \leq 2y$.



(f) *NE:* (X,Y,X,Y,Y,Y).
Requirement: $x \leq y$.

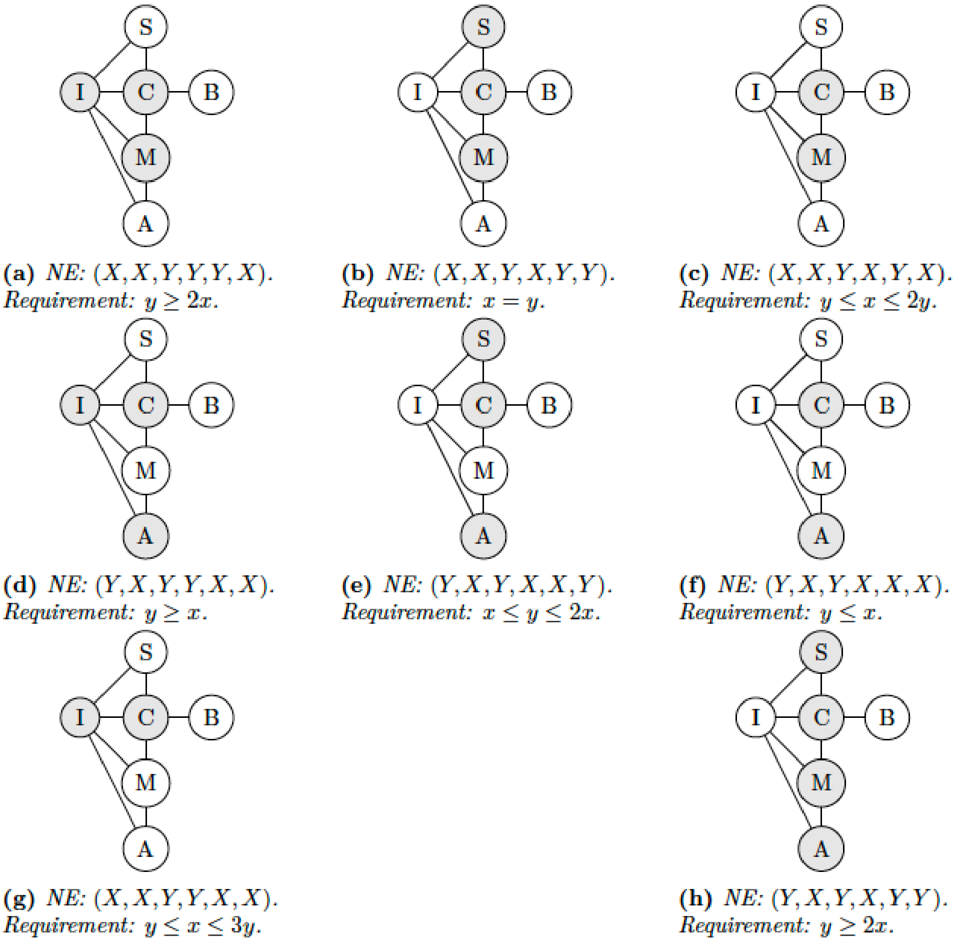


(g) *NE:* (Y,Y,X,X,Y,Y).
Requirement: $x \leq y \leq 3x$.



(h) *NE:* (X,Y,X,Y,X,X).
Requirement: $x \geq 2y$.

Figure 4. Nash equilibria in anti-coordination spatial games (Part I); $x = s - p$, $y = t - r$. White node corresponds to action X and gray node corresponds to action Y. In each NE, the first component (letter) corresponds to the action of Albania, followed by the action of Bosnia and Herzegovina, Croatia, Italy, Montenegro and Slovenia. The action profiles in the subfigures are Nash equilibria only if the required conditions are met.



(a) *NE:* (X, X, Y, Y, Y, X). *Requirement:* $y \geq 2x$.

(b) *NE:* (X, X, Y, X, Y, Y). *Requirement:* $x = y$.

(c) *NE:* (X, X, Y, X, Y, X). *Requirement:* $y \leq x \leq 2y$.

(d) *NE:* (Y, X, Y, Y, X, X). *Requirement:* $y \geq x$.

(e) *NE:* (Y, X, Y, X, X, Y). *Requirement:* $x \leq y \leq 2x$.

(f) *NE:* (Y, X, Y, X, X, X). *Requirement:* $y \leq x$.

(g) *NE:* (X, X, Y, Y, X, X). *Requirement:* $y \leq x \leq 3y$.

(h) *NE:* (Y, X, Y, X, Y, Y). *Requirement:* $y \geq 2x$.

Figure 5. Nash equilibria in anti-coordination spatial games (Part II); $x = s - p$, $y = t - r$. White node corresponds to action X and gray node corresponds to action Y. In each NE, the first component (letter) corresponds to the action of Albania, followed by the action of Bosnia and Herzegovina, Croatia, Italy, Montenegro and Slovenia. The action profiles in the subfigures are Nash equilibria only if the required conditions are met.

4.2.1. Illustration

To illustrate the use of Proposition 1, consider again the situation outlined at the beginning of the introduction: the coastal countries that must simultaneously decide whether or not to engage in pollution control and mitigation along their maritime borders. This pollution problem can be modelled as a spatial game on the Adriatic network where the underlying 2-player strategic games have two possible actions, to engage or not to engage in pollution control and mitigation. Using hypothetical data, we can determine all its Nash equilibria, and answer the unanswered questions, posed at the beginning of the introduction. In particular, suppose that the payoffs of the underlying 2-player strategic game correspond to the costs associated with pollution: country's costs are 1 monetary unit (MU) if it engages in pollution control and mitigation, and 0 otherwise. To successfully preserve the environment at a particular common maritime border (and eliminate any additional pollution costs), at least one of the two neighboring countries must undertake pollution control activities. Otherwise, the situation worsens and estimated potential pollution costs of both countries involved rise to 3.5 MU. For example, if countries do not immediately react to an oil spill, it will disperse and gradually reach the bottom of the sea or even the coast, resulting in higher cleaning costs.

The game played at each maritime border is an anti-coordination game that can be represented by the cost matrix shown in Figure 6. The numbers correspond to the costs associated with pollution and pollution control. It follows from Figure 3 that X corresponds to action *Engage*, Y to action *Not engage*, and that $(p, r, s, t) = (-3.5, -1, -1, 0)$. From subsection 4.2 it follows that $(x, y) = (s - p, t - r) = (2.5, 1)$. Since $y \in \left(\frac{x}{3}, \frac{x}{2}\right)$, the ninth row of Table 1 tells us that the anti-coordination spatial game has 5 Nash equilibria depicted in Subfigures 4a, 4d, 4h, 5f and 5g. These equilibria have different properties, yielding some interesting insights. First, in all but one equilibrium (i.e., one in Subfigure 5g), all 8 maritime borders are protected, as at least one of the neighboring countries always engages in pollution control and mitigation. In equilibrium from Subfigure 5g, however, there is no pollution control along the Croatian-Italian maritime border, which may cause major problems. Second, these equilibria differ in terms of efficiency which can be measured by calculating the average costs per country associated with a particular equilibrium (using formula from section 3). The most efficient equilibria are shown in Subfigures 4d and 5f, with average costs of 1.67 MU.[12] In these two equilibria the most efficient outcome of the underlying 2-player game (*Engage*, *Not*), or equivalently (*Not*, *Engage*), is played at 6 borders, and the second most efficient outcome (*Engage*, *Engage*) is played at 2 borders. The second most efficient equilibria are shown in Subfigures 4a and 4h, with average costs of 1.83 MU. In these two equilibria the most efficient outcome of the underlying 2-player game is played at 5 borders, and the second most efficient outcome is played at 3 borders. The remaining equilibrium, shown in Subfigure 5g, is the least efficient with average costs of 2.5 MU. While the most and second most efficient outcomes of the underlying 2-player game are played at 6 and 1 border, respectively, the most inefficient outcome (*Not*, *Not*) is also played at 1 (i.e, Croatian-Italian) border. Last but not least, assuming that each of the five equilibria is realized with equal probability (and all other action profiles with zero probability), the Montenegro is expected to be environmentally friendly in 4 out of 5 cases (80%), Bosnia and Herzegovina only in 2 out of 5 cases (40%), and the rest in 3 out of 5 cases (60%).

		Neighbor	
		Engage	Not
Country	Engage	$-1, -1$	$-1, 0$
	Not	$0, -1$	$-3.5, -3.5$

Figure 6. Anti-coordination game played at each common maritime border.

In the illustration example, the reason for not seeing more inefficient outcomes where both neighboring countries choose to free ride are high estimated potential pollution costs p. The assumption of high estimated pollution costs seems very realistic (see, e.g., Carić, 2010; McIlgorm et al., 2011). However, if countries underestimate these costs and for example assume they are 1.5 MU (instead of 3.5 MU), which still implies that the outcome where neither country engages in pollution control is the most inefficient, then x changes to 0.5 (and thus $y = 2x$), resulting in more inefficient outcomes in the underlying 2-player games (see the edges connecting the gray nodes, for example in Subfigures 4c, 4f, 5a and 5h; all Nash equilibria for this case are in the fourth row of Table 1).

The above consideration hints that our results can also be used for a more comprehensive analysis, which may lead to a better understanding of how the network might respond to continuous changes in the value of a specific payoff. For example, consider the game in Figure 6, but assume that if neither country engages in pollution control and mitigation, their payoff is $p < -1$.[13] Then, by varying the value of p, one can examine how the action profiles enter and leave the set of Nash equilibria, and how the number of Nash equilibria change

[12] More specifically, the efficiency of the Nash equilibrium (X, Y, X, X, Y, Y) from Subfigure 4d is $\frac{\sum_{i \in \mathcal{N}} \Pi_i((X,Y,X,X,Y,Y) \mid g)}{6} = \frac{-2+0-4-4+0+0}{6} = -\frac{10}{6} = -\frac{5}{3} \approx -1.67$. Payoffs are negative because in the example are considered costs rather than benefits.

[13] The value of p cannot be greater or equal -1, as otherwise the game would not be an anti-coordination game according to our definition.

(see Table 2 and Figure 7). In particular, if we start with $p \in \left(-\frac{4}{3}, -1\right)$ we have 4 Nash equilibria, listed in the first row of Table 1. By slightly lowering p to $-\frac{4}{3}$, one new equilibrium emerges, depicted in Subfigure 4g. The situation changes again at $p = -\frac{3}{2}$, when action profiles shown in Subfigures 4c and 5e also become equilibria. Lowering p further results in two equilibria less, namely those depicted in Subfigures 5a and 5h. At $p = -2$, a substantial change occurs, as the network obtains 7 new equilibria. This is the special case, when $y = x$, which is equivalent to $s - p = t - r$ or $r + s = t + p$, i.e., the average (or sum of) payoffs of actions X and Y are the same. A further decrease in p results in 7 equilibria less. At $p = -3$, two new equilibria emerge. When p becomes lower than -3, action profiles from Subfigures 4e and 5c are not equilibria anymore. When $p < -4$, the action profile shown in Subfigure 5g leaves the set of Nash equilibria. Decreasing p further does not change anything, and 4 action profiles listed in the last row of Table 1 remain the only Nash equilibria.

Case	Condition	Enter/Leave
1	$-\frac{4}{3} < p < -1$	
2	$p = -\frac{4}{3}$	4g
3	$-\frac{3}{2} < p < -\frac{4}{3}$	
4	$p = -\frac{3}{2}$	4c, 5e
5	$-2 < p < -\frac{3}{2}$	5a, 5h
6	$p = -2$	4b, 4d, 4e, 5b, 5c, 5f, 5g
7	$-3 < p < -2$	4b, 4c, 4f, 4g, 5b, 5d, 5e
8	$p = -3$	4a, 4h
9	$-4 < p < -3$	4e, 5c
10	$p = -4$	
11	$p < -4$	5g

Table 2. All Nash equilibria that enter (blue) and leave (red) the set of Nash equilibria when transitioning from Case $k - 1$ to Case k (right column).

By varying p, not only the equilibria and their number change, but also their efficiency, as can be seen from Table A4 in Appendix A. The main finding from that table is that if one has control over p, they should set it below -4 to promote efficiency,[14] as such p generates the highest efficiency. Reason for that is that in all four Nash equilibria (see the last row of Table 1) the most inefficient outcome is never played at any border.

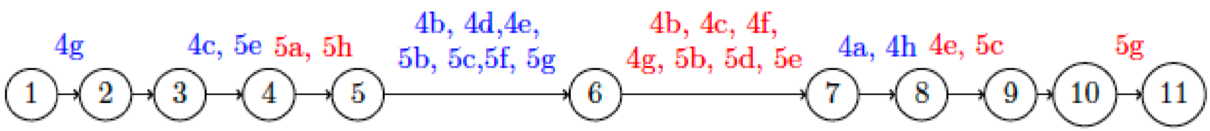


Figure 7. All Nash equilibria that enter (blue) and leave (red) the set of Nash equilibria during the transition.

[14]For example, by emphasizing to countries that failing to implement pollution control measures on both sides of the border could lead to environmental contamination, which could reduce tourism and lead to high cleanup costs, ultimately harming their economies.

4.3. Coordination spatial games

We conclude Section 4 with Nash equilibrium analysis of coordination spatial games. This subsection is short, as the analysis is simpler than that of the previous subsection, but concise. Our point of departure is Goyal (2007, Theorem 4.1) that states two important results, albeit for a more narrowly defined coordination games:[15] first, two trivial outcomes in which everyone chooses the same action are always equilibria, regardless of the network structure; and second, certain networks have additional equilibria in which players choose different actions. The first result can easily be verified on our network g. Suppose that everyone chooses action X. Then for an arbitrary country $i \in \mathcal{N}$, its payoffs are $a|N_i|$, which is more than $b|N_i|$, i.e., the payoffs that it gets by deviating to Y, implying that (X, X, X, X, X, X) is a Nash equilibrium (see Subfigure 8a). In the same way it can be shown that (Y, Y, Y, Y, Y, Y) is a Nash equilibrium (shown in Figure 8b). By choosing Y, country i's payoffs are $d|N_i|$, which is more than $c|N_i|$, i.e., the payoffs that it gets by deviating to X. Regarding the second result of Goyal (2007), we do not know a priori whether our network g possesses additional Nash equilibria. However, since g has a relatively simple structure, it is not difficult to determine all Nash equilibria. We have the following Proposition 2 whose proof is given in Appendix A.

Proposition 2. If $a + c = b + d$, coordination games on the Adriatic network have, in addition to two trivial Nash equilibria, two additional Nash equilibria: (Y, X, X, Y, Y, X) and (X, Y, Y, X, X, Y).

What this proposition says is that coordination games on the Adriatic network have only trivial equilibria, unless the average (or sum of) payoffs of actions X and Y are the same. In this case, there are two additional equilibria in which Bosnia and Herzegovina, Croatia and Slovenia choose one action and the other three countries (Albania, Italy, Montenegro) choose the opposite action (see Subfigures 8c and 8d).

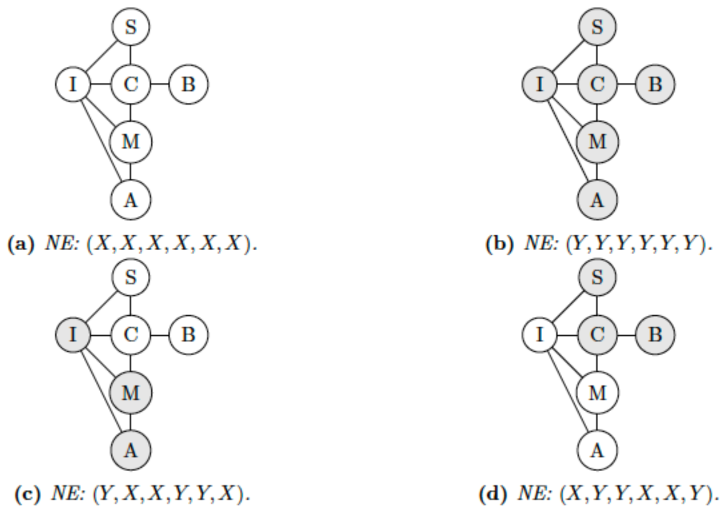


Figure 8. All Nash equilibria in coordination spatial games. NE in Subfigure 8b is also the unique NE in Prisoner's Dilemma spatial games. White node corresponds to action X and gray node corresponds to action Y. In each NE, the first component (letter) corresponds to the action of Albania, followed by the action of Bosnia and Herzegovina, Croatia, Italy, Montenegro and Slovenia. The action profiles in Subfigures 8c and 8d are Nash equilibria only if $a + c = b + d$.

[15] In our definition of coordination game, there are two restrictions regarding the payoffs: $a > b$ and $d > c$. In Goyal's (2007) definition, they additionally assume that $a > c > b$ and $a + c > b + d$ (according to Harsanyi and Selten (1988) the latter means that X is the risk-dominant action).

5. DISCUSSION

A concerted effort is needed to prevent maritime pollution from ships and plastic pollution which can have a long-lasting impact and lead to high cleanup costs, especially if pollutants reach the coastlines and waters of multiple countries. The Adriatic Sea is semi-enclosed, which affects water circulation and exchange rate, additionally complicating pollution management. Properly addressing pollution is particularly important for Adriatic countries, especially coastal regions, because they rely on tourism and fisheries (Morović et al., 2015). Ship monitoring, for example, may involve tracking and inspecting vessels, the exchange of information and coordination among coastal countries.

Maritime pollution also has a negative impact on biodiversity conservation which itself often requires transboundary cooperation, such as monitoring and protecting endangered and/or protected species, and better controlling of the invasive species, which are arriving in the Adriatic Sea, because it is getting warmer (Reuters, 2024). Effective monitoring of vessels and biodiversity relies on technological innovations, such as satellites and drones, which require successful interactions between neighboring countries and use of compatible software.

A powerful framework for analyzing the interactions between rational entities is game theory - a relatively new field, which was initially used mostly in mathematics, economics and computer science, but later regularly utilized outside these fields. While game-theoretic models vary in complexity, we decided to build our analysis on simple 2x2 strategic games, to which we added a realistic spatial component. Our spatial games consist of several identical 2x2 strategic games. Given that, at least to our knowledge, spatial games involving the Adriatic countries have not yet been considered in the literature, we decided to keep our analysis as simple as possible and thereby comprehensible to the general public. In the future, however, more complex and realistic models can be used for better capturing the strategic interactions among the Adriatic countries. Such models might include incomplete information or recurring interactions. A nice introduction to games with incomplete information and repeated games (without spatial component, though) can be found in Osborne's (2004) book.

Throughout the paper we provided a game-theoretic analysis on general Prisoner's Dilemma, coordination and anti-coordination spatial games, and also provided an illustrative pollution control example - modelled as an anti-coordination spatial game. The latter example is used to make our game-theoretic models, which some might find abstract since they rely on mathematics and are written in neutral language (e.g., actions are X and Y, payoffs are t, r, s and p), more accessible to marine and pollution scientists and the general audience.

We first show that on the Adriatic network cooperation (i.e., outcome (X, X) in Prisoner's Dilemma) cannot be attained as individual self-interest dominates over collective gain. This result, which in fact holds for any network, is not new (see, e.g., Goyal, 2007), but might be surprising at first glance because in real life we often see cooperation. However, the reason for this undesirable result lies in the classical game-theoretic assumptions, namely that our spatial games are played only once and that our players care only for their own payoffs. If we instead assumed that games are played infinitely many times and players give less importance to future payoffs (which may be a reasonable assumption, since countries "live" forever and future payoffs are less certain) or that countries care for their neighbors, then it can be shown that cooperation is an equilibrium outcome.

We then continued with anti-coordination spatial games, which were the most challenging to study on the Adriatic network, because they have multiple Nash equilibria for any set of payoffs, that is, for any underlying 2x2 anti-coordination game; on the Adriatic network always at least four. This poses equilibrium selection problem for the Adriatic countries in real life – even if countries know which are the equilibria and know these are common knowledge, it is still difficult to determine which one is to be played. On more complex networks, the anti-coordination problem becomes even more involved, and one may want to use Nash equilibrium refinements to retain equilibria that are more likely to be selected, like it was done for example by Bramoullé (2007). Since the Adriatic network is relatively small, we provided all Nash equilibria, but to facilitate equilibrium selection problem and to make better predictions from our results, we suggest to focus on the most efficient equilibria, as those should be more salient in our opinion. As we illustrated above, the equilibria vary in terms of efficiency, and for a particular value of p only one or two turned out to be the most efficient (can be deduced from Table A4 by counting the number of blue values in each row of the left column). The set of efficient Nash equilibria is much smaller than the set of all Nash equilibria, so focusing on the former set may facilitate the equilibrium selection problem and give sharper, less ambiguous predictions.

We also illustrated how our complete Nash equilibrium analysis can be relevant for predicting and understanding how the network might respond to continuous changes or disruptions. In particular, we showed how changes in pollution costs p, resulting from two neighboring countries refraining from pollution control and mitigation, impact the behavior on the network and efficiency. So, although our analysis focuses on *static* network configurations, by treating p as parameter, we can add to the analysis a *dynamical* component. In our illustration we saw that when p fell below -4, the set of Nash equilibria remained fixed and overall efficiency was maximal. Such results might help authorities to design campaigns that would align individual preferences with desired outcomes, potentially leading to higher collective benefit.

At last, we showed that in addition to full coordination (which is known to be a Nash equilibrium on any network; see, e.g., Goyal, 2007), the outcome where Albania, Italy and Montenegro choose one action, and Bosnia and Herzegovina, Croatia and Slovenia the other, also constitutes a Nash equilibrium but only under one specific condition, namely if the average (or total) payoffs of both available actions are the same, $a + c = b + d$. This finding suggests that to avoid miscoordination in the equilibrium at the borders between Italy and Slovenia, Italy and Croatia, and Croatia and Montenegro, the countries should be incentivized in such a way that the above equality does not hold, thereby halving the number of Nash equilibria and facilitating coordination.

We conclude with a final remark regarding the potential future improvements of the model. In our spatial games we defined total country's payoffs Π as the sum of country's payoffs π from all 2x2 games played with their neighbors. The payoffs were just summed, meaning that for a country each of their 2x2 games (and essentially neighbors) was equally important. However, to reflect a more realistic setup, the total payoff function could be reformulated, for example, by multiplying each payoff π by some weight. Such formulation can be found in Young (1998, subchapter 6.1) and captures different relative importance of the edges, which in our setup represent maritime borders. In this way one can account for the varying lengths of maritime borders, diverse existing environmental protection practices along these borders, or different ongoing relationships between neighboring countries.

6. CONCLUSION

Strategic interactions among rational entities can be challenging to study, especially when one adds a realistic spatial component. In the present paper we address cooperation, coordination and anti-coordination problems from a game-theoretic perspective. Using simple spatial games, we explore the decisions of six countries, which surround the Adriatic Sea, forming a core-periphery network. While it is already established that cooperation is unattainable and full coordination is achievable as the Nash equilibrium on any network, equilibrium partial coordination and anti-coordination do depend on the network structure. Regarding anti-coordination, we found that any anti-coordination spatial game played on the Adriatic network possesses several Nash equilibria, of which some are more efficient than others. However, since the Adriatic network contains an odd cycle (and therefore is not bipartite), in the equilibrium there is always some inefficiency along at least one maritime border. By exploring the equilibria of the studied spatial games, scientists, policy makers and stakeholders can find ways to intervene and align individual incentives with desired outcomes, potentially leading to more efficient behavior among Adriatic countries. Finally, our Adriatic network example can be used in game theory courses to teach students how to model spatial games based on some real-world network, and how to perform complete Nash equilibrium analysis, if the network is not to complex. We also demonstrate that the analysis can be simplified substantially by utilizing the properties of a given network, such as the core-periphery structure.

ACKNOWLEDGMENTS: Žiga Velkavrh gratefully acknowledges the support by the Slovenian Research and Innovation Agency (research program P1-0285).

CONFLICT OF INTEREST: The author declares that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

REFERENCES

Asratian, A.S., Denley, T.M. and Häggkvist, R., 1998. Bipartite Graphs and their Applications (Vol. 131), Cambridge: Cambridge University Press. Available at: https://doi.org/10.1017/CBO9780511984068.

Bondy, J.A. and Murty, U.S.R., 1976. Graph Theory with Applications (Vol. 290), New York: Elsevier. Available at: https://doi.org/10.1007/978-1-349-03521-2.

Borgatti, S.P. and Everett, M.G., 2000. Models of core/periphery structures. Social Networks, 21(4), pp. 375–395. Available at: https://doi.org/10.1016/j.geb.2005.12.006.

Bramoullé, Y., 2007. Anti-coordination and social interactions. Games and Economic Behavior, 58(1), pp. 30–49. Available at: https://doi.org/10.1016/j.geb.2005.12.006.

Carić, H., 2010. Direct pollution cost assessment of cruising tourism in the Croatian Adriatic. Financial Theory and Practice, 34(2), pp. 161–180. Available at: https://hrcak.srce.hr/53595.

Fernandez, L., 2002. Trade's Dynamic Solutions to Transboundary Pollution. Journal of Environmental Economics and Management, 43(3), pp. 386–411. Available at: https://doi.org/10.1006/jeem.2001.1187.

Fortuna, C.M., Holcer, D. and Mackelworth, P. (eds.), 2015. Conservation of cetaceans and sea turtles in the Adriatic Sea: status of species and potential conservation measures. Report produced under WP7 of the NETCET project, IPA Adriatic Cross-border Cooperation Programme. Available at https://www.netcet.eu/files/Conservation_measures/NETCET_WP7_Conservation_measures_for_CSTs.pdf, accessed September 5th, 2024.

Goyal, S., 2007. Connections: An Introduction to the Economics of Networks, Princeton and Oxford: Princeton University Press. Available at: https://doi.org/10.1515/9781400829163.

Grdović Gnip, A. and Velkavrh, Ž., 2022. To Pollute or Not To Pollute? Exploring MARPOL Efficiency in the Adriatic Sea. Transactions on Maritime Science, 11(1), pp. 219–236. Available at: https://doi.org/10.7225/toms.v11.n01.w13.

Harsanyi, J. C., and Selten, R., 1988. A General Theory of Equilibrium Selection in Games, Cambridge, MA: The MIT Press. Available at: https://doi.org/10.7551/mitpress/9780262081733.001.0001.

Klemenčić, M. and Topalović, D., 2009. The maritime boundaries of the Adriatic Sea. Geoadria, 14(2), pp. 311–324. Available at: https://doi.org/10.15291/geoadria.555.

Madani, K., 2010. Game theory and water resources. Journal of Hydrology, 381(3–4), pp. 225–238. Available at: https://doi.org/10.1016/j.jhydrol.2009.11.045.

McIlgorm, A., Campbell, H.F. and Rule, M.J., 2011. The economic cost and control of marine debris damage in the Asia-Pacific region. Ocean & Coastal Management, 54(9), pp. 643–651. Available at: https://doi.org/10.1016/j.ocecoaman.2011.05.007.

Morović, M., Ivanov, A., Oluić, M., Kovač, Ž., and Terleeva, N., 2015. Oil spills distribution in the Middle and Southern Adriatic Sea as a result of intensive ship traffic. Acta Adriatica, 56(2), pp. 145-156. Available at: https://hrcak.srce.hr/152236.

Muñoz, V.S. and Mc Gettrick, M., 2021. Nash Equilibria in certain two-choice multi-player games played on the ladder graph. International Game Theory Review, 23(03), 2050020. Available at: https://doi.org/10.1142/S0219198920500206.

Nash, J.F., 1950. Equilibrium points in N-person games. Proceedings of the National Academy of Sciences of the United States of America, 36(1), pp. 48–49. Available at: https://doi.org/10.1073/pnas.36.1.48.

Nash, J.F., 1951. Non-Cooperative Games. Annals of Mathematics, 54(2), pp. 286–295. Available at: https://doi.org/10.2307/1969529.

Reuters, 2024, March 21. New fish invade the Adriatic Sea, threatening local species. Available at: https://www.reuters.com/science/new-fish-invade-adriatic-sea-threatening-local-species-2024-03-20/, accessed September 5th, 2024

Vidas, D., 2009. The UN Convention on the Law of the Sea, the European Union and the Rule of Law: What is going on in the Adriatic Sea?. The International Journal of Marine and Coastal Law, 24(1), pp. 1–66. Available at: https://doi.org/10.1163/157180808X353902.

Osborne, M.J., 2004. An Introduction to Game Theory, New York: Oxford University Press.

Young, H.P., 1998. Individual Strategy and Social Structure: An Evolutionary Theory of Institutions. Princeton, NJ: Princeton University Press. Available at: https://doi.org/10.1515/9780691214252.

APPENDIX A. NASH EQUILIBRIA

Proof of Proposition 1.

An anti-coordination game on the Adriatic network involves 6 countries, each of which has 2 available actions, X and Y, implying that there are 64 candidates for Nash equilibrium. This number can be reduced substantially by exploiting the fact that if all neighbors of country i choose the same action, country i is always better off by choosing the opposite action.

Consider the periphery countries A, B and S. Since $|N_B| = 1$, B and its only neighbor C must choose different actions in Nash equilibrium, meaning that none of $(\_, X, X, \_, \_, \_)$ and $(\_, Y, Y, \_, \_, \_)$ can be Nash equilibrium. This leaves us with 32 candidates. Similarly, since $|N_S| = 2$, S and its two neighbors C and I must not choose the same action in Nash equilibrium, meaning that none of $(\_, X, Y, Y, \_, Y)$ and $(\_, Y, X, X, \_, X)$ can be Nash equilibrium. This leaves us with 24 candidates. Furthermore, since $|N_A| = 2$, A and its two neighbors I and M must not choose the same action in Nash equilibrium, meaning that none of $(X, \_, \_, X, X, \_)$ and $(Y, \_, \_, Y, Y, \_)$ can be Nash equilibrium. It can be checked that 10 of the latter 16 candidates were already eliminated in the first two steps. By eliminating the remaining 6 candidates, we are left with 18 candidates.

The conditions under which each of the remaining 18 candidates is a Nash equilibrium can be determined using a slightly weaker version of Bramoullé's (2007) Proposition 1, in which we allow weak inequalities (these are also considered in the Nash equilibrium definition). In fact, due to the symmetry, only half (9) of the remaining candidates must be examined (if for example (Y, Y, X, X, X, Y) is a Nash equilibrium for some x and y, then (X, X, Y, Y, Y, X) will also be a Nash equilibrium, but for some other x and y). Simple algebra then yields the required conditions.

Demonstration

In this paragraph we demonstrate how to determine the conditions under which (Y, Y, X, X, X, Y) is a Nash equilibrium. The conditions for other action profiles are determined analogously. The most transparent way is to construct a table, such as Table A1, where $|N_{i,X}|$ is the number of country i's neighbors that choose X and $p_X = \frac{x}{y+x} = \frac{s-p}{t-r+s-p}$ denotes the ratio between x and $y + x$.[16] In Nash equilibrium, for countries that choose X, $|N_{i,X}| \leq |N_i| p_X$ must hold, whereas for countries that choose Y, $|N_{i,X}| \geq |N_i| p_X$ must hold. All these inequalities are displayed in the sixth column of Table A1 and must hold simultaneously. Since p_X is a function of positive x and y, and $|N_i|$ and $|N_{i,X}|$ are numbers (i.e., constants), we must essentially solve a system of 6 linear inequalities given in the last column of Table A1. The solution to the system gives the Nash equilibrium condition for a Nash equilibrium candidate in question. If the system does not have a solution, a candidate can never be a Nash equilibrium. In our particular case, (Y, Y, X, X, X, Y) is a Nash equilibrium iff $0 \leq y \leq \frac{x}{2}$ (the lower bound is never attained, though, since $y = t - r > 0$ by assumption). This case is represented in Subfigure 4a. By exploiting the symmetry (i.e., by interchanging the roles of 1) X and Y, and 2) x and y) it follows immediately that (X, X, Y, Y, Y, X) is a Nash equilibrium iff $x \leq \frac{y}{2}$ or $2x \leq y$ (see Subfigure 5a).

After examining all 18 Nash equilibrium candidates we found that 2 of them are never Nash equilibria: (X, Y, X, Y, X, Y) and (Y, X, Y, X, Y, X). The remaining 16 candidates along with the required conditions are shown in Figures 4 and 5. As can be seen from Table 1 and Figure A1, the set and the number of Nash equilibria vary

[16] Although mixed Nash equilibria are not in the scope of the present paper it is worth mentioning that p_X is exactly the probability of choosing X in a mixed Nash equilibrium of a 2-player anti-coordination game presented in Figure 3, middle payoff matrix.

with x and y. Depending on the game payoffs, an anti-coordination game can have 4, 5, 7 or 12 Nash equilibria, i.e., $n^* \in \{4, 5, 7, 12\}$, with the maximum value of them being when $y = x$. ■

Country	$\|N_i\|$	$\|N_{i,X}\|$	$\|N_i\|p_X$	a_i	Inequality	Condition
A	2	2	$2x/(y+x)$	Y	$\geq$	$y \geq 0$
B	1	1	$x/(y+x)$	Y	$\geq$	$y \geq 0$
C	4	2	$4x/(y+x)$	X	$\leq$	$y \leq x$
I	4	2	$4x/(y+x)$	X	$\leq$	$y \leq x$
M	3	2	$3x/(y+x)$	X	$\leq$	$y \leq x/2$
S	2	2	$2x/(y+x)$	Y	$\geq$	$y \geq 0$

Table A1. Finding conditions under which (Y, Y, X, X, X, Y) is a Nash equilibrium.

A remark on the proof of Proposition 1.

18 Nash equilibrium candidates can alternatively be found by exploiting the core-periphery structure of the Adriatic network by fixing the actions of core countries and exploring best responses of the periphery countries (given the core countries' actions). As shown in Table A2, the core countries (C, I and M) can choose 8 different action combinations: all choose X, 2 choose X, 1 chooses X, or none chooses X. Now we take into account two facts: 1) if all neighbors of periphery country i choose the same action, country i is always better off by choosing the opposite action (i.e., it has only one best response); 2) if neighbors of periphery country i choose different actions, country i's best response depends on the game payoffs and may be either of the two actions. With these two facts in mind, we can easily determine the number of all potential best responses of the periphery countries (see the middle columns in Table A2).

So, for example, if all 3 core countries choose X, each of the periphery countries has only one best response, Y, whereas if C and M choose X, only B has one best response, while A and S have two potential best responses (because their neighboring countries choose different actions). The former case gives 1 Nash equilibrium candidate, and the latter gives $2 \cdot 1 \cdot 2 = 4$ candidates.

	# potential best responses			
a_C, a_I, a_M	A	B	S	# candidates
X, X, X	1	1	1	1
X, X, Y	2	1	1	2
X, Y, X	2	1	2	4
Y, X, X	1	1	2	2
X, Y, Y	1	1	2	2
Y, X, Y	2	1	2	4
Y, Y, X	2	1	1	2
Y, Y, Y	1	1	1	1
			Σ	18

Table A2. Finding Nash equilibrium candidates in anti-coordination games by exploiting the core-periphery structure of the Adriatic network.

Visual representation of all the cases from Table 1.

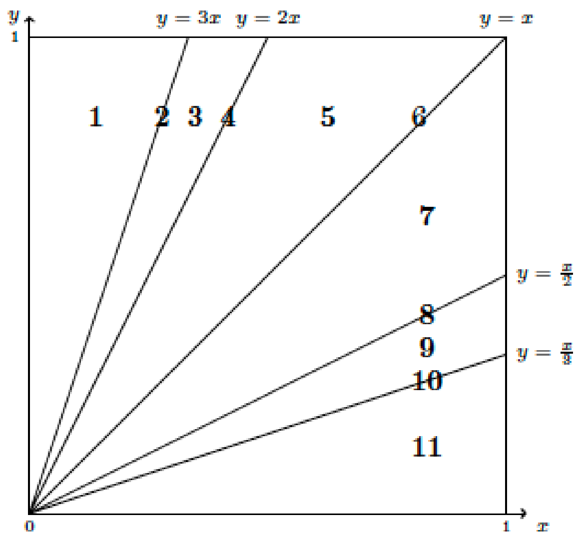


Figure A1. Visual representation of all the cases from Table 1; $x = s - p$, $y = t - r$. Numbers 1 to 11 correspond to different cases (rows) from Table 1.

Proof of Proposition 2.

The proof follows the same lines as the proof of Proposition 1, or more precisely, *A remark on the proof of Proposition 1*, as we again use the fact that the Adriatic network has the core-periphery structure and that the core countries can choose 8 different action combinations. We also take into account the following two facts: 1) if all neighbors of periphery country i choose the same action, country i prefers to choose it too. (i.e., it has only one best response); 2) if neighbors of periphery country i choose different actions, country i's best response depends on the game payoffs and may be either of the two actions. With these two facts in mind, we can easily determine the number of all potential best responses of the periphery countries (see the middle columns in Table A3). As it turns out, only 14 (out of 64) action profiles are actually Nash equilibrium candidates. Two of them are trivial outcomes in which everyone chooses the same action, and for these we have already seen that they are Nash equilibria (see the paragraph above Proposition 2).

Using the same procedure as in the proof of Proposition 1, we then examine the remaining candidates and find out that only two of them, (Y, X, X, Y, Y, X) and (X, Y, Y, X, X, Y), can be supported as Nash equilibria, but only under special condition, namely if $a + c = b + d$. ■

a_C, a_I, a_M	# potential best responses A	B	S	# candidates	
X,X,X	1	1	1	1	
X,X,Y	1	1	1	1	If A chose X instead of Y, M would prefer to choose X because all its neighbors would choose X.
X,Y,X	2	1	2	3	If A and S both chose X, I would prefer to choose X because all its neighbors would choose X.
Y,X,X	1	1	2	2	
X,Y,Y	1	1	2	2	
Y,X,Y	2	1	2	3	If A and S both chose Y, I would prefer to choose Y because all its neighbors would choose Y.
Y,Y,X	1	1	1	1	If A chose Y instead of X, M would prefer to choose Y because all its neighbors would choose Y.
Y,Y,Y	1	1	1	1	
			Σ	14	

Table A3. Finding Nash equilibrium candidates in coordination games by exploiting the core-periphery structure of the Adriatic network.

Efficiency of Nash equilibria in the example with a varying p.

Case	Efficiency (the same order as in Table 1)	Condition	Efficiency of the best/least efficient NE
1	$\frac{-6+4p}{6}, \frac{-5+6p}{6}, \frac{-6+4p}{6}, \frac{-5+6p}{6}$	$-\frac{4}{3}<p<-1$	$-\frac{17}{9}<\frac{-6+4p}{6}<-\frac{5}{3}, -\frac{13}{6}<\frac{-5+6p}{6}<-\frac{11}{6}$
2	$-\frac{17}{9}, -\frac{16}{9}, -\frac{13}{6}, -\frac{17}{9}, -\frac{13}{6}$	$p=-\frac{4}{3}$	$-\frac{16}{9}, -\frac{13}{6}$
3	$\frac{-6+4p}{6}, \frac{-8+2p}{6}, \frac{-5+6p}{6}, \frac{-6+4p}{6}, \frac{-5+6p}{6}$	$-\frac{3}{2}<p<-\frac{4}{3}$	$-\frac{11}{6}<\frac{-8+2p}{6}<-\frac{16}{9}, -\frac{7}{3}<\frac{-5+6p}{6}<-\frac{13}{6}$
4	$-\frac{13}{6}, -2, -\frac{11}{6}, -\frac{7}{3}, -2, -\frac{11}{6}, -\frac{7}{3}$	$p=-\frac{3}{2}$	$-\frac{11}{6}, -\frac{7}{3}$
5	$\frac{-7+4p}{6}, \frac{-6+4p}{6}, \frac{-8+2p}{6}, \frac{-6+4p}{6}, \frac{-8+2p}{6}$	$-2<p<-\frac{3}{2}$	$-2<\frac{-8+2p}{6}<-\frac{11}{6}, -\frac{17}{6}<\frac{-7+4p}{6}<-\frac{7}{3}$
6	$-\frac{13}{6}, -\frac{5}{2}, -\frac{5}{3}, -2, -\frac{7}{3}, -2, -\frac{5}{2}, -\frac{13}{6}, -\frac{7}{3}, -2, -\frac{5}{3}, -2$	$p=-2$	$-\frac{5}{3}, -\frac{5}{2}$
7	$-\frac{5}{3}, \frac{-8+2p}{6}, \frac{-9+2p}{6}, -\frac{5}{3}, \frac{-8+2p}{6}$	$-3<p<-2$	$-\frac{5}{3}, -\frac{5}{2}<\frac{-9+2p}{6}<-\frac{13}{6}$
8	$-\frac{11}{6}, -\frac{5}{3}, -\frac{7}{3}, -\frac{11}{6}, -\frac{5}{2}, -\frac{5}{3}, -\frac{7}{3}$	$p=-3$	$-\frac{5}{3}, -\frac{5}{2}$
9	$-\frac{11}{6}, -\frac{5}{3}, -\frac{11}{6}, -\frac{5}{3}, \frac{-8+2p}{6}$	$-4<p<-3$	$-\frac{5}{3}, -\frac{8}{3}<\frac{-8+2p}{6}<-\frac{7}{3}$
10	$-\frac{11}{6}, -\frac{5}{3}, -\frac{11}{6}, -\frac{5}{3}, -\frac{8}{3}$	$p=-4$	$-\frac{5}{3}, -\frac{8}{3}$
11	$-\frac{11}{6}, -\frac{5}{3}, -\frac{11}{6}, -\frac{5}{3}$	$p<-4$	$-\frac{5}{3}, -\frac{11}{6}$

Table A4. Efficiency of Nash equilibria in the example with a varying p.