

# THE CONCEPT OF INERTIA FROM CLASSICAL MECHANICS TO CONTEMPORARY PHYSICS

Lj. Novaković

Faculty of Science, Kragujevac

## ABSTRACT

One of basic quantities in physics, the concept of inertia, is reviewed from Newton's philosophy to Heisenberg's approach.

## 1. INTRODUCTION

There is a growing interest for relativistic cosmology and the problem of the space-time structure. Therefore one should be able to conduct experimental checks on various conclusions of the general theory of relativity by overthrowing the difficulties which arise in the interpretation of the measurements. Experiments are predominantly directed to observe the relativistic effects in relation to Newton's theory of gravitation (1687) which appears in Einstein's theory of relativity (1916) as the approximation for a weak gravitational field. In the framework of relevant observations a great deal of various theories of gravitation are analyzed which, on one hand, depart from the general theory of relativity and, on the other hand, contain Newton's mechanics as a limiting model. As a useful criterion the theorists quote Mach's idea about the conditional dependence of the inertial properties of material objects on the structure of the universe as a whole (Mach's principle, 1883).

## 2. NEWTON'S PHILOSOPHY

In the mathematical foundations of natural philosophy<sup>1-3</sup> Newton has taught that uniform motions, by which the time is measured, perhaps do not exist in nature and that all visible motions are

either accelerated or decelerated. However, according to Newton's conception, this difficulty is characteristic only for the motion in a relative space, i.e. the space relative to massive material objects. The mass is, in Newton's mechanics, a quantity of matter. In addition to the relative space Newton has introduced the concept of an absolute space, which remains "identical and motionless" all the time. A relative space is nothing but a motive part of the absolute space. Here in Newton's absolute space all processes take place in the absolute time which, compared to the relative time, is changeless in its course, whereas the relative time can be measured by the motion of material objects.

According to Newton, the relative and absolute motions are different as much as the translations and rotations, respectively. To illustrate his attitude Newton suggested the following thought experiment. Let us imagine that a metal container full of water is hanged by a long rope to the ceiling. If the rope together with the container is winded around its vertical axis in a certain direction and let suddenly alone then the container, under the influence of the elastic deformation of the rope, will rotate around the vertical axis in the opposite direction. In the beginning the water surface remains plane as the quantity of water remains motionless. At this moment the motion of the water relative to the metal walls of the container is increased to a maximum, whereas the absolute motion is equal to zero. However, as the walls drift water layers around the vertical axis, due to internal frictional forces, the relative motion will gradually diminish, whereas the absolute motion will become increased to a maximum. Finally the water surface gets a parabolic shape. Hence, an absolute motion, i.e. the motion relative to the absolute space, in Newton's philosophy, is the only genuine motion. It is very hard to distinguish the genuine motions from imaginary

motions because the portions of the absolute space, where material objects perform their motions, cannot be learned by our senses.

### 3. MACH'S PRINCIPLE

In Mach's system of mechanics<sup>4</sup> the absolute space and the absolute motion are abstract concepts which cannot be tested by experiment. Mach has taught that Newton's experiment with a container in rotation demonstrates that a relative rotation of water with respect to the walls of the container does not induce a centrifugal force of inertia because this force is created by the relative motion with respect to the earth and other celestial objects. It would be hard to anticipate, in Mach's opinion, the result of Newton's experiment if the walls of the container were very thick and very massive on a macroscopic scale.

In Mach's teaching mechanics can get rid of metaphysical concepts, such as the absolute space and absolute time, by a suitable selection of the reference frame. Using the objects in the outer space we may observe the motions relative to them, as the relative motions, by definition, may be uniform or accelerated with regard to a given standard motion. The structure of the space is tightly connected with the concept of inertia. Therefore, Mach has concentrated his critique on Newton's laws. Mach asked, whether in the nature there exist reference frames in which Newton's laws are satisfied, i.e. the inertial frames. Obviously, the frames in rotation are not inertial; so, Newton's laws do not hold. In addition, Mach pointed out, in an actual experiment one measures the relative quantities such as the relative position, velocity and acceleration. Therefore, forces just like masses must be determined with respect to other forces and masses.

The mass, in Mach's definition, must be determined as a dynami-

cal characteristic of the mutual interactions taking place between all bodies in the universe. A material particle of mass  $\mu$ , in Mach's mechanics, moves not with respect to some abstract absolute space, but with respect to the center of all masses in the universe,  $M_i$ , which are placed at  $R_i$  apart from the observed material particle. Hence, the uniform motion must be described by the following equation,

$$\frac{d^2}{dt^2} \left\{ \frac{\sum_i M_i \vec{R}_i}{\sum_i M_i} \right\} = 0 \quad (1)$$

In Mach's equation of motion (1) the test particle  $\mu$  will move at a constant speed if "sufficiently many, sufficiently remote and sufficiently heavy masses" are taken into account. Therefore, we may neglect the influence of small masses which are in a close vicinity of the test particle. Using Eq. (1) we can express Mach's principle as follows: The inertia of a material particle is determined by an attraction of heavy masses which exist at the infinity. In other words, the increase of inertia of a given material particle is conditioned by the accumulation of heavy masses in its neighbourhood.

The above principle may be used to determine the equilibrium condition for all masses in the universe. Let  $\vec{R}_c$  designate the center of all masses, and let  $\vec{R}_0$  be a distance separating the observed material particle from the center. We have

$$\vec{R}_i = \frac{\sum_i M_i \vec{R}_i + \mu \vec{R}_0}{\sum_i M_i + \mu} \quad (2)$$

If we insert  $\vec{R}_c = 0$  in Eq. (2) we obtain

$$\mu = \frac{|\sum_i M_i \vec{R}_i|}{|\vec{R}_0|} \quad (3)$$

Eq. (3) could be tested by experiment provided that all quantities appearing in it are available. However, this is impossible, since only a very limited number of those quantities are at our disposal, whereas

the greatest contributions come from the biggest and most distant masses in the universe. Therefore, Mach's principle has reduced the inertia of a test particle to the interaction with the distant stars and galaxies.

A good illustration for Mach's idea is a revolving circular platform<sup>5</sup> which is used as an amusement in our universe just because a centrifugal force of inertia is caused by the spinning stars. If Mach is right then the rotating roundabout in an empty space would not make much of an amusement since in this case the inertial forces would disappear.

#### 4. EINSTEIN'S RELATIVITY

Mach's definition of the mass contains a weakness as pointed out by Einstein<sup>6,7</sup>: "The weakness of the principle of inertia lies in this, that it involves a circle argument; a mass moves without acceleration if it is sufficiently far from other bodies, but we know that it is sufficiently far from other bodies only by the fact that it moves without acceleration". To break the vicious circle Einstein came to the idea of equalizing the role of all types of the motion and all reference frames not only in large space regions but also in the universe on the whole (general principle of relativity). He came to the conclusion that the principle of inertia is established with a high precision only in our planetary system, provided that perturbations owing to the motion of the sun and other planets are neglected. He considers that the laws of the special theory of relativity are perfect in local limits, provided that the space and the time depend mutually one on another, and so they must be unified into a single space-time continuum (Galilean regions).

In Einstein's opinion classical mechanics offers no explanation for the equality between the inertial and the gravitational mass of

a material object. For Einstein, the equation of motion in a gravitational field,

$$(\text{Inertial Mass}) \cdot \text{Acceleration} = (\text{Intensity of the Gravitational Field}) \cdot (\text{Gravitational Mass}), \quad (4)$$

and the experimental fact that the acceleration is independent of the nature of the material objects in a given gravitational field, must hold only if the inertial mass equals the gravitational mass (the principle of equivalence). Therefore, we may use, following this principle, noninertial frames in the Galilean regions, i.e. accelerated frames and those in rotation.

Einstein assumes that a distribution of material objects in the universe will influence and determine the metric laws of a spacetime continuum. In doing so, Einstein came to the conclusion that a formulation of the general theory of relativity requires a generalization of the theory of invariants and the theory of tensors. By following Mach's principle and Riemann's generalization of Euclid's geometry Einstein has postulated:

- 1) The space-time is a four-dimensional Riemann manifold;
- 2) The gravitational field is described by a symmetrical tensor field of the second rank, i.e. by the metric tensor  $g_{\mu\nu}$  of the space-time continuum;
- 3) Trajectories of test particles are geodesic lines in the space-time with the metric tensor  $g_{\mu\nu}$ ; and
- 4) A connection between the space-time metric and the distribution of masses is described by the gravitational field,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \kappa T_{\mu\nu} \quad (5a)$$

$$\kappa = 8\pi G/c^2 \quad (5b)$$

Here  $R_{\mu\nu}$  designates Ricci's tensor,  $R$  is a scalar curvature,  $T_{\mu\nu}$  is

the energy tensor of matter,  $\kappa$  is a gravitation constant. The mass of a material object becomes, by Einstein's equations, an element of the dynamical geometry.

Is Mach's idea fulfilled and materized in the general theory of relativity? To answer this question Einstein has solved the equations of the gravitational field, using a weak field approximation.

The result is

$$\frac{d}{dt} \{ (1 + \bar{c}) \vec{v} \} = \text{grad } \bar{c} + \frac{\partial \bar{c}}{\partial t} + [\text{rot } \bar{c} \times \vec{v}] \quad (6a)$$

$$\bar{c} = \frac{\kappa}{8\pi} \int \frac{\sigma dV_0}{r} \quad (6b)$$

$$\bar{c}_0 = \frac{\kappa}{2\pi} \int \frac{\sigma (dx_a/dt)}{r} dV_0 \quad (6c)$$

$$ds = \sqrt{g_{44}} dt, \quad (6d)$$

where  $a$  designates the space coordinates. Therefore,

(a) The inertial mass is proportional to the quantity  $1 + \bar{c}$ , in a suitably selected system of units, and will increase when the heavy masses accumulate around the test particle;

(b) A material object will experience an accelerating force when the neighbouring masses are accelerated, as given by the second term in Eq. (6a);

(c) A rotating hollow mass shell will generate, inside of its space, a Coriolis field of force which will deflect a moving test particle in the sense of the rotation, as given by the third term in Eq. (6a).

The conclusions (a) - (c) seem to agree, on a qualitative basis at least, with Mach's idea. Nevertheless, a full quantitative comprehension, in spite of the enormous progress in the general theory of relativity, is still lacking<sup>8</sup>.

## 5. HEISENBERG'S APPROACH

The concept of inertia is tightly connected with the spectrum of elementary particles. From experiments performed during the last ten years or so, we learned that various elementary particles are represented by stationary states of a general material system. The particles are characterized by quantum numbers or by the corresponding transformations of the fundamental groups. Our theoretical understanding of elementary particles is reduced to the calculation of the spectrum. One of the outstanding models to describe the world of elementary masses was the approach of Heisenberg<sup>9</sup> where the concept of an elementary length was introduced by connecting the fundamental nonlinear spinor equation with the spectrum. In doing so, Heisenberg considered the elementary masses as quantum states of a unified field of the primeval matter, i.e. the field which was created at the earliest time in world's history. Today, Heisenberg's result must be evaluated as a modest contribution to our understanding of the origin of the spectrum, as it is still separated from the general theory of relativity. Therefore, it has a limited quantitative value.

It is possible, Heisenberg pointed out on one of his latest days<sup>10</sup>, that the spectrum of elementary particles in our region of the universe is somewhat different from the spectrum inside of a neutron star because the boundary conditions are probably different from those we assume when we start solving Schrödinger's wave equation for a given atomic system. Therefore, the dynamics of matter is indispensable for our knowledge, but nobody knows, at the present time, the exact mathematical formulation for this dynamics.

## 6. CONCLUSIONS

We have given a brief review of the concept of inertia, represented by the mass of a material object, at important stages of its

historical development. First, it measured the quantity of matter (in Newton's philosophy); then it became a dynamical characteristic for the mutual interactions taking place among all objects in the universe (by Mach's principle); next it was defined as an element of the dynamical geometry (by Einstein's equations); finally, it was considered to be a quantum state of a unified field of the primeval matter (in Heisenberg's approach).

What is the next stage?

#### REFERENCES

- <sup>1</sup> I. Newton, *Philosophiae naturalis principia mathematica*, 1st ed., Streater, London (1687), cited according to Misner, Thorne and Wheeler, Reference 7
- <sup>2</sup> K.A. Putilov, *Kurs fiziki*, volume I, FM, Moskva (1962).
- <sup>3</sup> D.N. Burghes and A.M. Downs, *Modern Introduction to Classical Mechanics and Control*, Ellis Horwood Ltd., Chichester, England (1975)
- <sup>4</sup> E. Mach, *Mechanics*, cited according to Misner, Thorne and Wheeler, Reference 7, and according to Konopleva, Reference 8.
- <sup>5</sup> P.C. Davies, *Space and Time in the Modern Universe*, Cambridge University Press, Cambridge (1977).
- <sup>6</sup> A. Einstein, *The Meaning of Relativity*, Chapman and Hall, Science Paperbacks, London (1978).
- <sup>7</sup> C.W. Misner, K.S. Thorne, and J.A. Wheeler, *Gravitation*, W.H. Freeman and Co., San Francisco (1973).
- <sup>8</sup> N.P. Konopleva, *Ajnštajnovskij zbornik*, Izdatelstvo "Nauka", Moskva (1978).
- <sup>9</sup> W. Heisenberg, *Introduction to the Unified Field Theory of Elementary Particles*, Interscience Publishers, London (1966).
- <sup>10</sup> W. Heisenberg, *Cosmic Radiation and Fundamental Problems in Physics*, *Naturwissenschaften* 63 (1976) 63.