ON BELL'S INEQUALITY

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The aim of this note is to examine the influence of structure of underlying probability space, on the Bell's inequality[1], whose standard form[2] is

$$|P(a,b) - P(a,c)| + |P(d,b) + P(d,c)| \leq 2 \quad (1)$$

where, $P(a,b) = \int \rho(\lambda)\ f(a,\lambda)\ f(b,\lambda)\ d\lambda$ i.e. $P(a,b)$ is the expected value of the product of a stohastic variables $f(a,\lambda) = \pm 1$, $\rho(\lambda)$ is certain probabillity distribution.

From (1) one may obtain

$$p(a\wedge b) - p(a\wedge c) + p(d\wedge b) + p(d\wedge c) \leq 1 \quad (2)$$

where $p(a\wedge b)$ is probabillity of event $a\wedge b \in \mathfrak{B}$ from $(\Omega,\mathfrak{B},p)$.

(2) may be obtained directly, using next properties of the proba bilyties of events from $(\Omega,\mathfrak{B},p)$ space,

$$p(a) + p(\bar{a}) = 1 \quad (3)$$

$$p(a\vee b) + p(a\wedge b) = p(a) + p(b) \quad (4)$$

$$p(a\wedge b) = p(a) + p(\bar{a}\wedge b) \quad (5)$$

$$p(a\wedge b) + p(\bar{a}\wedge c) \leq p(b\wedge c) \quad (6)$$

which are valid when $\mathfrak{B}$ have properties of the distributive lattice i.e $(\Omega,\mathfrak{B},p)$ is the classical probability space.

To obtain the same sence as in (2) one must assume that $\Omega = \Omega_1 \times \Omega_2$, $\mathfrak{B} = \mathfrak{B}_1 \times \mathfrak{B}_2$, $a,d \in \mathfrak{B}_1$ and $b,c \in \mathfrak{B}_2$.

Experimental violation of (2),[3] discards the validity of the proposed scheme. Violation may be explained introducing the nonlocality in classical probability space, or using non-distributive probability space.

Non-local assumption asks for χ_a(set function of event a) to be dependent on the event b., i.e. χ_a $\chi_{a(b)}$ and consequently

$$p(a(b)) \neq p(a(c)) \tag{7}$$

In this case inequality simillar to (2) is

$$p(a(b)\wedge b(a)) - p(a(c)\wedge c(a)) + p(d(b)\wedge b(d)) + p(d(c)\wedge c(d)) \tag{8}$$

$$\leq 1 + f(p(a(b)\wedge \bar{a}(c)),\ p(b(a)\wedge \bar{b}(d))$$

where f in (8) may be such to include experimental resultes, and (2) is special case of (8).

For non-distributive lattices and probability distributions on them, (5) is no more valid and should be replaced with

$$p(a\vee b) \geq p(a) + p(\bar{a}\wedge \bar{b}). \tag{5'}$$

In this case (2) may be deduced only if a,b,c and d belong to the center of lattice. This type of scheme is realised in quantum mechanics, so the non-locality is necessary only when the classical probability scheme is used. One of the ways to include experimental resultes in some Bell's type inequality, on classical probability scheme and without non-local assumption, is to change value space of $f(a;\Lambda)$.[4]

REFERENCES

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