COMPETITION OF CHARGE DENSITY WAVE AND SUPERCONDUCTING INSTABILITY IN A TWO BAND MODELL

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Abstract

A one dimensional-model of interacting electrons forming two bands with different masses is considered. Both intraband and interband interactions are assumed. If the interband charge transfer processes become dominant, the two Fermi velocities scale to a common value yielding an effective one band model. Otherwise, depending on the initial parameters, charge density wave or superconducting instability may appear, formed dominantly from one of the bands or from both bands.

Introduction

The Hubbard model and its extensions including interactions between further neighbours or allowing for two bands near the Fermi energy have been in the focus of interest recently. The simplest two band model that appears in this context is the periodic Anderson model studied in detail to understand the behaviour of valence fluctuating or heavy fermion systems [1]. The electronic states form a broad conduction band mixed to a narrow band with heavier effective mass. Depending on the parameters of the model (position and width of the bands, hybridization matrix element and Coulomb energy) strong valence fluctuation, heavy fermion behaviour, magnetic or superconducting ordering may be obtained.

Most calculations have been done on two or three dimensional models. The one dimensional case is easier to threat mathematically since in a renormalization group transformation the number of couplings does not increase. It is therefore of interest to study the properties of the two band model in one spatial dimension. Varma and Zawadowski [2] calculated the first logarithmic corrections to the vertices and derived scaling equtions for the couplings. Here we go beyond the leading logarithmic approximation and show that in the next order a Fermi velocity renormalization appears, that in some cases eliminates the difference between the two species of electrons. In most cases, however, the interband charge transfer processes become irrelevant and the dominant instability is determined by the Kondo type interband exchange or the intraband backscattering processes, leading to a charge density wave or superconducting state.

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Model

Assuming that the Fermi energy crosses two bands at momenta $\pm k_A$ and $\pm k_B$, linearizing the spectrum near the Fermi points and cutting off the states far from them, the electron spectrum will consist of four distinct pieces characterized by the velocities $\pm v_A$ and $\pm v_B$, respectively. The creation and annihilation operators for the A and B particles are $a^\dagger_{k\pm k_A,\sigma}$ and $a_{k\pm k_A,\sigma}$, $b^\dagger_{k\pm k_B,\sigma}$ and $b_{k\pm k_B,\sigma}$, respectively. In the diagrams solid lines with label A or B stand for the particles near $+k_A$ or $+k_B$, respectively, while dashed lines indicate particles near $-k_A$ or $-k_B$.

The non-interacting part of the Hamiltonian has four terms corresponding to the four kinds of particles

$$\begin{aligned} H_0 = & \sum_{k,\sigma} v_A k a^\dagger_{k+k_A,\sigma} a_{k+k_A,\sigma} - \sum_{k,\sigma} v_A k a^\dagger_{k-k_A,\sigma} a_{k-k_A,\sigma} \\ & + \sum_{k,\sigma} v_B k b^\dagger_{k+k_B,\sigma} b_{k+k_B,\sigma} - \sum_{k,\sigma} v_B k b^\dagger_{k-k_B,\sigma} b_{k-k_B,\sigma}. \end{aligned} \tag{1}$$

The summation over k is restricted to $|k| < k_C$ in all the four terms.

From the possible interaction processes we consider only those that lead to logarithmically singular contributions in the lowest order bubble diagrams already. When the interactions are spin dependent 12 couplings should characterize the scattering processes as shown in Fig.1.

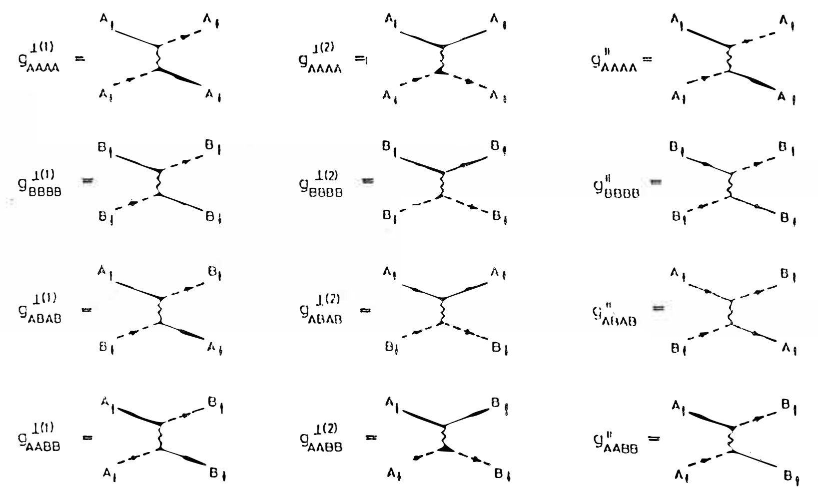

Fig.1. The couplings leading to singular corrections in the vertices. The lines are labelled by a band index (A or B) and the spin ($\uparrow$ and $\downarrow$). Solid line denotes particles near $+k_A$ or $+k_B$, dashed line indicates particles near $-k_A$ or $-k_B$.

Varma and Zawadowski performed a renormalization group calculation on this model by collecting the leading logarithmic singular contributions to the vertices. We have extended the calculation to the next order by calculating the next to leading logaritmic corrections to the vertices and the self energy as well.

Results

In deriving the second order scaling equations it turned out that this model satisfies scaling only if a simultaneous Fermi velocity renormalization is performed. For the dimensionless quantity

$$\gamma = \frac{(v_A + v_B)^2}{4 v_A v_B}$$

the scaling equation has the form

$$\mathrm{d}\,\gamma/\mathrm{d}\,\ln S = 2\gamma(\gamma - 1)\tilde{g}^2_{AABB}$$

Where S is the scale change, $\tilde{g}$ denotes dimensionless couplings and $\tilde{g}^2_{AABB} = (\tilde{g}^{\perp(1)}_{AABB})^2 + (\tilde{g}^{\perp(2)}_{AABB})^2 + (\tilde{g}^{\parallel}_{AABB})^2$.

The fixed points of this equation are $\gamma = 1$, i.e. $v_A = v_B$, or $\tilde{g}^2_{AABB} = 0$. In the first case the distinction between the two bands disappears and an effective one band model is recovered with the known results. We therefore concentrate on the second possibility, when v_A and v_B remain different and all the charge transfer couplings $\tilde{g}^{\perp(1)}_{AABB}$, $\tilde{g}^{\perp(2)}_{AABB}$ and $\tilde{g}^{\parallel}_{AABB}$ vanish.

In this case the scaling equations give three invariants, namely

$$\tilde{g}^{\parallel}_{AAAA} - \tilde{g}^{\perp(2)}_{AAAA},\ \tilde{g}^{\parallel}_{BBBB} - \tilde{g}^{\perp(2)}_{BBBB},\ \tilde{g}^{\parallel}_{ABAB} - \tilde{g}^{\perp(2)}_{ABBA}.$$

Introducing the combination $\tilde{g}^{\parallel(1)} = \tilde{g}^{\parallel} + \tilde{g}^{\perp(2)}$ for $AAAA$, $BBBB$ and $ABAB$ channels, the scaling equations for the remaining 6 couplings simplify such that their fixed poits can be determined. They are shown on Table 1.

	$\tilde{g}^{*\perp(1)}_{AAAA}$	$\tilde{g}^{*\parallel(1)}_{AAAA}$	$\tilde{g}^{*\perp(1)}_{BBBB}$	$\tilde{g}^{*\parallel(1)}_{BBBB}$	$\tilde{g}^{*\perp(1)}_{ABAB}$	$\tilde{g}^{*\parallel(1)}_{ABAB}$
F_1	0	*arb.*	0	*arb.*	0	*arb.*
F_2	0	*arb.*	± 1	-1	0	0
F_3	± 1	-1	0	*arb.*	0	0
F_4	± 1	-1	± 1	-1	0	0
F_5	0	0	0	0	± 1	-1
F_6	0	0	$\pm u$	$-u$	$\pm w$	$-w$
F_7	$\pm u$	$-u$	0	0	$\pm w$	$-w$
F_8	$\pm 1/2$	$-1/2$	$\pm 1/2$	$-1/2$	$\pm 1/2$	$-1/2$

Table 1: The fixed points if $\gamma \neq 1$. arb. means arbitrary, $u = 2w - w^2$ and $w = 4/3 - 1/3(\sqrt[3]{17 + 3\sqrt{33}} + \sqrt[3]{17 - 3\sqrt{33}})$.

Only the first five fixed points are attractive as seen in Fig.2., where the scaling trajectories are shown for the isotropic case $\tilde{g}^{\parallel(1)} = \tilde{g}^{\perp(1)}$.

F_1 is the weak coupling fixed point, available if the bare couplings are positive. For negative, attractive bare couplings the model scales to one of the strong coupling fixed points. The value given in Table 1. is certainly a wrong estimate of the strength of the coupling. It only indicates that the couplings scales to the strong coupling limit, where the higher order corrections will determine the fixed point value.

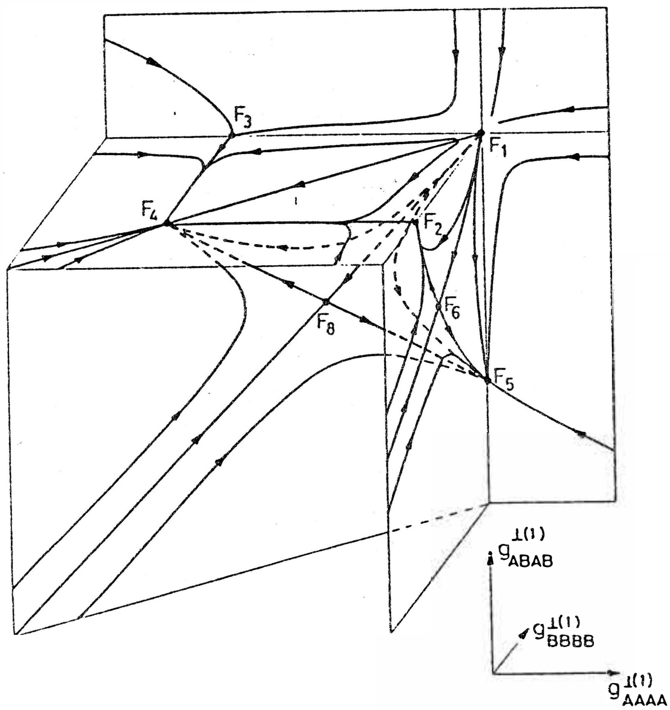

Fig. 2. The scaling trajectories for the special case $\tilde{g}_{AABB}=0$.

In the cases F_2 and F_3 particles in one of the bands become dominant and will determine the properties of the model. A charge density wave (CDW) or superconducting state instability may occur, as in the one band model, the CDW or Cooper pair being formed from one band only. For F_4, both bands form a CDW or Cooper pair, but there is no coupling between the bands. The new feature appears in F_5, where the interband exchange (Kondo coupling) becomes dominant and the CDW or Cooper pair is formed from particles of the two bands.

Summary

We have performed a second order scaling calculation for a one dimensional model of interacting fermions forming two bands with different Fermi velocities. It was shown that the interband charge transfer processes tend to eliminate the difference between the velocities leading to an effective one band model. It was found, however, that quite often the interband charge transfer couplings scales out of the problem, leaving the two Fermi velocities different. In this case charge density wave or superconducting instability may appear if the intraband backscattering or the interband exhange couplings are repulsive. Depending on which one is stronger, the CDW or Cooper pair will be formed from one band or from both.

References

1. See e.g. the reviews: C. M. Varma, Comments Solid State Phys. **11** , 221 (1985) and P. A. Lee *et al.*, Comments Cond. Mat. Phys. **12**, 99 (1986).
2. C. M. Varma and A. Zawadowski, Phys. Rev. B **32**, 7399 (1985).