SPIN, STATISTICS and CHARGE of SOLITONS in (2+1)-DIMENSIONAL THEORIES

Victor M. YAKOVENKO

L.D. Landau Institute for Theoretical Physics, 2 Kosygin St., Moscow 117940, USSR

Abstract. General topologically invariant microscopical expressions for quantum numbers of particle-like solitons ("skyrmions") are derived for a class of (2+1)D models. Skyrmions are either half-integer spin fermions with odd electric charge or integer spin bosons with even charge. So they cannot be Anderson's spinons or holons. General results are exemplified by a square lattice model reminiscenting high-T_c models.

It was conjectured by P.W.Anderson [1] that unusial particles — spinons and holons — may exist in condensed matter. Spinons are neutral fermions with spin $\hbar/2$ while holons are spinless bosons with charge e. In polyacetylene, which is (1+1)D system, solitons realy have such quantum numbers [2]. In (2+1)D case it is intersting to consider [3] quantum numbers of "skyrmions" — particle-like solitons of $\vec{n}$-field. To describe skyrmion let us map (x, y) plane on the sphere S^2 by stereographic projection. The skyrmion configuration is the hedgehog configuration of $\vec{n}$-field on the sphere S^2. At the (x, y) plane this configuration looks as follows: $\vec{n}$ is down at the center, $\vec{n}$ is up at the infinity and there is a concentric domain wall in between.

Quantum numbers of skyrmions can be calculated in a general form [4], [5] for a class of models which have the action

$$S(\psi(\vec{r}), \vec{n}(\vec{r})) = Tr \int d^3r_1 d^3r_2 \psi^+(\vec{r_1}) \tilde{G}^{-1}(\vec{r_1}, \vec{r_2}) \psi(\vec{r_2}), \tag{1}$$

$$\tilde{G}^{-1}(\vec{r}_1, \vec{r}_2) = G_0^{-1}(\vec{r}_1, \vec{r}_2) + \vec{\sigma}\,[\vec{n}(\vec{r}_1) + \vec{n}(\vec{r}_2)]\, G_1^{-1}(\vec{r}_1, \vec{r}_2). \tag{2}$$

Here $\vec{r}_1, \vec{r}_2$ are (2+1)D space-time coordinates (continuous or discrete), $\psi(\vec{r})$ is an electron annihilation operator, $\tilde{G}^{-1}$ is the inverse Green function taken in the real-space representation, $\vec{\sigma}$ are Pauli matrices which act on the spin indices of electrons. Matrix functions G_0^{-1} and G_1^{-1} are the coefficients of decomposition of the inversed Green function $\tilde{G}^{-1}$ over the unit and the Pauli spin matrices. Functions G_0^{-1} and G_1^{-1} have translational symmetry appropriate to uniform or periodic potential, while $\vec{n}(\vec{r})$ is any vector function slowly varying in space-time with the constrain $|\vec{n}(\vec{r})| = 1$. Trace is taken over spin and other indices of electrons which are assumed. The system of units , where $e, \hbar$ and c are set to unity, is used. Dimensional factors are restored only in final formulas.

Physically action (1), (2) describe electrons which do not interact between each other but interact with external $\vec{n}$- field. So (1), (2) is typical mean-field action which frequently appears in weak-coupling models. Some people try to describe even strongly correlated electrons by effective action of this type.

Now let us perform functional integration over $\psi(\vec{r})$ field and find effective action $S_{eff}(\vec{n}(\vec{r}))$ for $\vec{n}$-field. This action contain the term

$$S_1 = \frac{C_1 \varepsilon_{\mu\nu\lambda}}{32\pi} \int d^3r\ B_\mu P_{\nu\lambda}, \quad P_{\mu\nu} = \vec{n}\left[\frac{\partial \vec{n}}{\partial r_\mu} \times \frac{\partial \vec{n}}{\partial r_\nu}\right], \quad \frac{\partial B_\nu}{\partial r_\mu} - \frac{\partial B_\mu}{\partial r_\nu} = P_{\mu\nu}. \tag{3}$$

(received November 15, 1989)

The coefficient C_1 is given [4] by the following expression

$$C_1 = N(G), \quad N(G) = \frac{\varepsilon_{\mu\nu\lambda}}{24\pi^2} Tr \int d^3k \, G \frac{\partial G^{-1}}{\partial k_\mu} G \frac{\partial G^{-1}}{\partial k_\nu} G \frac{\partial G^{-1}}{\partial k_\lambda}. \tag{4}$$

In formula (4) we use Green function (2) in the momentum representation $G(\vec{k})$ with $\vec{n}(\vec{r})$ field set to uniform distribution $\vec{n}(\vec{r}) = \vec{n}_0 = \vec{e}_z$ where $\vec{e}_z$ is a unit vector along z-axis in spin space. Integral (4), taken over momenta and frequency, is a topological invariant of the matrix function $G(\vec{k})$. It takes integer values. As was shown in [6] skyrmions have spin

$$S = C_1\hbar/2 \tag{5}$$

and corresponding statistics. Thus they may have either fermion or boson but not fractional statistics depending on the parity of integer (4). Taking into account that $G(\vec{k})$ is diagonal in spin indices expression (4) can be rewritten as

$$C_1 = N(G_\uparrow) + N(G_\downarrow), \tag{6}$$

where $G^{-1}_{\uparrow,\downarrow}(\vec{k}) = G^{-1}_0(\vec{k}) \pm 2G^{-1}_1(\vec{k})$.

To find electrical charge of the skyrmion the electromagnetic potential $A_\mu(\vec{r})$ is introduced in (1) in standard way. After functional integration over fermions two additional to (3) terms appear in $S_{eff}(\vec{n}(\vec{r}), A_\mu(\vec{r}))$:

$$S_2 = \frac{C_2 \varepsilon_{\mu\nu\lambda}}{4\pi} \int d^3r \, A_\mu \frac{\partial B_\lambda}{\partial r_\nu}, \quad S_3 = \frac{C_3 \varepsilon_{\mu\nu\lambda}}{4\pi} \int d^3r \, A_\mu \frac{\partial A_\lambda}{\partial r_\nu}. \tag{7}$$

Varying S_2 over A_0 we find expression for the electric charge of the skyrmion:

$$e^* = C_2 e. \tag{8}$$

Varying S_3 over A_x we find nondiagonal component of conductivity

$$\sigma_{xy} = C_3 e^2/h. \tag{9}$$

Microscopical expressions for C_2 and C_3 are as follows [4], [5], [7]:

$$C_2 = N(G_\uparrow) - N(G_\downarrow), \quad C_3 = C_1 = N(G_\uparrow) + N(G_\downarrow). \tag{10}$$

From (10) we see that fermions always have odd charge while bosons have even charge. So skyrmions cannot be spinons or holons.

Now let us consider a paricular model where C_1, C_2 and C_3 have non-zero values. It is square lattice model with the period doubling in both directions. Green functions $G_\alpha(\vec{k})$, $\alpha = \uparrow, \downarrow$ can be expanded over Pauli matrices $\vec{\tau}$ which act on the doublet of electrons $(\psi(k_x, k_y), \psi(k_x + \pi, k_y + \pi))$:

$$G_\alpha(\vec{k}) = i\omega + \vec{\tau}\vec{w}(k_x, k_y), \tag{11}$$

$$w_1 = M_\alpha + t_2(\cos(k_x + k_y) - \cos(k_x - k_y)), \tag{12}$$

$$w_2 = J(\cos k_x - \cos k_y), \quad w_3 = t_1(\cos k_x + \cos k_y). \tag{13}$$

In (13) the term proportional to t_1 describes electron hopping between the nearest neighbouring sites. The term proportional to J describes current density wave that is spontaneous staggering currents along the links of square lattice. It was suggested in [8] and recently discussed in [9]. With $J = t_1$ and $t_2 = M = 0$ model (11)–(13) is very close to the "flux phase" model [10]. But in weak coupling limit relation between J and t_1 is arbitrary. The term proportional to t_2 was suggested in [11] and describe staggering hopping amplitude between the next-nearest neighbouring sites. This term breaks macroscopical time-reversal and inversion symmetries of the model. The terms proportional to M_α, suggested in [12], describe staggering site energy. There are charge density wave with the amplitude $(M_\uparrow + M_\downarrow)/2$ and spin density wave with the amplitude $(M_\uparrow - M_\downarrow)/2$ in the model. Polarization of SDW is determined by $\vec{n}$-field, which is set to the constant value $\vec{n_0} = \vec{e_z}$ in (11)–(13).

Applying the methods of [4], [5] and [7] to model (11)–(13) we find: $N(G_\alpha)$=sign(t_2) if $|2t_2| > |M_\alpha|$ and $N(G_\alpha) = 0$ otherwise. Let $|M_\uparrow| > |M_\downarrow|$. Then from (10) it follows that in the region $|2t_2| > |M_\uparrow|$ skyrmions are neutral bosons with spin $\hbar$, in the region $|M_\downarrow| < |2t_2| < |M_\uparrow|$ skyrmions are fermions with spin $\hbar/2$ and charge e, in the region $|2t_2| < |M_\downarrow|$ skyrmions are neutral bosons with spin 0. If $M_\alpha = 0$ and J has opposite signs for spins up and down then skyrmions are spinless bosons with charge $2e$. In this case the staggering currents are the spin currents and their polarization is given by $\vec{n}$-field.

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