# Quantum Dynamics of Dissipative Josephson Chains

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## 1 Introduction

Recent experiments on granular superconducting films[1] have revealed a surprising feature, namely that the low temperature properties of the films seem to be determined solely by the *normal state* sheet resistance (per square); more precisely, the films become superconducting for $T \to 0$ only when the normal state resistance is less than a critical value of $\sim 6.5\,k\Omega$. Since, in a *single* Josephson junction shunted by a resisitor $R_s$, a diffusion-localization transition is found[2] (at $T = 0$) at the critical value $R_s^c = R_0 \equiv \pi\hbar/2e^2 \simeq 6.45\,k\Omega$, it has been speculated that the appropriate generalization of the single junction model to a network could provide an explanation of the experimental results. However, I find it difficult to imagine how shunt resistors can be motivated microscopically for a granular film.

Nevertheless, it is a challenging theoretical question, as an extension of the single junction result,[2] to inquire about the zero temperature properties of an array or a chain of identical Josephson junctions,[3-6] each of which is shunted by the same $R_s$. In particular, the phase diagram for chains is highly controversial.[4-6] In this note, I do not wish to discuss specific results for the phase diagram. Rather, I wish to illuminate some aspects of the underlying theoretical formulation, which essentially consists of transforming the standard model (given in terms of an Euclidean action) to a system of vortices (or charges) interacting via a highly anisotropic interaction – on this aspect, Refs. 4-6 agree. Using methods developed in the investigation of the quantum dynamics of vortices in a network of ideal (i.e. unshunted) junctions,[7,8] it is straightforward (avoiding the Villain transformation and the discretization of the time variable) to confirm the basic expression for the interaction between charges.[4-6] However, in this approach, an ambiguity remains concerning the continuum limit, which is explained and discussed below.

## 2 The Model

The Euclidean action describing the quantum statistical properties of a chain of dissipative Josephson junctions is given by $S = S_0 + S_D$, where

$$S_0 = \sum_j \int d\tau \left[\frac{1}{2} m \dot{\varphi}_j^2 - E_J \cos(\varphi_j - \varphi_{j-1})\right] \ . \qquad (1)$$

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Here $\varphi_j$ denotes the order parameter phase on the $j$-th superconducting grain, $E_J$ is the Josephson coupling energy, and $m = \hbar^2 c_0/4e^2$, where $c_0$ is the ground capacitance of a grain. In addition, the dissipative part $S_D$ is given by

$$\begin{aligned} S_D &= \frac{1}{8}\sum_j \int d\tau d\tau' \, A_s(\tau-\tau')\big[(\varphi_j-\varphi_{j-1})_\tau - (\varphi_j-\varphi_{j-1})_{\tau'}\big]^2 \\ &= -\frac{1}{4}\sum_j \int d\tau d\tau' B_s(\tau-\tau')(\dot\varphi_j-\dot\varphi_{j-1})_\tau(\dot\varphi_j-\dot\varphi_{j-1})_{\tau'} \;, \end{aligned} \tag{2}$$

where I used subscripts to indicate the time argument. The second form in (2), which I call choice *II*, is obtained by a partial integration from the first one (choice *I*). Accordingly, with $\alpha_s = R_0/R_s$, one has

$$A_s(\tau) = (\hbar\alpha_s/\pi^2)\cdot\tau^{-2} \;, \;\; B_s(\tau) = (\hbar\alpha_s/\pi^2)\cdot\ln|\tau| \tag{3}$$

and for the Fourier transforms:

$$A_s(\omega) = -\hbar\alpha_s|\omega|/\pi \;, \;\; B_s(\omega) = A_s(\omega)/\omega^2 \;. \tag{4}$$

In the continuum limit, in which $\varphi_j$ is replaced by $\varphi(x)$, and

$$\varphi_j - \varphi_{j-1} \to \partial_x\varphi(x) \ll 1 \;, \tag{5}$$

it is convenient to employ the obvious (after expansion of the cosine) space-time symmetry of $S_0$ and define $y = c\tau$, where $c^2 = E_J/m$. Then $S_0$ is given by

$$S_0 = \frac{1}{2}\frac{E_J}{c}\int d^2r \, (\nabla\varphi)^2 \;. \tag{6}$$

On the other hand, being a bit careful about when the continuum limit is taken, $S_D$ is given by

$$S_D^I = \frac{1}{8}\int dx dy dy' \, A_s(y-y')\Big[\partial_x\varphi(x,y) - \partial_x\varphi(x,y')\Big]^2 \;; \tag{7}$$

$$S_D^{II} = -\frac{1}{4}\int dx dy dy' \, B_s(y-y')\,[\partial_x\partial_y\varphi(x,y)]\,[\partial_x\partial_{y'}\varphi(x,y')] \;. \tag{8}$$

Clearly, both forms are equivalent if space and time derivatives can be interchanged. *However, this is not the case for the vortex solution to be discussed below.*

## 3 The Vortex (Charge) Picture

The transformation to the vortex picture is achieved by noting that, since the action is quadratic in the phase, it is sufficient to study special solutions of the equation of motion, $\delta S/\delta\varphi = 0$. I use the following ansatz (see e.g. Refs. 7 and 8):

$$\varphi = \varphi^V + \varphi^S \;\; ; \;\; \varphi^V = \sum_j e_j \arctan\frac{y-y_j}{x-x_j} \tag{9}$$

where $\vec{r}_j = (x_j, y_j)$ denotes the center of the $j$-th vortex, and $e_j = \pm 1$ its vorticity. Since the equation of motion is linear, the spin wave part $\varphi^S$ is readily determined in Fourier representation. Explicitly, define

$$\delta S_0/\delta\varphi = \mathcal{L}_0 * \varphi \; ; \; \delta S_D/\delta\varphi = \mathcal{L}_D * \varphi \; ; \; \mathcal{L} = \mathcal{L}_0 + \mathcal{L}_D \; , \tag{10}$$

and note that $\mathcal{L}_0 * \varphi^V = 0$; thus

$$\varphi^S = -\mathcal{L}^{-1} * \mathcal{L}_D * \varphi^V \; . \tag{11}$$

A brief inspection shows that $\mathcal{A}[\{\vec{r}_j\}] \equiv S[\varphi^V + \varphi^S]$ is of the form of the potential energy of a system of interacting charges, namely

$$\hbar^{-1} \cdot \mathcal{A}[\{\vec{r}_j\}] = \frac{1}{2}\sum_{i,j} e_i e_j U(\vec{r}_i - \vec{r}_j) \; . \tag{12}$$

The Fourier transforms of the interaction potential, $U(\vec{q})$, for case $I$ and case $II$ are found to be given by the following expressions:

$$U^I(\vec{q}) = (2\pi)^2 \cdot \frac{J + (\alpha_s/2\pi)|q_y|}{\vec{q}^2 + (\alpha_s/2\pi)|q_y| q_x^2/J} \tag{13}$$

$$U^{II}(\vec{q}) = (2\pi)^2 \cdot \frac{J + (\alpha_s/2\pi) q_x^2/|q_y|}{\vec{q}^2 + (\alpha_s/2\pi)|q_y| q_x^2/J} \tag{14}$$

where $J = E_J/\hbar c$. The result $U^{-1}(\vec{q} = 0) = 0$ implies that only charge-neutral configurations contribute to the partition function, i.e. $\sum_j e_j = 0$. Of course, for $\alpha_s = 0$, both expressions given above are identical, and the model shows the well known Berezinskii-Kosterlitz-Thouless[9] transition for $J \sim 1$. In addition, it is clear from $U^I(\vec{q})$ that the long-distance logarithmic interaction between charges is *not* modified by the dissipative contribution. On the other hand, note that the main features (for long distances) of $U^{II}(\vec{q})$ are described by the approximation[4-6]

$$U^{II}(\vec{q}) \simeq (2\pi)^2 \cdot \left( \frac{J}{\vec{q}^2} + \frac{\alpha_s}{2\pi|q_y|} \right) \; , \tag{15}$$

which shows, besides the isotropic logarithmic interaction, that there is an anisotropic part, local in $x$ and increasing logarithmically in $y$; i.e. charges on the same superconducting grain interact logarithmically for different times, which is the relevant result known from a single junction: The second part of (15) is precisely the interaction obtained in Ref. 2 in the tight binding limit $J > 1$ by the instanton technique. Note that the long distance behavior of the potential can be obtained by the approximation $\varphi \simeq \varphi^V$, i.e. by neglecting the spin wave part $\varphi^S$. I emphasize that the expression (14) agrees with the one obtained earlier[4-6] by use of the Villain transformation and discretization of the time (i.e. $y$) variable. As mentioned above, the athors of Refs. 4-6, however, disagree on the conclusions to be drawn from this result.

## 4 Discussion

The difference between case $I$ and case $II$ is due to the different order of $x$- and $y$-derivatives; however, for the vortex part $\varphi^V$, one finds

$$[\partial_x\partial_y - \partial_y\partial_x]\varphi^V = 2\pi\sum_j e_j\delta(\vec{r}-\vec{r}_j) \ , \tag{16}$$

which is most easily confirmed by noting that

$$[\nabla\varphi^V]_{\vec{q}} = 2\pi i\sum_j e_j\frac{(q_y, -q_x)}{q^2}e^{-i\vec{q}\cdot\vec{r}_j} \ . \tag{17}$$

But what is the correct interpretation, (7) or (8)? A similar question arises when including a nearest-neighbor capacitance, which adds the following contribution to the action:

$$\frac{M}{2}\sum_j\int d\tau\ (\dot{\varphi}_j - \dot{\varphi}_{j-1})^2 \rightarrow \frac{M}{2}\int dx d\tau\ (\partial_x\dot{\varphi})^2 \tag{18}$$

Recalling that $\dot{\varphi}_j$ is related to the voltage according to Josephson's relation, $\hbar\dot{\varphi}_j = 2eV_j$, it seems that the interpretation given in (18) is the correct one.[7] Similarly, it is plausible that the choice $II$ (see (8)), i.e. first writing the dissipative part of the action in terms of the voltage, and then taking the spatial continuum limit, leads to the correct answer.[10] Of course, this interpretation is supported by the agreement with the results obtained by different techniques.[4-6]

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