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VACUUM INSTABILITIES FOR MODEL FIELD THEORIES

Thomas D. Cohen

Department of Physics and Astronomy

University of Maryland, College Park, Maryland 20742 U.S.A.

ABSTRACT

The fermion loop contribution to the energy for all renormalizable field theories which contain fermions leads to negative energies for boson background field configurations of sufficiently small scales. Such configurations indicate that, to this level, the vacuum is unstable. Boson loops cannot stabilize the vacuum unless there are boson-boson derivative couplings. Thus, models such as the σ-model, the Friedberg-Lee soliton, or the Walecka model all suffer from this instability. Moreover, due to the strong couplings in this models, the instability occurs at distances scales comparable to the inverse nucleon mass. This instability is related to the famous Landau ghost problem in QED.

QCD is, to the best of our knowledge, the correct theory for strong interactions. It is also quite nasty to deal with in the low q regime. Unfortunately, it is precisely this regime which is of interest to nuclear physics and we are, in a sense, driven to consider model theories. Such model theories, it is hoped, capture the essential features of QCD relevant for low q physics. Examples include the σ model with quarks, the Skyrme model, the Nambu-Jona-Lasinio model, bag models, the color dielectric model, the Friedberg-Lee model, and, in the context of the nuclear many-body problem, the Walecka model. Many of these models are given in terms of field theoretic lagrangians.

One is faced with the difficult question of how such models should be interpreted. Are they true quantum field theories or effective theories to be used only for mean-field calculations or, perhaps, something else? Some of these models (the linear σ model, some versions of the Friedberg-Lee model and various versions of the Walecka model) are of the form of renormalizable field theories and it is tempting to interpret these models as true field theories.

Such temptations should be avoided! In the first place, the interpretation of these models as true field theories is physically unreasonable as it implies that, at least in loops, the models work up to arbitrarily high q and, at high q we know that perturbative QCD is the correct description of strong interactions. Secondly, such models suffer from a peculiar affliction--the vacuum is unstable, at least at the one loop level. It is this second problem which is the subject of this talk.

The vacuum instability disease was discovered by Vikram Soni[1,2] in the context of topologically nontrivial configurations in the nonlinear σ model. He showed that such configurations, when considered at the one fermion loop level, lead to a negative energy compared to the "vacuum" about which one is calculating. His calculation only assumes that the size of the configuration is sufficiently small. Ripka and Kahana subsequently derived Soni's result from a slightly more transparent formalism[3]. Actually, the Soni disease is much more general. *Any* renormalizable field theory with fermions will suffer from the vacuum instability unless every boson in the theory has derivative couplings (*e.g.* non-abelian gauge theories)[4,5]. To illustrate the issues let us consider the following toy model with a scalar boson coupled to fermions

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - M + g\sigma)\psi + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - U(\sigma) \quad , \tag{1}$$

where U is an arbitrary potential of up to fourth order.

It is relatively easy to show that the one loop fermion energy in a background σ field is

$$E_{\text{fermion}} = - \frac{1}{2} \text{ Tr } \log[1+G\ V] \ , \tag{2}$$

where $G=1/(\partial^2+M^2)$ and $V=-2gM\sigma+g^2\sigma^2+ig(\not{\partial}\sigma)$ and the trace is over four momenta and Dirac indices. The metric in this expression is Euclidean. This energy is just the difference of the energy of the Dirac sea in the background σ field configuration from the energy of the sea in the vacuum, *i.e.* the Casimir energy. Expanding the log yields

$$E_{\text{fermion}} = - \frac{1}{2} \text{ Tr } [GV - \frac{1}{2}(GV)^2 + \frac{1}{3}(GV)^3 - \frac{1}{4}(GV)^4 + \ldots] \quad . \tag{3}$$

The first two terms in this sum are divergent. The total expression can be made finite by as subtraction of the form of $- \frac{1}{2}\text{ Tr}[GV-\frac{1}{2}G^2V^2]$. This amounts to a renormalization of U and a wavefunction renormalization.

Now consider a background σ field configuration, $\sigma=f(x/R)$, where f is an arbitrary functional form and R will be treated as a variational parameter which characterizes the spacial extent of the configuration. Note that for small R, G will scale like R^2 and V will scale 1/R. Thus, for small R, the first terms in the expansion should dominate. At sufficiently small R, the first non-vanishing term, $\frac{1}{4}\text{ Tr}[(GV)^2-G^2V^2]$

should dominate. The energy associated with the dominant part of the one fermion loop energy can be explicitly calculated by summing of plane waves. The result for small R is $E = A R \log(MR)$, where

$$A \approx (g^2 N/64\pi^5)\int d^3q' |f(q')|^2 q'^2 \quad ,$$

$f(q')$ is the fourier transform of $f(x/R)$ and N is the number of internal degrees of freedom (colors, flavors etc.). Note that A is manifestly positive. The kinetic energy for σ obviously scales with R and the potential energy goes like R^3. It is clear that for small enough R the 1 fermion loop energy will dominate and will lead to a negative energy. This is the vacuum instability as such negative energy "bubbles" can energetically be formed out of the vacuum.

The value of R which minimizes the energy including the fermion loop is $M^{-1} \exp(-a-8\pi^2/g^2N)$, where a is a constant of order unity which depends on the form of f. For weak coupling, the value of R_{min} is obviously very short. For large g R_{min} can occur at scale not very different from the inverse fermion mass.

Let us now consider boson loops. For models such as this one where there is no boson-boson derivative coupling the boson one loop energy will be of the form $\frac{1}{2}$ Tr $\log[1+GV]$, where $G=1/(\partial^2+M_\sigma^2)$ and V is the second functional derivative of U with respect to σ. This Tr log is of the same form as the fermion one loop energy but of the opposite sign. This might suggest that the boson loop might be able to overcome the fermion loop and stabilize the vacuum. Note however, that for the fermion loop V had a derivative part which scaled like 1/R while for the boson loop V is independent of R. Thus, the boson loop energy will, for small R, be two powers of R suppressed compared to the fermion loop and thus cannot stabilize the vacuum. Derivative coupling, will, of course, change this and can lead to R log(R) contributions to the energy and thus can possibly stabilize the vacuum.

How is one to understand this mysterious vacuum instability? The simplest way is to study the boson propagator to one loop[5]. One can write the inverse propagator as the bare inverse propagator plus a polarization (or boson self energy). The polarization can be calculated at the one loop level by using Feynman rules. It is easy to show that for large negative (Minkowski) q^2 that the fermion contribution to the polarization goes like $|q^2| \log|q^2/M^2|$ while the boson loop contributes like $-M_\sigma^2 \log|q^2/M_\sigma|$. Note that for sufficiently large $|q^2|$ the fermion loop contribution will dominate over the boson loop contribution. Moreover, the fermion loop will for large $|q^2|$ eventually become larger in magnitude and opposite in sign from the bare propagator $(-|q^2| + M^2)$. Since the inverse propagator goes through zero, the propagator develops a pole. This pole is in a tachyonic (*i.e.* space-like) region. Such tachyonic poles indicate a breakdown of a theory or an approximation. The integrals needed to calculate the polarization are precisely those needed to calculate the one loop energy in a background field. Moreover, the value of q at which this tachyonic pole occurs is of order R_{min}^{-1} and has the same dependence on g. An exact association of q at the tachyonic pole and R_{min} is, of course, impossible because the precise value of R_{min} depends on the precise form of the arbitrary function f(x/R). Nevertheless, it is quite clear that the vacuum instability is strongly associated with the tachyonic pole in the boson propagator. It should be observed that in the context of QED this tachyonic pole is known as a Landau ghost. Due to the Ward identities this tachyonic pole, in QED, is associated with a diverging running coupling constant. Because the QED coupling is weak, the Landau ghost occurs at absurdly high momentum.

What does this vacuum instability tell us about the interpretation of the various models of baryon and nuclear structure? It tells us that we must be very careful in our treatment of effective models. We cannot calculate loops up to arbitrarily high momenta. Indeed, for the strong coupling theories we must restrict q to values not much larger than the inverse baryon mass. Thus we can interpret the models as being valid for mean-field calculations or, perhaps, for loop calculations with an explicit, and reasonably small, momentum cutoff. In short, we cannot just take our lagrangians and blindly grind away using the techniques of renormalizable field theories. Instead we must rely on good taste.

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