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TIME-DEPENDENT HARTREE-FOCK AND THE EXCITATIONS OF THE DIRAC SEA

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Abstract

An investigation on chiral phase transitions, the meson spectrum and pion properties, realized in the Nambu-Jona-Lassinio model and using a semi-classical approximaton based on the TDHF method is reported.

1- INTRODUCTION

The investigation of the mechanism of chiral symmetry breaking and its implications for the properties of hadrons is of fundamental interest in hadron physics.

In Quantum Chromodynamics (QCD), numerical calculations indicate that the underlying chiral symmetry is spontaneously broken, supposedly via the Nambu-Goldstone mechanism, leading to a $q\bar{q}$ pair condensate into the vacuum.

It is also expected that in hadronic matter at high temperatures and/or densities chiral symmetry is restored, being realized in the Wigner-Weyl mode. Lattice QCD calculations support the concept of such a phase transition.

Chiral effective models provide an alternative way of investigating these problems at a phenomenological level. In this concern the Nambu-Jona-Lasínio (NJL)[1] model provides an adequate framework from which much physical insight can be obtained. The NJL model illustrates the mechanism of chiral symmetry breaking and the occurrence of an associated Goldstone Boson. The light meson spectrum can be obtained as collective excitations of $q\bar{q}$ pairs.

The analogy with the theoretical description of low-lying particle-hole states in Nuclear Physics suggests the use of many-boby techniques developed in this field.[2]

2-THE FORMALISM

The TDHF equations, which govern the time evolution of a many-fermion system, may be derived from the minimum action principle:

$$\delta\int_{t_1}^{t_2}\langle\psi(t)|\,i\hbar\frac{\partial}{\partial t}-H\,|\psi(t)\rangle\, dt=0 \qquad (1)$$

where $|\psi(t)\rangle$ is a Slater determinant and H is the many-body Hamiltonian.

For an equilibrium configuration the functional of the energy should obey the equilibrium condition $\mathcal{E}(\rho_0) \leq \mathcal{E}(U\rho_0 U^+)$, where ρ_0 is the density matrix of the equilibium configuration and U any arbitrary unitary operator.

The time evolution of the density matrix is derived from the action principle (1). In terms of the density matrix the Lagrangian may be written as:

$$\mathcal{L} = i\hbar\, tr(\dot{U}(t)\,\rho_0\, U^+(t)) - \mathcal{E}(U(t)\,\rho_0\, U^+(t)) \ . \tag{2}$$

U(t) is chosen of the form $U(t)=\exp(i/\hbar)s(t)$ where s(t) is a time dependent Hermitian single particle operator which generates deviations from the equilibrium configuration.

The classical limit of this Lagrangian will be considered, retaining only the leading order terms in a Wigner-Kirkwood expansion.

3-CHIRAL PHASE TRANSITIONS

The NJL model describes a system of many fermions interacting through the chiral invariant Hamiltonian:

$$H = \sum_{i=1}^{N} \gamma_5(i)\,\vec{\sigma}(i)\cdot\vec{p}_i - g\sum_{i\neq j}\delta(\vec{x}_i-\vec{x}_j)\big(\beta(i)\beta(j) - \beta(i)\gamma_5(i)\beta(j)\gamma_5(j)\big) \ . \tag{3}$$

We will study different equilibrium configurations using the formalism presented before.

- **Dynamical chiral symmetry breaking**

Within a mean field approach we define the effective single particle Hamiltonian $h = \beta M + \gamma_5\vec{\sigma}\cdot\vec{p}$ where βM is a mean field potential, **M** being a variational parameter which will be interpreted as the mass of the constituent quarks. We consider as trial states Slater determinants of negative energy states. The following expression for the Wigner transform of the density matrix is obtained:

$$\rho_0(\vec{p}) = \frac{1}{2}\Big(I - \beta\frac{M}{E} - \gamma_5\frac{\vec{\sigma}\cdot\vec{p}}{E}\Big)\,\theta(\Lambda^2 - p^2) \tag{4}$$

where Λ is a cutoff in momentum space.

The equilibrium condition leads to the self-consistent equation for the mass:

$$M = 4gM\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{E}\,\theta(\Lambda^2 - p^2) \ . \tag{5}$$

By computing this expression one easily sees that there is a phase transition from a Dirac sea of massless particles to a Dirac sea of massive particles, provided that the critical condition $g\Lambda^2 > \pi^2$ is satisfied. Chiral symmetry is therefore dynamically broken and, in accordance with the Goldstone theorem, a pseudoscalar zero energy mode shoud appear. As a matter of fact there is a solution of the TDHF equations which corresponds to a pseudoscalar zero energy state.

- **Restoration of chiral symmetry**

In order to illustrate a simple mechanism of restoration of chiral symmetry with density we consider as trial state a Slater determinant of positive energy states built on the top of the chiral deformed phase previously described. The Wigner transform of the density matrix is now:

$$\rho(\vec{p})=I\,\theta(\Lambda^2-p^2)+\frac{1}{2}\left(I-\beta\frac{M}{E}-\gamma_5\frac{\vec{\sigma}.\vec{p}}{E}\right)\left(\theta(\Lambda^2-p^2)-\theta(\lambda^2-p^2)\right) \tag{6}$$

where λ is the momentum of the highest positive energy level occupied. The following self-consistent equation for the mass is obtained:

$$M=4gM\int\frac{d^3p}{(2\pi)^3}\,\frac{1}{E}\,\left(\theta(\Lambda^2-p^2)-\theta(\lambda^2-p^2)\right)\ . \tag{7}$$

It is found that for fixed values of g and Λ the system undergoes a phase transition for $(\lambda_{cr})^2=\Lambda^2-\pi^2/g$.

We conclude that for enough positive energy levels occupied it is not energetically advantageous for the quarks to acquire a mass. The $q\bar{q}$ pairs decouple and chiral symmetry is restored.

4-MESON SPECTRUM

The study of the time evolution of a slightly disturbed chiral deformed equilibrium configuration will lead us to the equations of motion of collective modes of qq pairs, with boson quantum numbers.

Following the general lines presented before, the operator which generates deviations from the equilibrium configuration is chosen in such a way that its Wigner transform is:

$$s(\vec{p},t)=\gamma_5L_1-i\beta\gamma_5L_2+\gamma_5\vec{\sigma}.\vec{V}_1-i\beta\gamma_5\vec{\sigma}.\vec{V}_2 \tag{8}$$

The Wigner transform of the density matrix is:

$$\begin{aligned}\rho(\vec{p},t)=\ &\rho_0(\vec{p})+\Big(\gamma_5\frac{M}{E}(L_2+\vec{\sigma}.\vec{V}_2)-i\,\frac{\vec{\sigma}.\vec{p}}{E}\times\vec{V}_1\\&-\beta\,\frac{1}{E}(\vec{\sigma}.\vec{p}\,L_2+\vec{p}.\vec{V}_2)+i\beta\gamma_5\frac{M}{E}(L_1+\vec{\sigma}.\vec{V}_1)\Big)\,\theta(\Lambda^2-p^2)\end{aligned} \tag{9}$$

By performing arbitrary variations with respect to the variational functions L_i and $\vec{V}_i$, and assuming harmonic dependence on time, one easily obtains the equations of motion corresponding to pseudoscalar, scalar, axial vector and vector modes. Looking for low energy solutions ($0\lesssim\omega\lesssim2M$) one obtains:

- A pseudoscalar mode with frequency $\omega=0$
- A scalar mode with frequency $\omega=2M$

- An axial vector mode which has no low energy solution
- A vector mode with frequency $0<\omega<2M$.

The first two solutions are cutoff independent, contrarily to the last ones. These results are in agreement with those obtained by Nambu and Jona-Lasínio at a qualitative level.

5-PION PROPERTIES

The zero frequency pseudoscalar mode found before is interpreted as the chiral limit for the pion. Its dispersion relation, in the limit of the long wavelenght, is derived within a semi-classical approximation. The pion decay constant is also computed.

In order to obtain dispersion relations we allow the generator s to become $\vec{x}$ dependent. Lenghty calculations are avoided if we perform an expansion of the generating functions in powers of $\vec{p}$ and keep only the leading order terms. An adequate choice of the generators allows, in principle, to obtain dispersion relations for all bosons. The simplest approximation in order to obtain the pion dispersion relation is to retain only the first term of the expansion, which means to take into account only the generating functions L_1 and L_2 considered as functions of $\vec{x}$ and t alone.

Assuming plane wave solutions and harmonic dependence on time, the semi-classical limit of the quantum action principle (1) leads us straightforwardly to a phonon-like dispersion relation for the pion: $\omega(\vec{k})=f(\Lambda/M)\vec{k}$.

The time-like pion decay constant is obtained from the expression

$$\langle 0|\, j_5^{(0)}\, |\pi\rangle = f_\pi \left(\omega(\vec{k})\right)^{1/2} \tag{10}$$

In order to compute the axial vector current matrix element, we identify a quantal pionic state $|\pi\rangle$ with a ramdom phase approximation (RPA) excited state and the vacuum $|0\rangle$ with the RPA vacuum. Performing standard calculations the pion decay constant is easily obtained: $f_\pi = M\, h(\Lambda/M)$.

A non zero value of f_π is therefore related with a non zero value for the mass of the constituent quarks and, consequently, with dynamical symmetry breaking. For values of Λ/M which allow to fit relevant values of M and of the order parameter $-\langle\bar\psi\psi\rangle^{1/3}$, the value of f_π lies around its experimental value, if also a colour factor is included.

REFERENCES

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[2] J. da Providência, M.C. Ruivo and C.A. de Sousa, Phys.Rev. D *36* (1987), in print.