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DETERMINATION OF THE ENERGY LEVEL OF THE TWO DIMENSIONAL ELECTRON GAS IN THE JUNCTION FIELD EFFECT TRANSISTOR BY A VARIATIONAL CALCULUS

During the last ten yearsthe influence of quantum effects has been studied intensively theoretically and experimentally in the cases of strong inversion and accumulation in the MOSFET transistors. Only few papers were devoted to this problem for the case of the transistor with p-n junction based field effect (JFET).

The assumed wave function $\psi(z)$ (the z-axis is normal to the direction of the source-drain) in the channel has the form: $\psi(z)=\text{const}(z^2-\alpha^2)$ where n is a natural number ($n\geqslant 2$) and α the variational parameter.

Comparing the basic energy levels obtained through the variational calculawith those obtained by means of the exact self-consistent solution of the Schrödinger and Poisson equation (1), the possibility of applying the variational method depending on the channel width W_k and parameter n, was discussed. It was shown that the variational method yields more accurate results at smaller n, while the error, increses fast, at greater W_k.

1. Introduction

A large number of papers were devoted to the problem on the influence of quantum effects on the MOSFET structures. On the other hand the quantum effects in the p-n junction based field effect transitors (JFET) were extensively studied [1,2]. In paper [2] the energy levels were determined by the approximation methods: the WKB and the variational methods. Let us mention that determination of the basic energy level by a variational method [2] contains the definite incorrectness. The complete self-consistent solution of the JFET was presented in [1].

The present paper will deal with the variational method of determining the lowest energy level, assuming a more complex wave function form than that in [2]. Besides, it will be pointed out to errors appearning in paper [2]. The numerical results will be compared with those obtained through self-consistent solution.

2. Determination of the ground energy level by a variational method

Let us observe the JFET, the diagram of which is displayed in Fig.1. The coordinate system from Fig.1 and symmetrical conditions with respect to plane z=0, the trial wave function of the ground level may be presented in the form:

$$\psi(z)=\begin{cases} A(z^2-\alpha^2)^n & |z|\leq\alpha \\ 0 & |z|>\alpha \end{cases} \tag{1}$$

In (1) α is a variational parameter, n the natural number, ($n\geqslant 2$) and A a constant determined from the normalizing conditions.

Introducing $\psi(z)$ from (1) into the Poisson's equation:

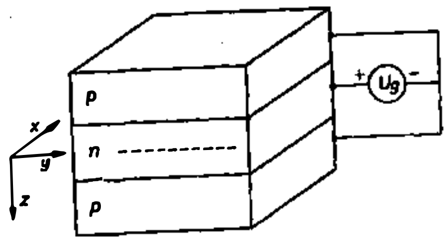

Fig.1. Schematic diagram of JFET.

$$\frac{d^2\phi(z)}{dz^2} = \frac{e}{\varepsilon_s}\left[N_D W_k \psi^2(z) - N_D\right] \text{ , } \phi(0)=0 \text{ and } \left|\frac{d\phi}{dx}\right|_{x=0} = 0 \text{ ,} \qquad (2,3)$$

the expression for the dependence of potential $\phi(z)$ along the JFET, is obtained:

$$\phi(z) = \frac{eN_D}{\varepsilon_S}\left[W_k A^2 \sum_{k=0}^{2n} \binom{2n}{k} \frac{z^{2k+2}\,\alpha^{4n-2k}\,(-)^k}{(2k+1)(2k+2)} - \frac{z^2}{2}\right] \qquad (4)$$

In expressions (2)-(4) ε_s represents the dielectric constant of the semiconductor, e is the electron charge (e>0), N_D - the donors' concentration (which is in our considerations uniform), while W_k is the channel width.

The integration of the Schrödinger equation leads to the expression for energy of ground level (and other levels) [3,4]:

$$E = \frac{\hbar^2}{2m_{zz}} \int_{-\alpha}^{\alpha} \left(\frac{d\psi}{dz}\right)^2 dz - e \int_{-\alpha}^{\alpha} \psi^2 \phi dz \qquad (5)$$

where $\hbar$ is the Dirac constant, and m_{zz} - the effective electron mass describing the motion in the z-axis direction.

Introducing (1) and (4) into (5) we get, for the energy of ground level E_0:

$$E_0 = \frac{\hbar^2}{m_{zz}} \alpha^{-2} \cdot A_1 - \frac{e^2}{\varepsilon_s} N_D W_k \alpha A_2 + \frac{e^2}{\varepsilon_s} \alpha^2 N_D A_3 \qquad (6)$$

A_1, A_2 and A_3 are constants depending only on number n, and determined by expressions:

$$A_1(n) \equiv 8R^{-1} \sum_{k=0}^{2n-2} \frac{\binom{2n-2}{k}(-)^k}{2k+3} \qquad (7)$$

$$A_2(n) = \frac{1}{2} R^{-2} \sum_{k=0}^{2n} \binom{2n}{k} \frac{(-)^k}{(2k+1)(2k+2)} \sum_{m=0}^{2n} \binom{2n}{m} \frac{(-)^m}{2k+2m+3} \tag{8}$$

$$A_3(n) = \frac{1}{2} R^{-1} \sum_{k=0}^{2n} \binom{2n}{k} \frac{(-)^k}{2k+1}, \tag{9}$$

where

$$R \equiv \sum_{k=0}^{2n} \binom{2n}{k} \frac{(-)^k}{2k+1}. \tag{10}$$

The variational parameter α is determined from condition (4):

$$\frac{dE_0}{d\alpha} = 0 , \tag{11}$$

which, in case (6) reduces to the equation of fourth order:

$$C^4 + AC - B = 0, \quad C \equiv \frac{W_k}{2\alpha}. \tag{12}$$

Constants A and B (A,B>0) depend on number n, effective mass m_{zz}, donors´ concentration N_D and channel width W_k. By a very simple analysis, it is possible to show that equation (12) has, for all practical values and parameters, one and only one solution in the interval (0.1). In paper [2] dealing only with the case n=2, the boundary conditions in solving the Poisson equation were incorrectly taken and instead of equation (12) the equation of the eleventh order appears Eqn. (14) in [2].)

Numerical results are given for p-n-p JFET with (111)Si, the uniform donors´ concentration $N_D=10^{24}m^{-3}$, at the room temperature (T=300K). The channel width was assumed to vary from $W_k=0$ to $W_k=240Å$.

Fig.2 presents the dependence of constants A_1, A_2 and A_3 on n. With the increase of number n these constants decrease. It is easy to see that constant A_3 could be written in a simpler form:

$$A_3(n)=\left|2|11+4(n-2)|\right|^{-1} \tag{13}$$

Table 1 contains the results obtained by a self-consistent solving of the Poisson and Schrödinger equation |1| and the results obtained by the variational method (n=2,3) for the channel width from 2Å to 204Å. At W_k 180Å the deviations from the "accurate" self consistent solution are big; therefore they are not included in Table 1. At $W_k=0$, by a self consistent solution we find that energy of ground level is $E_0=1/2\ \hbar\omega_0$ (the energy of ground level the linear harmonic oscillator), while, by the variational method it is:

$$E_o = \sqrt{\frac{12}{\pi}}\ \frac{1}{2}\ \hbar\omega_o ,\quad \omega_o^2 \equiv \frac{e^2 N_D}{\varepsilon_s\, m_{zz}} \qquad (14).$$

W_k |A^o|	E_o^{sc} |meV|	$E_o^{n=2}$ |meV|	$E_o^{n=3}$ |meV|
2	10,54	10,38	10,35
6	10,35	10,15	10,12
18	9,81	9,72	9,63
30	9,29	8,20	8,04
42	8,71	8,44	8,31
60	8,07	7,60	7,30
90	6,97	6,03	5,96
120	5,97	4,80	4,62
150	5,14	3,54	3,14
180	4,35	-	-
192	4,09	-	-
204	3,84	-	-

Table 1 - Dependece of the energy of ground level E^o on W_k for the cases: of self-consistent solution (first column), variational solution at n=2 and n=3 (the second and the third columns).

3. Conclusion

The paper presents the theoretical procedure of determining the basic energy level in the JFET-s by the variational calculus method. The comparison of the numerical results leads to the conclusion that the variational method is more accurate, if quantities $W_k N_D$ are smaller. At W_k=0 the error amounts to 4.5%. When W_k reaches a certain definite value, the error drastically grows. On the orther hand, the error increases with the increase of number n.

Apart from that, the variational method can be applied to the problems not requiring the exceptional accuracy. The reason lies in the fact that the variational method demands from the numerical procedure only the solving of equations of the fourth order; the self-consistent solving is essentially more complex from the point of view of numerical side.

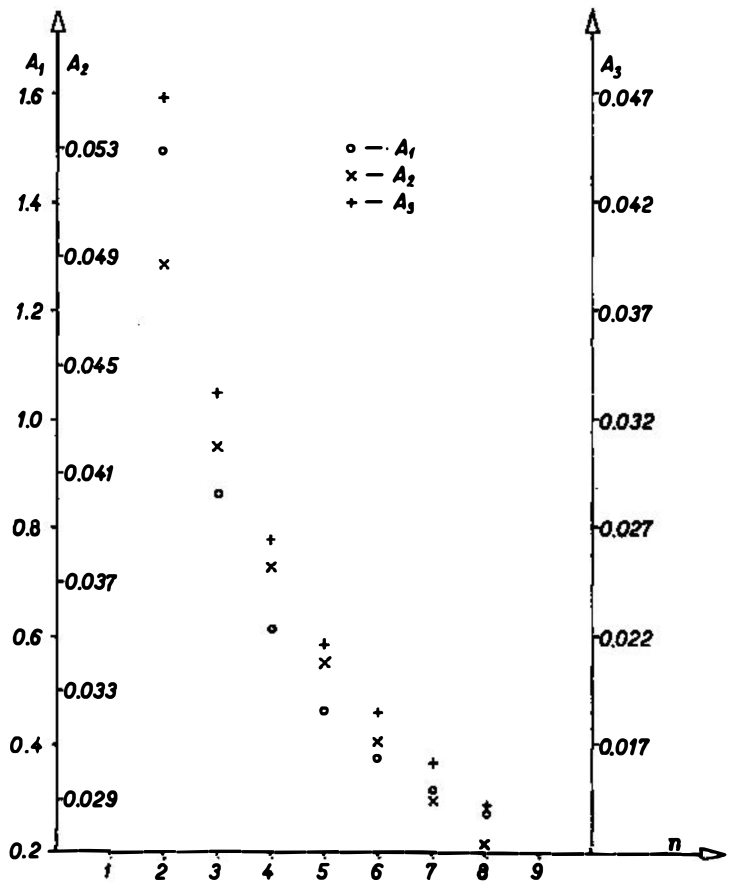

Fig.2. Dependences $A_2(n)$, $A_2(n)$ and $A_3(n)$.

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