A TWO-DIMENSIONAL ANALYSIS OF CARRIERS´ TRANSPORT IN THE MOS STRUCTURES

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Starting from the carriers´transport equation in semiconductors, a mathematical model was proposed for the two-dimensional analysis of the potential distribution (ψ) and the carriers´ concentration in the semiconductor of a MOS transistor structures. The model proposed provides a possibility of a full analysis of the influence of particular parameters on the important quantities characterizing the real structures.

1. The problem of a wide analysis of the transport phenomena in semiconducting inhomogeneous structures (as those appearing in the chanel of uni-polar semiconducting devices) is very complex. There is no analytical solution to this problem, so that adequate models are made and numerically treated.

Starting from the continuity equations, equation for the density of carriers´ currents and the Poison´s equation as the basic relations for the carriers´transport in semiconductor structures and using the equations obtained by the Boltzmann approximation of the Fermi statistics, as well as the Einstein relation, the system of equations is obtained which, in a two-dimensional representation and in the normalized form have the shape [1,2]:

$$\frac{\partial}{\partial x}\left(\mu_n(x,y)\cdot\frac{\partial\lambda(x,y)}{\partial x}\cdot e^{\psi(x,y)}\right)+\frac{\partial}{\partial y}\left(\mu_n(x,y)\cdot\frac{\partial\lambda(x,y)}{\partial y}\cdot e^{\psi(x,y)}\right)=R(x,y) \qquad (1)$$

$$\frac{\partial}{\partial x}\left(\mu_p(x,y)\cdot\frac{\partial\nu(x,y)}{\partial x}\cdot e^{-\psi(x,y)}\right)+\frac{\partial}{\partial y}\left(\mu_p(x,y)\cdot\frac{\partial\nu(x,y)}{\partial y}\cdot e^{-\psi(x,y)}\right)=R(x,y) \qquad (2)$$

$$\frac{\partial^2\psi(x,y)}{\partial x^2}+\frac{\partial^2\psi(x,y)}{\partial y^2}=\lambda(x,y)\cdot e^{\psi(x,y)}-\nu(x,y)\cdot e^{-\psi(x,y)}+N_A(x,y)-N_D(x,y) \qquad (3)$$

with the usual notations.

The boundary conditions for the structure presented in Fig.1 in the normalized form can be expressed by the following expressions:

$$\frac{\partial \Psi(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial y} = \frac{\partial P(x,y)}{\partial y} = 0 \tag{4}$$

on $\overline{EI}$ and $\overline{HJ}$,

$$P(x,y) \cdot N(x,y) = 1 \tag{5}$$

$$P(x,y) - N(x,y) + N_D(x,y) - N_A(x,y) = 0 \tag{6}$$

on $\overline{EF}$, $\overline{GH}$ and $\overline{IJ}$,

$$J_{nx}(x,y) = \mu_n(x,y)\left(\frac{\partial N(x,y)}{\partial x} - N(x,y)\cdot\frac{\partial \Psi(x,y)}{\partial x}\right) = 0 \tag{7}$$

$$J_{px}(x,y) = -\mu_p(x,y)\left(\frac{\partial P(x,y)}{\partial x} + P(x,y)\cdot\frac{\partial \Psi(x,y)}{\partial x}\right) = 0 \tag{8}$$

$$\varepsilon_{SiO_2}\left(\frac{\partial \Psi(x,y)}{\partial x}\right)_{SiO_2} - \left(\frac{\partial \Psi(x,y)}{\partial x}\right)_{Si} = \left(\Psi(x,y) - \Psi_o(x,y)\right)\cdot D_{st} \tag{9}$$

on $\overline{FG}$,

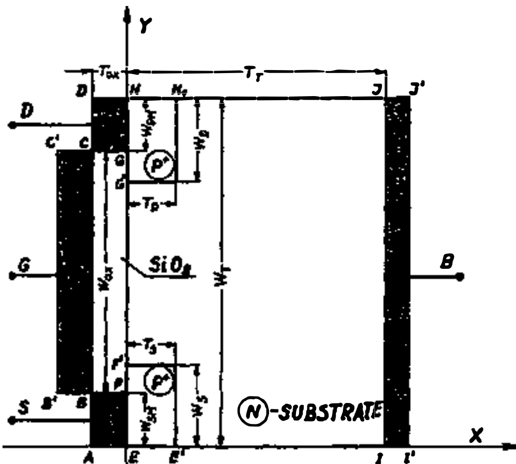

Fig.1. The cross-section of a p-channel heterogeneous semiconducting structure .

assuming that the potential ψ is known along the lines $\overline{EF}$, $\overline{GH}$, $\overline{IJ}$, $\overline{BF}$, $\overline{BC}$, $\overline{CG}$.

2. Introducing the auxiliary functions $S(x,y)$, $u(x,y)$, $\varphi(x,y)$, $\chi(x,y)$, $\beta(x,y)$ and $\gamma(x,y)$, equations (1), (2) and (3) can be written in a unique general form

$$\frac{\partial}{\partial x}\left(S(x,y)\cdot\frac{\partial u(x,y)}{\partial x}\right)+\frac{\partial}{\partial y}\left(S(x,y)\cdot\frac{\partial u(x,y)}{\partial y}\right)=\varphi\big(x,y,u(x,y)\big),\qquad (x,y)\in\Delta \tag{10}$$

with the general boundary condition

$$\chi(x,y)\cdot u(x,y)+\beta(x,y)\cdot\frac{\partial u(x,y)}{\partial u}=\gamma(x,y)\,,\qquad (x,y)\in\Gamma \tag{11}$$

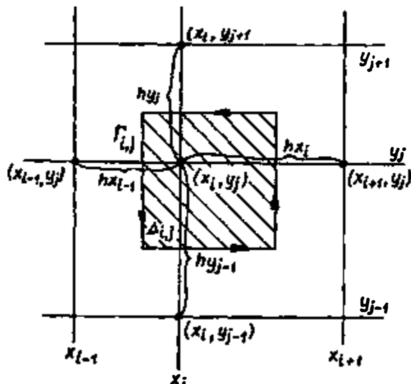

Fig.2 The rectangular, non-uniform MESH at domain Δ.

In the above expressions Δ denotes the domain EHIJ, Γ the bounds of the domain, and $\frac{\partial u(x,y)}{\partial u}$ represents the derivative of function $u(x,y)$ in the direction normal to the contour Γ. Functions $\chi(x,y)$ and $\beta(x,y)$ appearing in the generalized boundary condition are piecewise continuous. For numerical solving of the problem, the method of finite differences and the nonuniform rectangular mesh were used.

If every point (x_i, y_j) of the mesh (Fig.2) is joined by the closed region Δ_{ij} and if the integration of equation (10) over that region is made, and afterwards the Green theorem is applied, with the corresponding approximation of the double integral, we get:

$$\begin{aligned}
&u_{i-1,j}\left(S_{i-\frac{1}{2},j}\,\frac{hy_{j-1}+hy_j}{2hx_{i-1}}\right)+u_{i,j-1}\left(S_{i,j-\frac{1}{2}}\,\frac{hx_{i-1}+hx_i}{2hy_{j-1}}\right)-\\
&-u_{i,j}\left(S_{i-\frac{1}{2},j}\,\frac{hy_{j-1}+hy_j}{2hx_{i-1}}+S_{i,j-\frac{1}{2}}\,\frac{hx_{i-1}+hx_i}{2hy_{j-1}}+S_{i,j+\frac{1}{2}}\,\frac{hx_{i-1}+hx_i}{2hy_j}+S_{i+\frac{1}{2},j}\,\frac{hy_{j-1}+hy_j}{2hx_i}\right)+\\
&+u_{i,j+1}\left(S_{i,j+\frac{1}{2}}\,\frac{hx_{i-1}+hx_i}{2hy_j}\right)+u_{i+1,j}\left(S_{i+\frac{1}{2},j}\,\frac{hy_{j-1}+hy_j}{2hx_i}\right)=\varphi_{i,j}\,\frac{hx_{i-1}+hx_i}{2}\cdot\frac{hy_{j-1}+hy_j}{2}
\end{aligned} \tag{12}$$

where

$$u_{i,j}=u(x_i,y_j) \tag{13}$$

$$S_{i+\frac{1}{2},j}=S\left(x_i+\frac{hx_i}{2},\,y_j\right) \tag{14}$$

The remaing quantities with the respective indices have the analogous meaning.

For point (x_1, y_i) along line FG there is the following relation

$$\begin{aligned} & u_{1,j-1}\left(S_{1,j-1/2}\frac{hx_1}{2hy_{j-1}}\right) - u_{1,j}\left(-S_{1,j}\frac{\chi_{1,j}}{\beta_{1,j}}\cdot\frac{hy_{j-1}+hy_j}{2} + S_{1,j+1/2}\frac{hx_1}{2hy_j} + \right. \\ & \left. + S_{1,j-1/2}\frac{hx_1}{2hy_{j-1}} + S_{1+1/2,j}\frac{hy_{j-1}+hy_j}{2hx_1}\right) + u_{1,j+1}\left(S_{1,j+1/2}\frac{hx_1}{2hy_j}\right) + \\ & + u_{2,j}\left(S_{1+1/2,j}\frac{hy_{j-1}+hy_j}{2hx_1}\right) = \varphi_{1,j}\frac{hx_1}{2}\cdot\frac{hy_{j-1}+hy_j}{2} + S_{1,j}\frac{\gamma_{1,j}}{\beta_{1,j}}\cdot\frac{hy_{j-1}+hy_j}{2} \end{aligned} \tag{15}$$

Analogously, we get the expressions for points located at other parts of the contour Γ.

The presented procedure for discretization of partial differential equations (1),(2),(3), yields the system of equations (with respect to variable $u_{i,j}$; $i=1,2...l$; $j=1,2,..k$) which, in the matrix form, can be written as:

$$A_\lambda(\hat{\psi})\cdot\hat{\lambda} = B_\lambda(\hat{\lambda},\hat{\nu},\hat{\psi}) \tag{16}$$

$$A_\nu(\hat{\psi})\cdot\hat{\nu} = B_\nu(\hat{\lambda},\hat{\nu},\hat{\psi}) \tag{17}$$

$$A_\psi \quad \hat{\psi} = B_\psi(\hat{\lambda},\hat{\nu},\hat{\psi}) \tag{18}$$

where, $\hat{\lambda},\hat{\nu},\hat{\psi}$ are vectors the components of which are the values of unknown functions at points of non-uniform rectangular mesh which are to be determined. A_λ, A_ν, and A_ψ are the matrices where the matrix elements depend on vectors ψ. B_λ, B_ν and B_ψ are vectors whose components are nonlinear functions of vectors $\hat{\lambda}$, $\hat{\nu}$, and $\hat{\psi}$.

For mobility (μ) the relation according to [3] was used, while for the recombination, the Hall-Shockley-Reed recombination model was used.

The mathematical model presented differes from the model described in [1] by the chosen discretization and in solving the initial system of partial nonlinear differential equations, as well as by the models used for the recombination and mobility of carriers and by the treatment of the boundary conditions.

The problems of the analysis of the processes in semiconductors were also treated in [4,5] but less general than in the present paper.

The desired solutions of ψ, λ and ν i.e. ψ, N and P can be obtained by applying the Gummel's algorithm for the cyclic solution

of the system of equations [6,7,8] and the corresponding numerical methods [9,10].

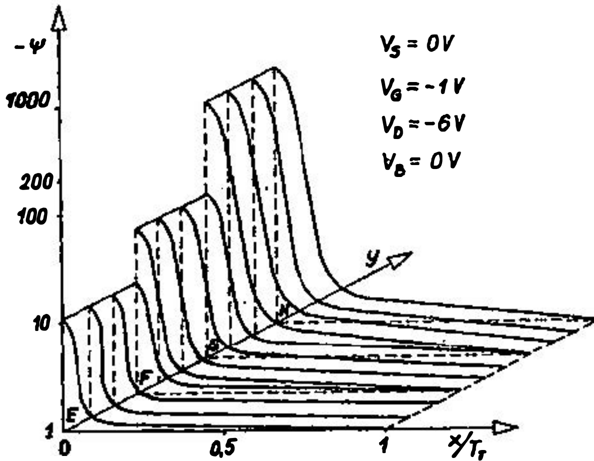

Fig.3 The potential distribution in EHIJ domain

Fig.3 displays $\psi(x,y)$, where the normalized values according to [1] were used, as well as the following dimensions and carriers' concentrations: W_{OX}=35μm, $W_{MS}=W_{MD}$=50μm; $W_S=W_D$=50μm; W_T=105μm, T_{OX}=0,1μm, T_T=100μm; $T_D=T_S$=2μm; $N_{ASB}=N_{DS}=N_{DD}=0$; $N_{AS}=N_{AD}=2\cdot10^{19}cm^{-3}$; $N_{DSB}=8\cdot10^{14}cm^{-3}$.

5. Conclusion. The elaborated mathematical model may be used to show the influence of doping of particular region of the non-homogeneous semiconductor (MOS structure) on the electronic processes in it. The probable point of the occurrence of avalanche (and other) breakdowns could be foreseen and their sizes could be determined which is of particular importance for the physics of the failure process. Such a treatment yields a reasonable and real behaviour of the device and the possibility to select an optimal structure for the given technology.

The assumption that the mobility is dependent on the electric field makes the model even more exact and such an approach to the problem can be connected to the problem of diffusion distribution of impurities.

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