FIZIKA 19, (1987), Supplement 2, 21-26
(Proc. Workshop on Mesonic Degrees of Freedom in Hadrons, Bled/Ljubljana, June/July 1987)

QUARK LOOP CORRECTIONS IN THE LEE FRIEDBERG SOLITON BAG MODEL†

H. Forkel
Institute of Theoretical Physics
University of Regensburg
D-8400 Regensburg, Federal Republic of Germany

Abstract

Using a proper time heat kernel expansion of the fermion determinant, we derive a local approximation to the effective action of the Lee-Friedberg model. It is given as a functional of the classical scalar background field which is supposed to represent the nonperturbative vacuum structure of QCD. The field equation obtained by varying this action may be employed to study the structure and dynamics of the soliton solutions beyond the previously used valence quark approximation.

Our present understanding of hadronic structure and its links with the underlying theory of strong interactions is to a large extent based upon QCD-inspired models. In particular the bag-like models, characterized by a relativistic, perturbative quark core, confined in the physical vacuum, have been intensively investigated and applied to the calculation of hadronic properties with remarkable success.

The majority of these calculations up to now restrict themselves to the so-called quasiclassical approximation, which amounts to neglecting the Dirac sea of the quarks. Recent attempts to incorporate the quark vacuum sector of the original MIT bag model [1] and it s chiral descendants [2] are seriously hindered by the sharp, static boundary conditions of these models. They indicate, however, important contributions from quark vacuum fluctuations to the calculated observables [3].

In view of this situation, soliton bag models which confine the quarks dynamically in a meson field configuration and therefore do not exhibit an unphysically sharp surface, provide a useful starting point from which one can address quark loop effects. A first numerical investigation of these effects in the linear σ model with quarks [4] was based on a fixed, parametrized background field configuration. This complex calculation, although not selfconsistent, indicated important contributions from the virtual quarks to the energy and size of the soliton.

In this contribution we would like to investigate the quark vacuum sector of the original soliton bag model, developed by Lee and Friedberg (LF) [5]. Using a heat kernel expansion of the effective action we derive field equations which can be used to study selfconsistently the dynamical influence of leading quark vacuum effects on the structure and stability of the soliton. This procedure amounts to a partial bosonization of the original model, providing a local approximation to the bosonized action in the form of an effective Lagrange density[1].

We start from the Lagrangian of the LF model in its simplest form with massless quarks and without vector fields:

$$\mathcal{L} = \overline{\Psi}(i\gamma_\mu\partial^\mu - g\sigma)\Psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma). \tag{1}$$

† supported in part by Studienstiftung des deutschen Volkes
[1] Related methods have been used in other quark models [6].

Here Ψ are the quark fields, $U(\sigma)$ is the self coupling of the real scalar field σ with its absolute minimum at $\sigma = \sigma_v$, a relative minimum for $\sigma = 0$ and

$$U(\sigma_v) = 0, \qquad U(0) \equiv p > 0. \tag{2}$$

The most general renormalizable ansatz for $U(\sigma)$ is a polynomial of fourth order in σ:

$$U(\sigma) = p + \frac{a}{2}\sigma^2 + \frac{b}{3!}\sigma^3 + \frac{c}{4!}\sigma^4 \tag{3}$$

with $a, c > 0$, $b < 0$ und $b^2 > 3ac$. The qualitative shape of the σ field in quasiclassical approximation (only valence quarks) is determined by the minimum of the interaction energy density

$$\mathcal{H}_{\mathrm{int},\sigma} = g\overline{\Psi}\Psi\sigma + U(\sigma). \tag{4}$$

For suitably chosen parameters in $U(\sigma)$ and a sufficiently large coupling strength g, $\mathcal{H}_{\mathrm{int},\sigma}$ takes its absolute minimum in regions of nonvanishing quark density at $\sigma \simeq 0$, otherwise at σ_v. Therefore the quark density is dynamically concentrated in a bag-like soliton configuration, with $\sigma \simeq 0$, surrounded by the physical vacuum phase corresponding to $\sigma = \sigma_v$.

The interpretation as an order parameter of the QCD vacuum phase structure suggests treating σ as a classical field, with no physical meaning assigned to it's short wavelength components. On the other hand the quantum fluctuations of the fermion sector have to be taken into account, which we shall do by functionally integrating the quarks with the σ-field as an external source. The average baryon number of the system may be fixed by coupling a chemical potential μ to the conserved fermion number [7,8] in the Euclidean version[2] of the path integral[3]

$$Z[\sigma,\mu] = \int \mathcal{D}\overline{\Psi}\mathcal{D}\Psi e^{-\int d^4x \overline{\Psi}(\gamma_\mu\partial^\mu + ig\sigma - \mu\gamma^4)\Psi} \equiv e^{-\Gamma[\sigma,\mu]}. \tag{5}$$

Thanks to the bilinear fermionic action, the path integral (5) can be formally evaluated to give the determinant of the Dirac operator [10]. So the effective action $\Gamma[\sigma,\mu]$, which describes the nonlocal functional dependence of the quantized fermion system on the background field configuration, becomes

$$\Gamma[\sigma,\mu] = -N_c \mathrm{Tr}\ln(\gamma_\mu\partial^\mu + ig\sigma - \mu\gamma^4) \tag{6}$$

Using the chemical potential we may now separate the valence quark and vacuum contributions:

$$\Gamma[\sigma,\mu] = \Gamma_{val}[\sigma,\mu] + \Gamma_{vac}[\sigma] \tag{7}$$

with

$$\Gamma_{val}[\sigma,\mu] = \Gamma[\sigma,\mu] - \Gamma[\sigma,0], \qquad \Gamma_{vac}[\sigma] = \Gamma[\sigma,0]. \tag{8}$$

[2]The analytic continuation to Euclidean spacetime is performed using the following conventions: the time components of all four-vectors are continued like $x^0 \longrightarrow -ix^4$, the (antihermitean) Euclidean gamma matrices are defined analogously ($\gamma^0 \equiv -i\gamma^4$).
[3]which is equivalent to the grandcanonical partition function in the limit $T \longrightarrow 0$ [9]

The valence contribution $\Gamma_{val}[\sigma,\mu]$ can be further evaluated in closed form [8]. In the following, we are interested in the hadron ground state only, so that we may limit ourselves to static σ fields. Then the eigenfunctions of the Dirac operator in (6) can be constructed out of the eigenfunctions of the static Dirac Hamiltonian, defined by

$$[-i\vec{\alpha}\vec{\nabla}+g\beta\sigma(\vec{x})]\,\Psi_k(\vec{x})=\varepsilon_k\Psi_k(\vec{x}). \tag{9}$$

They allow us to write (7) in the spectral representation, so that the valence part of the effective action can be finally cast into the form

$$\Gamma_{val}[\sigma,\mu]=-N_c\int dx_4\sum_{0<\varepsilon_k<|\mu|}(|\mu|-\varepsilon_k). \tag{10}$$

(The absence of states with $\varepsilon_k = 0$, as proven in [5], has been used in this derivation.) As would be expected, the chemical potential acts as a Fermi level (at $T=0$). In order to describe hadron ground states it has to be adjusted between the two lowest quark energy levels.

We proceed now to the approximate evaluation of the vacuum part of the effective action. From (6) and (8) we have

$$\Gamma_{vac}[\sigma]=-N_c\mathrm{Tr}\ln(D) \tag{11}$$

with

$$D=\gamma_\mu\partial^\mu+ig\sigma. \tag{12}$$

The modulus of the action functional,

$$\Gamma[\sigma]=-\frac{1}{2}N_c\mathrm{Tr}\ln(D^\dagger D), \tag{13}$$

may be regularized in Schwinger's proper time representation [11]

$$\Gamma[\sigma]=\ \frac{1}{2}N_c\int\limits_{1/\Lambda^2}^{\infty}\frac{d\tau}{\tau}\mathrm{Tr}\,K(\tau), \tag{14}$$

because the eigenvalue spectrum of the Hermitean operator $D^\dagger D$ is real and positive. (Λ is the UV-cutoff.) It's heat kernel

$$K(\tau)\equiv e^{-D^\dagger D\tau} \tag{15}$$

satifies the heat equation

$$-\frac{\partial K(\tau)}{\partial\tau}=D^\dagger D\,K(\tau), \qquad K(0)=1. \tag{16}$$

Separating the heat kernel of the free Klein-Gordon operator,

$$K_0(x,y;\tau)\equiv<x|K_0(\tau)|y>=\frac{1}{(4\pi\tau)^2}e^{-m^2\tau+(x-y)^2/4\tau}. \tag{17}$$

with the ansatz

$$K(\tau) = K_0(\tau) H(\tau), \tag{18}$$

(m is the free fermion mass $m \equiv g\sigma_v$) and putting (18) into (16) one arrives at the corresponding equation for the interaction part H:

$$\left(\frac{\partial}{\partial \tau} + \frac{1}{\tau}(x_\mu - y_\mu)\partial^\mu + D^\dagger D - m^2 \right) H(x, y; \tau) = 0. \tag{19}$$

(The derivative operators act on the x coordinates.) In order to solve this equation (approximately), one inserts the WKB-like power series ansatz

$$H(x, y; \tau) = \sum_{n=0}^{\infty} h_n(x, y)\, \tau^n \tag{20}$$

into (19) and obtains the following recursion relations

$$[(n+1) + (x_\mu - y_\mu)\partial^\mu] h_{n+1}(x, y) + (D^\dagger D - m^2) h_n(x, y) = 0 \tag{21}$$

$$(x_\mu - y_\mu)\partial^\mu h_0(x, y) = 0 \tag{22}$$

for the heat coefficients $h_n(x, y)$. The vacuum part of the effective action can now be expressed in terms of the diagonal heat coefficients $h_n(x) \equiv h_n(x, x)$:

$$\Gamma[\sigma] = \frac{N}{2(4\pi)^2} \sum_{n=0}^{\infty} m^{4-2n}\, \Gamma(n-2, \frac{m^2}{\Lambda^2})\, \mathrm{Tr}\, h_n(x). \tag{23}$$

$\Gamma(k, m^2/\Lambda^2)$ are the incomplete gamma functions [12]. They diverge for non-positive k in the limit $\Lambda \to \infty$, so that all infinities of (23) are contained in the first three terms of the sum. From (23) it becomes obvious, that the series is equivalent to an expansion in the inverse free fermion mass.

The contributions of the first four heat coefficients lead after analytic continuation into Minkowski space, combination with the original mesonic part of the LF Lagrangian and renormalization to the following approximation for the effective Lagrange density

$$\begin{aligned} \mathcal{L} = {} & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \tilde{U}(\sigma) \\ & + \frac{G^2}{2(4\pi)^2 m^2}\left[(\partial_\mu\partial^\mu\sigma)(\partial_\nu\partial^\nu\sigma) - 10G^2\sigma^2\partial_\mu\sigma\partial^\mu\sigma + 2G^4\sigma^6 \right] \end{aligned} \tag{24}$$

(irrelevant constants have been omitted) with the renormalized selfinteraction

$$\tilde{U}(\sigma) = p + \frac{a'}{2}\sigma^2 + \frac{b}{3!}\sigma^3 + \frac{c'}{4!}\sigma^4. \tag{25}$$

(The parameter b does not get renormalized.) Combining the valence part (10) and the mesonic and vacuum contributions from (24), one obtains the complete effective action in the chosen approximation. Variation of its static part with respect to σ leads to the field equation

$$\begin{aligned} \vec{\nabla}^2\vec{\nabla}^2\sigma - 10G^2\sigma\vec{\nabla}\sigma\vec{\nabla}\sigma + \left[(4\pi)^2\sigma_v^2 - 10G^2\sigma^2\right]\vec{\nabla}^2\sigma \\ - (4\pi)^2\sigma_v^2\widetilde{U}'(\sigma) + 6G^4\sigma^5 \;=\; \left[(4\pi)^2\sigma_v^2 N_c G\right]\overline{\Psi}\Psi. \end{aligned} \tag{26}$$

Because of the four-gradient term in (24), originating from the $n = 3$ heat coefficient, this equation is of fourth order and therefore already quite difficult to solve. The contribution of each further heat coefficient h_n ($n \geq 4$) would increase it s order by two, so that the usefulness of the equation in practical applications restricts n. Retaining the first four h_n seems to be a resonable compromise between practicality and accuracy of the approximation.

Via an approximate bosonization of the Lee Friedberg soliton model by a heat kernel expansion, we have derived a local and still managable approximation to it s effective action. In this approach the valence quark contributions can be separated by introducing a chemical potential. As would be expected, they lead to the expressions used in the quasiclassical approximation.

The leading effects of the quark vacuum are expressed as additional interactions of the σ field, induced by virtual quark loops. The local effective Lagrangian allows us to derive an equation for the background field, which governs the soliton dynamics in the presence of quark vacuum fluctuations. This equation, together with the corresponding Dirac equation for the quarks provides the basis for a selfconsistent, dynamical investigation of existence, structure and stability of the LF soliton solutions beyond the usual tree level approximation.

The numerical investigation of (radially symmetric) solutions is in progress. The same methods may be also applied to the modified LF model [13] with a running quark σ coupling strength.

References

[1] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, V.F. Weisskopf, Phys. Rev. **D9** (1974) 3471

[2] A. Chodos, C.B. Thorn, Phys. Rev. **D12** (1975) 2733

G.E. Brown, M. Rho, Phys. Lett. **82B** (1979) 177

G.E. Brown, M. Rho, V. Vento, Phys. Lett. **84B** (1979) 83

V. Vento, M. Rho, F.M. Nyman, J.H. Jun, G.E. Brown, Nucl. Phys. **A345** (1980) 413

R.L. Jaffe, in *Pointlike Structures Inside and Outside of Hadrons*, ed. by A. Zichichi, Plenum Press, New York 1981

A.W. Thomas, Adv. in Nucl. Phys. **13** (1984) 1

[3] K.A. Milton, Phys. Rev. **D22** (1980) 1444

K.A. Milton, Phys. Rev. **D25** (1982) 3441

J. Goldstone, R.L. Jaffe, Phys. Rev. Lett. **51** (1983) 1518

P.J. Mulders, Phys. Rev. **D30** (1984) 1073

I. Zahed, U.G. Meissner, A. Wirzba, Phys. Lett. **145B** (1984) 117

[4] G. Ripka, S. Kahana, Phys. Lett. **155B** (1985) 327

G. Ripka, S. Kahana, R. Perry, Phys. Lett. **163B** (1985) 37

[5] R. Friedberg, T.D. Lee, Phys. Rev. **D15** (1977) 1694

[6] I.J.R. Aitchison, C.M. Fraser, Phys. Rev. **D31** (1985) 2605

A. Dhar, R. Shankar, S.R. Wadia, Phys. Rev. **D31** (1985) 3256

D. Ebert, H. Reinhardt, Nucl. Phys. **B271** (1986) 188

[7] C.W. Bernard, Phys. Rev. **D9** (1974) 3312

B.A. Freedman, L.D. Mc Lerran, Phys. Rev. **D16** (1977) 1130

[8] A.G. Williams, R.T. Cahill, Phys. Rev. **D28** (1983) 1966

[9] R.P. Feynman, A.R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York 1965

[10] F.A. Berezin, *The Method of Second Quantization*, Academic Press, New York & London 1966

[11] J. Schwinger, Phys. Rev. **82** (1951) 664

[12] M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions*, Dover Publications, New York 1970

[13] L. Bayer, H. Forkel, W. Weise, Z. Phys. **A324** (1986) 365