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NUCLEONS AND NUCLEAR MATTER IN A GENERALIZED LEE FRIEDBERG MODEL+)

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ABSTRACT

Some deficiencies showing up on the mean field level of the Lee Friedberg soliton model can be cured by allowing for a running coupling constant $G(\sigma)$ which guarantees absolute quark confinement. In the first part of the talk we introduce this modified Lee Friedberg model and show that it reproduces static nucleon observables equally well as the standard version, but with better implementation of confinement and asymptotic freedom properties.
In the second part we apply our model to hot and dense nuclear matter, using a Wigner Seitz cell description and a relativistic Thomas Fermi approximation. We encounter a 1st order phase transition from the nuclear to a homogeneous phase at a critical temperature of $\sim$ 150 MeV, and critical densities that depend sensitively on parameters and boundary conditions. Finally we study the influence of the nuclear medium on some nucleon observables.

1. Standard and Modified Lee Friedberg Model

The non-topological soliton bag model of Lee and Friedberg (LF model) [1] is defined in its simplest approximation (without gauge fields, massless quarks) by a Lagrangian density

$$\mathcal{L}_{LF} = \overline{\Psi}\left(i\gamma_\mu\partial^\mu - G\sigma\right)\Psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma). \tag{1.1}$$

As usual, Ψ denotes a four component Dirac spinor, σ a phenomenological scalar field assumed to simulate non-perturbative gluon effects, $U(\sigma)$ a selfinteraction potential with a local minimum at $\sigma = 0$ and a global minimum at some positive vacuum expectation value $\sigma = \sigma_v$. Unlike the earlier bag models the LF model contains a dynamical mechanism for the fermions to generate their bag selfconsistently as a limited region of "perturbative" vacuum embedded in the non-trivial, non-perturbative, physical vacuum. Despite lacking chiral invariance, the model enjoys popularity for being intuitive and easy to handle.

Mean field solutions of the LF model have been thoroughly investigated by many people, including ourselves. Although they allow a decent description of nucleon properties, they are always afflicted with two major shortcomings [2]:

(i) Either the resulting quark densities are unphysically surface peaked, or the confining scalar potential well $G\sigma_v$ is very low, allowing for free quarks at energies of the order of the nucleon mass (see fig. 1).

(ii) The "effective fermion mass" $G\sigma(0)$ in the bag center takes on pronounced negative values (–10 to –50 % of the vacuum value), which is not in line with the original bag model idea of quarks being free and (almost) massless in the deep interior of hadrons.

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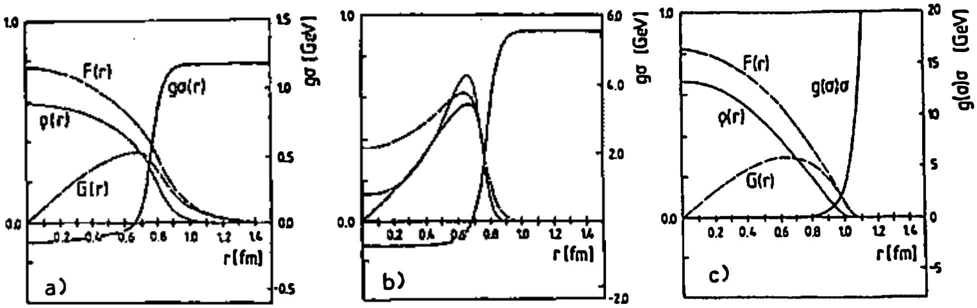

Fig.1. Typical solutions of the LF model (a,b) and the MLF model (c). The plots show the upper and lower components $F(r)$ and $G(r)$ of the quark wave functions and the charge density $\varrho(r) = F^2(r) + G^2(r)$ (in arb. units), together with the "effective mass" $G\sigma(r)$ (in GeV; note the different scales!).

These difficulties are resolved in a modified version of the model, as suggested by us in a previous paper [2]. The Lagrangian density of this Modified Lee Friedberg (MLF) model is (in simplest approximation) given by:

$$\mathcal{L}_{MLF} = \overline{\Psi}\big(i\gamma_\mu\partial^\mu - G(\sigma)\sigma\big)\Psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma). \tag{1.2}$$

In contrast to the original LF model, the quark σ coupling strength is now itself a (nonlinear) function of σ, interpolating between weak coupling in the perturbative vacuum sector ($\sigma \approx 0$) and strong coupling in the non-perturbative vacuum ($\sigma = \sigma_v$).

In the MLF model we write

$$G(\sigma) = \frac{G_0}{\sqrt{\kappa(\sigma)}}, \tag{1.3}$$

assuming that the coupling constant G suffers an "anti-screening" due to vacuum polarization effects, which are describable in an effective theory by a colour dielectric function $\kappa(\sigma)$. In (1.3) G_0 is supposed to be a small number characteristic of the perturbative limit. $\kappa(\sigma)$ is parametrized such as to vary monotonically between[1)]

$$\kappa(\sigma) = \begin{cases} 1, & \text{for } \sigma = 0, \\ 0, & \text{for } \sigma = \sigma_v. \end{cases} \tag{1.4}$$

As a nice feature of this model the detailed form of the parametrization of $\kappa(\sigma)$ turns out to be almost irrelevant for the results. In fact, if only (1.4) is satisfied, the fields always end up in the same bag-like σ configuration as depicted in fig. 1c, implying small effective quark masses $|G(\sigma)\sigma| \leq 10$ MeV in the bag interior and absolute quark confinement ($G(\sigma)\sigma \to \infty$ for $\sigma \to \sigma_v$. In particular, surface peaked quark distributions are no longer possible.

Fixing the (recoil corrected) rms radius $\langle r^2\rangle_c^{1/2}$ at 0.55 fm and specifying $U(\sigma)$ by a sensible choice of parameters ("bag constant" $B := U(0) - U(\sigma_v) \approx 200$ MeV, "glue ball mass" $M_{GB} := \sqrt{U''(\sigma_v)} \approx$ 1–2 GeV), one obtains a reasonable description of nucleon quark core properties with a cm corrected energy of 1500 MeV, a g_A between 1.10 and 1.15, and roughly 60 % of the empirical proton magnetic moment.[2)]

[1)]Defining $\kappa(\sigma) = (1 - \frac{\sigma}{\sigma_v})^4$, our model closely resembles the "chromodielectric model" [3].

[2)]Remember that the pion cloud, which is missing in our model, is expected to decrease the energy by several hundreds of MeV, and to carry roughly one half of the total magnetic moment.

2. Nuclear Matter in the Modified Lee Friedberg Model

In applying the model to nucleons in a hot and dense nuclear medium, we follow the method developed by H. Reinhardt and H. Schulz [4,5]. A thermodynamic potential Ω is defined via the grand canonical partition function Z_β:

$$e^{-\beta\Omega} := Z_\beta := \mathrm{Tr}\, e^{-\beta(H-\mu N)}. \tag{2.1}$$

Here H is the Dirac Hamiltonian of the MLF model, N the particle number operator, μ the quark chemical potential and $\beta = T^{-1}$ the inverse temperature. The trace is taken only over the fermion fields, while σ is regarded as classical.

Integrating out the fermion fields, one arrives at an effective meson theory with a thermodynamic potential of the form

$$\Omega = \Omega(\sigma, \beta, \mu; \epsilon_\alpha), \tag{2.2}$$

which in fact still contains the complete energy spectrum $\{\epsilon_\alpha\}$ of H.

To get a manageable expression for Ω, we proceed in the spirit of a relativistic *Thomas Fermi* approximation (TFA), replacing

$$\epsilon_\alpha \longrightarrow E_{\vec{k}} = \pm\sqrt{m^2(\sigma(x_0)) + \vec{k}^2} \qquad (\text{with } m(\sigma) = g(\sigma)\sigma)$$

and then averaging out the artificial x_0 coordinate dependence. Finally substituting the sum over $\vec{k}$-states by an integral, and minimizing the resulting expression for Ω with respect to σ, we end up with the (static) equation of motion

$$\nabla^2\sigma - \frac{dU(\sigma)}{d\sigma} = \frac{dm(\sigma)}{d\sigma}\cdot\varrho_s(r) \tag{2.3}$$

which has to be solved together with the baryon number constraint

$$N_B = 1 = \frac{1}{3}\int \varrho_v(r)\, d^3r\,. \tag{2.4}$$

The densities in the above equations are defined as

$$\begin{aligned} \varrho_s &= N_s \int \tfrac{d^3k}{(2\pi)^3}\tfrac{m(\sigma)}{|E_{\vec{k}}|}(n_k + n_{\bar{k}}), && \text{scalar quark density,} \\ \varrho_v &= N_s \int \tfrac{d^3k}{(2\pi)^3}(n_k - n_{\bar{k}}), && \text{vector quark density,} \end{aligned} \tag{2.5}$$

with a degeneracy factor $N_s = 12$ and the finite temperature quark (antiquark) occupation numbers

$$\left.\begin{matrix} n_k \\ n_{\bar{k}} \end{matrix}\right\} = (1 + e^{\beta(E_k \mp \mu)})^{-1}\,. \tag{2.6}$$

A nuclear medium of density $\bar{\rho}$ is simulated by putting the system into a spherical *Wigner Seitz* cell of radius

$$R_{ws} = \left(\frac{4\pi}{3}\bar{\rho}\right)^{-\frac{1}{3}}\,. \tag{2.7}$$

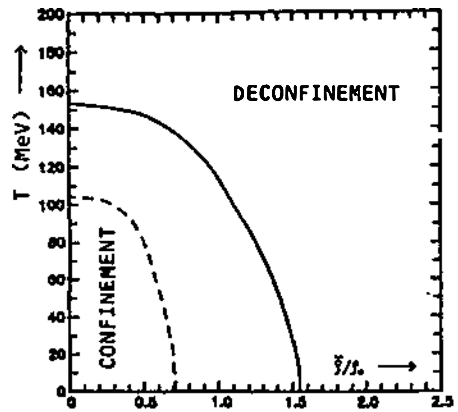

Fig.2. Phase diagrams as calculated in the generalized Lee Friedberg model. The density is given in units of the empirical nuclear saturation density $\rho_0 = 0.16\ \mathrm{fm}^{-3}$. The straight line refers to the MLF model, with an isolated nucleon rms radius $\langle r^2\rangle_c^{1/2} = 0.55$ fm, the dashed line exhibits our results in the LF model with parameters as in [5], thence $\langle r^2\rangle_c^{1/2} = 0.83$ fm.

This picture provides us the boundary conditions needed:

$$\sigma'(R_{ws}) = \sigma'(0) = 0. \tag{2.8}$$

Studying numerically the solutions of eqs. (2.3,4) for various values of R_{ws} and T, we find the following behaviour:

(i) For large R_{ws} and small T we get two types of bag-like solutions: a stable one, transforming asymptotically into the isolated nucleon solution with increasing WS radius, and an unstable one that maximizes Ω and has finite energy only for finite R_{ws}.

(ii) At some (temperature dependent) critical radius $R_{ws}^{crit}(T)$ the two types of solutions become identical, and they abruptly cease to exist, if R_{ws} is further decreased.

(iii) For $R_{ws} < R_{ws}^{crit}(T)$ we find no bag-like solutions, whatsoever. Only the trivial solution, $\sigma(r) \equiv 0$, exists for arbitrary R_{ws} and T.

The break-down of bag-like solutions is interpreted as a phase transition from the nuclear ("bubble") phase to the deconfined (homogeneous) phase.[3] In disagreement with Reinhardt et al. [5], who reported a smooth (2nd order) phase transition, the transition happens discontinuously (1st order), according to our results.[4] Plotted in the T–$\bar{\rho}$ plane, the critical values give the phase diagrams of fig. 2. The curves change only insignificantly with parameters, once the rms charge radius (for the free nucleon) is fixed. With $\langle r^2\rangle_c^{1/2} = 0.55$ fm we get

$$\begin{aligned} T^{crit}(\bar{\rho} \to 0) &\approx 150\text{–}160\ \mathrm{MeV}, \\ \bar{\rho}^{crit}(T = 0) &\approx 1\text{–}2\ \rho_0, \end{aligned}$$

$\rho_0 \approx 0.16\ \mathrm{fm}^{-3}$ being the empirical nuclear matter density.
With parameters used in [5] ($\langle r^2\rangle_c^{1/2} = 0.83$ fm), the critical density at zero temperature comes out even below ρ_0. Hence, although the model produces nice qualitative results, our approximations seem to be to crude for quantitative predictions – at least the critical densities obtained are obviously much too low!

[3] A very similar break-down of solutions has also been observed by Banerjee et al. in their calculations of the two baryon system in a chiral model (see also their contribution to this volume).
[4] On correspondence with the authors of [5], their solutions at small R_{ws} turned out to be numerical artifacts.

To check, how far the TFA is responsible for this failure, the calculations can be repeated in the $T = 0$ case without TFA, using explicit quark spinors

$$q(r) = \begin{pmatrix} F(r) \\ i\,\vec{\sigma}\cdot\hat{r}\,G(r) \end{pmatrix}. \tag{2.10}$$

For this purpose boundary conditions have to be specified for the upper and lower components of the spinor; we consider two sets of boundary conditions,

$$G(R_{ws}) = 0, \; G(0) = 0, \qquad \text{or alternatively} \qquad F(R_{ws}) = 0, \; G(0) = 0, \tag{2.11}$$

corresponding to the lower and upper limit of the first Brillouin zone (cf. [6]). Unfortunately these two sets yield completely different solutions at small R_{ws}, the break-down occurring around ρ_0 in the first case, yet at much higher densities (6–30 ρ_0) in the latter case. With such large uncertainties we obviously cannot draw quantitative conclusions concerning the critical densities.

Nevertheless we can use the explicit quark wave functions to calculate nucleon observables and at least qualitatively observe them change as a function of nuclear matter density (fig. 3).

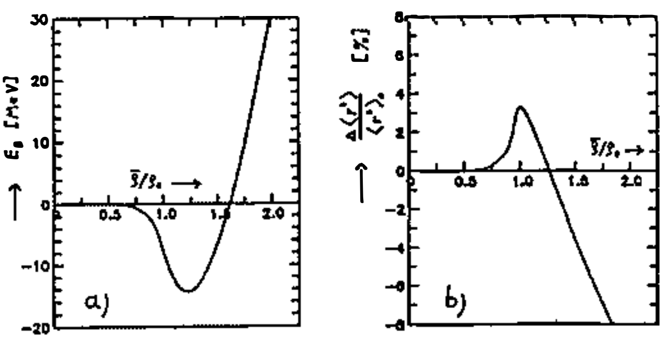

Fig.3. Nucleon properties as a function of nuclear matter density at zero temperature.
(a) binding energy
(b) rms radius: deviation from the isolated nucleon value in %

3. Summary

Summing up, we conclude that by taking into account vacuum polarization effects phenomenologically in the form of a running coupling constant $G(\sigma)$, the Modified Lee Friedberg model successfully describes the quark core of isolated nucleons on the mean field level. It is superior to the original LF model in generating absolute quark confinement and dispensing with highly negative effective quark masses in the hadron interior. As another advantage the results are virtually independent of all parameters, once the rms radius is fixed. Putting the nucleon into a spherical Wigner Seitz cell and introducing finite temperature in a relativistic Thomas Fermi approximation, one can simulate hot and dense nuclear matter. We find that bag-like solutions exist only within a limited low temperature low density region of the T–$\bar{\rho}$ plane; the break-down of solutions occurs along a critical curve in the T–$\bar{\rho}$ diagram which separates the "normal" nuclear phase from the deconfined phase. Its intersections with the T and $\bar{\rho}$ axes define a critical temperature of 150–160 MeV and a critical density that depends strongly on parameters and boundary conditions.

The model allows to study the *qualitative* response of the quark wave functions (and hence of nucleon observables) to the presence of a nuclear medium, but for more reliable quantitative statements the unphysical WS boundary conditions should be replaced by a more realistic picture.

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