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A MODEL OF BAG FORMATION

Manoj K. Banerjee
University of Maryland, College Park, Maryland 20742 (USA)

Abstract

A model of bag formation based on the interaction of quarks with the non-perturbative vacuum of QCD is described. This talk is based on work done by Wojchiech Broniowski, Thomas Cohen and myself.

The central idea was already proposed by Nielsen and Patkos[2] and models of bag formation, similar to the one I will describe, were developed by Pirner and his collaborators[3], by Thomas and his collaborators[4] and by Ian Duck[5]. These models are known as chromodielectric models. Despite the existence of so many similar models, I hope that you will find that our phenomenological approach contains some new ideas.

It is generally believed that QCD is the correct theory of strong interaction. It is also believed that colored objects do not exist in nature. Therefore, by implication, one believes that QCD predicts that (1) quarks are absolutely confined and (2) colored states cannot exist. Absolute confinement means that no amount of energy can separate a quark or an antiquark from a hadron. A mathematical implication is that the momentum space wave function of an absolutely confined quark is an entire function of $|\vec{q}|$.

Absolute confinement is a long range effect. So, it is reasonable to suggest that it should be describable in a mean field theory. We all know that this can be done if we could invent a rising potential for the quarks. Unfortunately, one cannot generate a rising potential by exchanging bosons massive or massless. What then is a possible alternative? Perhaps we can try to make the kinetic energy term vanish. Then, relative to the kinetic energy the potential would seem to rise. Let me illustrate the point with a simple example from nonrelativistic physics. Consider the Schrödinger equation with $E < 0$.

$$[-\nabla^2 + 2M(V(r)-E)]\psi = 0, \quad E < 0, \qquad (1)$$

where $V(r)$ is an attractive, finite ranged potential. By making $V(r)$ arbitrarily deep we can make the binding energy, $-E$, as large as we wish, but never achieve absolute confinement. However, if we can modify the equation by multiplying the kinetic energy term with a function $f(r)$,

$$[-\frac{1}{2}\{f(r)\nabla^2 + \nabla^2 f(r)\} + 2M(V(r)-E)]\psi = 0,$$

and require f(r) to be non-negative and to go to zero as $r \to \infty$, we can achieve absolute confinement. One can check that the function $\phi(\vec{r}) = [f(r)]^{1/2}\psi(\vec{r})$ satisfies the equation

$$\left[-\nabla^2 + \frac{1}{2}\left(\frac{1}{f(r)}\frac{df(r)}{dr}\right)^2 + \frac{2M(V(r)-E)}{f(r)}\right]\phi(\vec{r}) = 0, \qquad (3)$$

and that the asymptotic behaviour of $\phi(\vec{r})$ is of the form $\exp\{-\int^r dr'[\frac{2M(V(r')-E)}{f(r')}]^{1/2}\}$, i.e., it goes to zero faster than any exponential, $\exp(-\lambda r)$. One can then verify that the fourier transform of $\psi(\vec{r}) = \phi(\vec{r})[f(r)]^{-1/2}$ is an entire function of $|\vec{q}|$. Of course, all this is true only if 2M(V(r)-E) goes to a positive constant asymptotically. In a relativistic field theory the analog of this plan would be to (a) generate a factor multiplying the free quark term, $\bar{\psi}i\partial^\mu\gamma_\mu\psi$, in the lagrangian, and (b) ensure that the quark experiences interaction everywhere. Can we do this? Amazingly, the answer is yes--with the help of the 0^{++} glueball.

The 0^{++} glueball is a scalar under chiral SU(2)×SU(2) transformations in sharp contrast to the σ meson which belongs to the (1/2,1/2) representation. There are only two permissible forms for coupling Γ, the 0^{++} glueball field, linearly to the quarks which are chiral invariant, viz,

$$\text{(a)}\quad g_\Gamma\, \Gamma\, \frac{i}{2}\,\bar{\psi}\,\partial\,\psi \quad\text{and}\quad \text{(b)}\quad g'_\Gamma\, \bar{\psi}\,\gamma_\mu\,\psi\,\partial^\mu\,\Gamma.$$

The second form will be recognized as a gauge transformation of the ω coupling and can be removed from our considerations. Retaining only the form (a) we may write the phenomenological mean field lagrangian as

$$\mathcal{L} = (1+g_\Gamma\Gamma)\,\frac{i}{2}\,\bar{\psi}\,\partial\,\psi - g_\pi\bar{\psi}\,[\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}]\psi + \mathcal{L}_{meson} + \mathcal{L}_\Gamma. \qquad (4)$$

Thus we have succeeded in acquiring, at least, a factor multiplying the free quark term in the lagrangian. The next step is pure conjecture. We suggest that the nonperturbative QCD vacuum is such that

$$1 + g_\Gamma\langle\Gamma\rangle_v = 0, \qquad (5)$$

where $\langle\Gamma\rangle_v$ is the vacuum expectation value of Γ. The field variable Γ is neccessarily related to the quantity $\frac{\alpha}{\pi} G^a_{\mu\nu}G^{\mu\nu,a}$, the two having identical transformation properties. The latter is believed to have vacuum expectation value[6], $\langle\frac{\alpha}{\pi} G^a_{\mu\nu}G^{\mu\nu,a}\rangle_v = (330\ \text{MeV})^4$. Thus it is reasonable to suggest that $1 + g_\Gamma\langle\Gamma\rangle_v \neq 1$. But Eq. (5) is a very special requirement. It demands an unusual relation between the vacuum expectation value of a field and its coupling strength to the quark. No similar suggestion is ever made for any meson field But we should remember that the 0^{++} glueball is indeed a unique and curious object. It is not like any other meson.

Next, we note that $\langle\sigma\rangle_V \neq 0$, specifically, $\langle\sigma\rangle_V = -F_\pi = -93$ MeV. This is referred to as the chiral symmetry breaking of the vacuum. Therefore the quark always experiences an interaction giving rise to its dynamical mass-- $g_\pi F_\pi$. Now we are in a position to see how a bag may be formed. A collection of quarks can act as a source for the glueball field and thus can polarize the vacuum to alter the value of Γ away from $\langle\Gamma\rangle_V$ in its immediate neighborhood. In this region the quark kinetic energy term is not zero and the quarks can exist. If a quark tends to stray away from this region $1 + g_\Gamma\Gamma$ will tend to its normal vacuum expectation value, viz, 0, while the quark continues to interact with the σ field which goes to a constant value. This interaction then appears to increase in strength continuously as the quark moves farther away. The result is absolute confinement of the quarks.

However, the scenario described above does not prevent occurence of single quarks or groups of quarks in color non-singlet state in nature. One simple way of preventing these undesirable states is to suggest that the color dielectric constant of the nonperturbative vacuum is zero--a suggestion made in the early days of QCD by several authors[7]. If there is a color charge or any color 2^ℓ multipole moment somewhere it will produce displacement fields $\vec{D}^a$ which will fall off, away from the source, with the power law, $r^{-\ell-1}$. Now if the color dielectric constant, $\epsilon(r)$, goes to zero faster than any power law the electric field energy, $\int d^3r\vec{D}^a\cdot\vec{D}^a/\epsilon(r)$, will become infinite and the state cannot exist.

Nielsen and Patkós[2] suggested a very interesting set of collective variables which help produce both bag formation and vanishing of the chromodielectric constant. They introduced their idea for the color SU(2) group. One hopes that with appropriate modification of the algorithms the basic idea can be extended to the color SU(3) group. Briefly their idea was to consider the gauge invariant quantity

$$U(x,\delta x) = \text{average}\{P \exp [\int_x^{x+\delta x} dy^\mu A_\mu(y)]\}, \tag{6}$$

where P stands for path ordering and the average is over all possible paths in the four-dimensional box of sides L_0. Thus an averaging over a small distance scale is performed. Expansion in a power series of δx gives

$$U(x,\delta x) = \mathcal{K}(x) + i\delta x^\mu B_\mu(x) \cdots . \tag{7}$$

They chose K(x), the color singlet part of $\mathcal{K}(x)$, and $B_\mu(x)$ as their new collective variables. Integration over the gluon fields in favor of these collective variables converts the term $\bar{\psi}(x)(i\not\partial - A(x))\psi(x)$ of the QCD lagrangian into the form $\bar{\psi}(x)(K(x)i\partial\!\!\!/ - B(x))\psi(x)$. The reader will recognize that we may equate K(x) to our $1 + g_\Gamma\Gamma(x)$. The color octet field B(x) was termed as the coarse-grained gluon field by the authors. The vector fields, $B_\mu(x)/K(x)$ were taken to be nonabelian gauge fields, which they were in the SU(2) example they treated. Then it follows automatically that the chromodielectric constant, $\epsilon(x) = K(x)^4$. The authors conjectured that K(x) tended to zero at large distances, just as we do for $(1 + g_\Gamma\Gamma)$, and since it must must go to zero as $e^{-m_\Gamma r}/r$ where m_Γ is the lowest glueball mass, $\epsilon(x)$ goes to zero faster than any power law. Thus the requirements for the absence of colored states in nature are met.

Let us now return to the lagrangian (4). Upon use of the standard procedure we find that the variable canonically conjugate to ψ is not $i\psi^\dagger$, but $(1 + g_\Gamma\Gamma)i\psi^\dagger$. The quark sector of every Noether current also acquires the factor $(1 + g_\Gamma\Gamma)$. Thus the fermion current is $j_\mu = (1 + g_\Gamma\Gamma)\bar{\psi}\gamma_\mu\psi$. It is very convenient to introduce a new fermion field

$$\xi = (1 + g_\Gamma\Gamma)^{1/2}\psi, \tag{8}$$

whereupon the lagrangian becomes

$$\mathcal{L} = \frac{i}{2}\bar{\xi}\partial\xi - \frac{g_\pi\bar{\xi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\xi}{(1 + g_\Gamma\Gamma)} + \mathcal{L}_{meson} + \mathcal{L}_\Gamma. \tag{9}$$

This lagrangian is the basis of our chiral invariant model of bag formation. Wojcieck Broniowski will describe the actual calculations we have done with this model. I will end with a summary. We have proposed a model which rests on three principal ingredients. First, it requires the QCD vacuum to have a nonzero vacuum expectation value for the glueball field. Only in a nonabelian gauge theory where there can be attractive interaction among the vector bosons can we expect gluon condensate to occur. And, of course, the occurence of gluon condensate is essential for a nonzero $\langle\Gamma\rangle_v$. Second, we require Eq. (5) to be valid. This is the trickiest part of the model. Our scenario of bag formation demands it. But we cannot offer any proof nor any independent argument why it should be true. Finally, breaking of the chiral symmetry of the vacuum is central to our model. Without this we will have to depend on the current quark mass for confinement. It seems more natural to suggest that there is confinement even in the chiral limit.

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References

1. M. K. Banerjee, Proc. of Workshop on Nuclear Chromodynamics - Quarks and Gluons in Particles and Nuceli, Santa Barbara, 1985, ed. S. Brodsky and E. Moniz (World Scientific, Singapore, 1985).
W. Broniowski, M. K. Banerjee and T. D. Cohen, U. of Maryland preprint ORO 5126-273 (1985) and ORO 5126-298 (1986).
2. H. B. Nielsen and A. Patkós, Nucl. Phys. B195, 137 (1982).
3. G. Chanfrey, O. Nachtmann and H. J. Pirner, Phys. Lett. 147B, 249 (1984).
A. Schuh and H. J. Pirner, Phys. Lett. B173, 19 (1986).
4. A. G. Williams and A. W. Thomas, Phys. Lett 154B, 320 (1985).
A. G. Williams and A. W. Thomas, Phys. Rev. C33, 1070 (1986).
5. I. Duck, Phys. Rev. D34, 1493 (1986).
6. M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B147, 385, 448, 519 (1979).
7. J. B. Kogut and L. Susskind, Phys. Rev. D9, 3501 (1974).
P. Hasenfratz and J. Kuti, Phys. Rep. 40, 75 (1978).