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NUCLEON IN CONFINING MODELS WITH GLUEBALLS

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ABSTRACT

Solutions to non-chiral and chiral color dielectric models are discussed. The coupling of glueballs produces absolute quark confinement and generates selfconsistently a bag.

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We have heard in Manoj Banerjee's talk a justification of color dielectric models based on the concept of glueball exchange between quarks [1]. The idea of color dielectrics has also been described by Ludwig Bayer and Mitja Rosina. Here I will discuss in some greater detail the mean-field solutions to the color dielectric models of ref. [2,3].

We will consider two cases: one is a non-chiral toy model with quark and glueball fields only, the other is a chiral model with quarks, glueball and σ and π meson fields. In the first case the lagrangian is

$$\mathcal{L} = \bar{\psi}\, i\not{\partial}\, \psi - \frac{\alpha\bar{\psi}\psi}{\chi} + \frac{1}{2}\,(\partial_\mu\chi)^2 - U(\chi) \quad , \qquad (1)$$

where χ is the glueball field related to Γ of ref. [1] through $\chi = (1/g_\Gamma + \Gamma)$, α is a coupling constant of dimension $(\mathrm{mass})^2$ and U is a glueball potential parameterized in the following manner:

$$U(\chi) = \frac{1}{2}\, M^2\chi^2\left[1 + [8\eta^4/\gamma^2-2](\chi/\gamma M) + [1-6\eta^4/\gamma^2](\chi/\gamma M)^2\right] \quad . \qquad (2)$$

The global minimum of U is at $\chi = 0$, with the curvature given by the glueball mass M. The other minimum of U is located at $\chi = \gamma M$, with a height of $U(\gamma M) = M^4\eta^4$. The interaction term in

(1) breaks explicitly the chiral symmetry, hence the model cannot be treated realistically. However, the toy model illustrates cleanly the role of the glueball in the system and we analyze it for this reason.

To restore the chiral symmetry one introduces additionally the σ and π meson fields. The lagrangian has the following form:

$$\mathcal{L} = \bar{\psi}\, i\not{\partial}\, \psi - \frac{g\bar{\psi}(\sigma + i\gamma_5 \vec{\tau}\cdot\vec{\pi})\psi}{\chi} + \frac{1}{2}(\partial_\mu \chi)^2 - U(\chi) + \mathcal{L}(\sigma,\vec{\pi}) \quad , \qquad (3)$$

where g is a coupling constant of dimension (mass) and $\mathcal{L}(\sigma,\vec{\pi})$ is the linear σ-model lagrangian

$$\mathcal{L}(\sigma,\vec{\pi}) = \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - \lambda^2/4\, (\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - c\sigma \quad . \qquad (4)$$

The constants are defined as

$$\lambda^2 = (m_\sigma^2 - m_\pi^2)/2F_\pi^2 \ , \quad \nu^2 = F_\pi^2 - m_\pi^2/\lambda^2 \ , \quad c = F_\pi m_\pi^2 \ , \qquad (5)$$

where m_σ = 1200 MeV, m_π = 139 MeV and F_π = 93 MeV. Lagrangian (4) leads to spontaneous chiral symmetry breaking with the σ acquiring a non-zero vacuum expectation value $\langle\sigma\rangle_{vac} = -F_\pi$. The term in (4) proportional to σ breaks explicitly the chiral symmetry in order to give the pion its physical mass.

To construct a baryon we put three valence quarks, coupled to a color singlet, in a single space-spin-flavor state described by a spinor of the form

$$q(\vec{r}) = \begin{pmatrix} G(r) \\ i\vec{\sigma}\cdot\hat{r}\, F(r) \end{pmatrix} |sf\rangle \quad , \qquad (6)$$

Radial functions G and F are the quark upper and lower components and $|sf\rangle$ is the spin-flavor part of the wave function. For the non-chiral model (1) $|sf\rangle$ does not have to be specified, since the interaction is scalar-isoscalar. For the chiral model (2) it has a familiar hedgehog form

$$|sf\rangle = 2^{-1/2}(|u\downarrow\rangle - |d\uparrow\rangle) \quad . \qquad (7)$$

Correspondingly, the boson fields are described by three radial functions:

$$\chi = \chi(r) \ , \quad \sigma = \sigma(r) \ , \quad \vec{\pi} = \hat{r}\, \pi(r) \ . \qquad (8)$$

The Euler-Lagrange equations following from (1) and (3) are solved numerically. The boundary conditions at the origin are

$$F(0) = 0 , \quad \chi(0) = 0 , \quad \frac{d\sigma}{dr}(0) = 0 , \quad \pi(0) = 0 . \qquad (9)$$

At $r \longrightarrow \infty$ they follow from the forms of the asymptotic solutions, which for boson fields have standard Yukawa tails, and for the quark one finds

$$G \sim F \sim \exp\left[-\alpha/A \int^{r} dr' r' \exp(Mr')\right] \sim \exp\left[-\alpha/A \; r^2 \exp(Mr)\right] , \qquad (10)$$

where A is some constant. Thus G and F drop to zero extremely rapidly, as a double exponent. This behavior reflects the fact that there are no plane wave solutions in the system. An appropriate boundary condition is $G = F$ as $r \longrightarrow \infty$.

Our numerical solutions for the nonchiral and chiral models are shown in fig. (1) and (2), respectively. Authors of refs. [4-6] have also found solutions to models similar to ours. We have chosen the following values of parameters in (1-3):

$M = 1400$ MeV, $\alpha = (39 \text{ MeV})^2$, $\gamma = .022$, $\eta = .040$ fig. (1)
$M = 1400$ MeV, $g = 112$ MeV, $\gamma = .022$, $\eta = 0$ fig. (2)

Near the origin the χ field remains close to the local minimum of the glueball potential (2). Around $r = 1.5$ fm it drops to the vacuum value of zero. At this point the effective quark mass, which is proportional to $1/\chi$, tends to infinity, and as a result the quarks are confined in a selfconsistently generated bag.

The meson fields in the chiral solution do not depart significantly from their vacuum values. The σ field remains close to $-F_\pi$ everywhere. The winding number of the solution, defined as $\tan^{-1}[\pi(r)/\sigma(r)]\Big|_{r=0}^{r=\infty}$, is equal to zero. This is in contrast to the chiral soliton model [7], where σ flips the sign and the winding number of the solution is equal to one. The solution of fig. (2) reminds of the Cloudy Bag model, where the chiral fields are build perturbatively on the MIT bag.

We have verified stability of the solutions to the nonchiral model with respect to monopole vibrations using a generalized RPA method [8]. Also, the Roper resonance in color dielectric models was examined [8].

The energy decomposition and some properties of the solutions are presented in Table I. We note that the dimensionless product of soliton energy and quark rms radius is 6.7 for the nonchiral model and 6.6 for the chiral model. Hence the solutions are too heavy (or too large). To cure this problem center of mass corrections have to be incorporated, and/or other degrees of freedom have to be added (e.g. vector mesons). CM corrections are straightforward to evaluate for the valence quarks. The inclusion of sea quark and glueball contributions is a serious problem, since the model is nonrenormalizable. Before dealing with the problem of CM corrections one has to learn how to treat quantum corrections in such models in general. One has to bear in mind, however, that the short distance behavior has been "integrated out" in the approach of Nielsen and Patkòs and thus bulk of quantum effects may already be present in our mean-field treatment of the model.

We note a very much improved value of the pion-nucleon σ commutator in the case of the chiral model. This quantity is an important test of phenomenological models. The experimental value is 25 MeV

Summarizing, we have shown that the scenario in which the 0^{++} glueball is responsible for confinement is possible. The models discussed lead to a selfconsistent quark bag formation and absolute confinement.

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Fig. 1

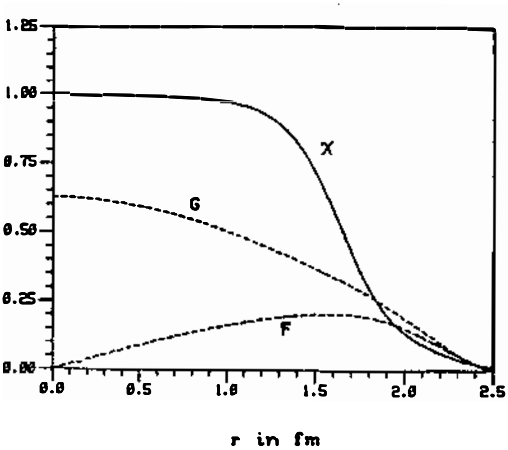

Fig. 2

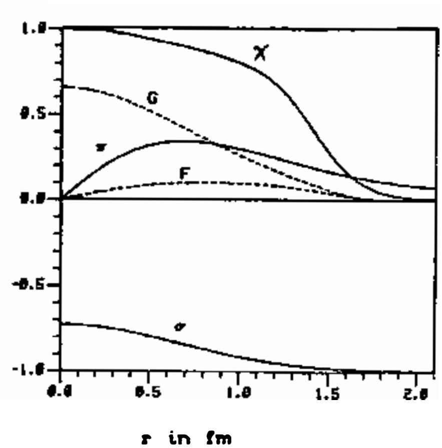

Fig. 1. Solution to the nonchiral model. G and F in arbitrary units, χ in units of its central value of 32 MeV.

Fig.2. Solution to the chiral model. G and F in arbitrary units, χ in units of its central value of 42 MeV, σ and π in units of F_π.

Table I

Quantity	Nonchiral Model	Chiral Model
glueball kinetic energy	80 MeV	92 MeV
glueball potential energy	151 MeV	101 MeV
meson kinetic energy	-	189 MeV
meson potential energy	-	48 MeV
quark eigenvalue	224 MeV	355 MeV
soliton mass	903 MeV	1496 MeV
quark rms radius	1.46 fm	0.88 fm
$\int F^2 / \int G^2$	0.27	0.11
π-N sigma commutator	141 MeV	26 MeV