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FERMI GAS OF QUARKS IN THE CHROMODIELECTRIC MODEL

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Abstract. The mechanism how the chromodielectric field binds quarks is illustrated in a simplified model. Although the system is described as a Fermi gas of quarks, the dependence of binding energy and equilibrium density on model parameters seems relevant also for nuclear matter.

1. Introduction

In Yang-Mills equations of chromodynamics the nonlinear terms can be viewed as additional sources of the chromoelectric and chromomagnetic field. This is somewhat similar to the electrodynamics of polarizable media where the displaced ions and electrons act as additional sources and their effect can be described by additional fields $(E - D) = (\frac{1}{\varepsilon}-1)D$ and $(B-H) = (\mu-1)H$. It is tempting to construct a model of chromodynamics using this analogy. There are,however, two basic differences. (i) The vacuum itself acts as a dielectric. (ii) The dielectric constant is $\varepsilon \leq 1$ since the vacuum exerts antiscreening rather than screening (for a nice pedagogical picture at a classical level see ref. [1]). In most versions of the chromodielectric model one assumes $\mu = 1/\varepsilon$ so that the velocity of light is c=1 for all values of ε (not only in the physical vacuum). It is usually assumed that $\varepsilon(\mathbf{r})$ is a dynamical field; this additional degree of freedom represents the nonperturbative effects of gluons and should not double-count the effects of the remaining "perturbative gluons". The chromodielectric model has not yet been truely derived from QCD as an approximation. The suggestive ideas of Nielsen and Patkos [2] and the calculations of Pirner and collaborators [3] give a good qualitative start. At present, it is still the best for a practitioner of the model to make an educated guess of the Lagrangian, for example

$$\mathcal{L} = -\frac{1}{4}\ell^4 B_a^{\mu\nu} B^a_{\mu\nu} + \bar{\Psi}\{\ell\, \gamma^\mu (i\partial_\mu - gB_\mu \hat{C}) - m\}\Psi + \frac{1}{2}w^2(\partial_m \ell)^2 - U(\ell) \qquad (1)$$

It is obtained from the QCD Lagrangian by the gauge invariant replacement $A^a_\mu \to \ell B^a_\mu$, $\partial \to \ell\,\partial$; by adding the terms for thr ℓ field; and by reducing in importance (or dropping completely) the cubic and quartic terms in B_μ. The factor ℓ^4 plays the role of dielectric"constant". The parameter w appears to make ℓ dimensionless. For simplicity we choose $U = M^2 w\, \ell^2 (2-\ell)^2/8$.

2. The mechanism of confinement

The mechanism of absolute confinement has been discussed very clearly by M.K.Banerjee in these Proceedings [4]. I shall repeat the main argument. It is convenient to transform the Lagrangian (eq.1) so that the new quark fields ψ, $\overline{\psi}$ become canonically conjugate to each other: $\psi = \sqrt{\chi}\,\Psi$, $\overline{\psi} = \overline{\Psi}\sqrt{\chi}$.

$$\mathcal{L} = -\frac{1}{4}\ell^4 B_a^{\mu\nu} B_{\mu\nu}^a + \overline{\psi}\left\{\gamma^\mu(i\partial_\mu - gB_\mu \hat{C}) - m/\ell\right\}\psi + \frac{1}{2}w^2(\partial_\mu \ell)^2 - U(\ell) \qquad (2)$$

There are three conditions for absolute confinement which are satisfied in the present model:

(i)There is a term in the Lagrangian,$U(\ell)$, which allows the ℓ field to be noticably different from the vacuum value $\ell=0$ (where U is minimal) only in a finite volume - the "bag".

(ii) There is a term, m/ℓ, which would cost infinite energy if quarks went too far outside the "bag" into the region with $\ell\to 0$. This still allows the bag to split with a single quark sitting in its private bag.

(iii) There is a term, $D^2/\mathcal{E}$ (with $\mathcal{E}=\ell^4$, $D^a=\mathcal{E}E^a$) , requiring infinite amount of energy if the field lines due to the Poisson equation should cross regions with $\ell\to 0$ in order to connect an isolated quark far away.

3. How the chromodielectric field binds quarks

The Fermi gas of quarks is in a way simpler than three quarks in the nucleon and we shall describe this system to point out some interesting features of the binding mechanism. Since the presentation is primarily pedagogical, we shall simplify the derivation considerably.

Using the Lagrangian (2) the energy of N quarks can be written as

$$NE = \frac{1}{2} M^2 w^2 \ell^2 V + \overline{\sqrt{k^2+m^2/\ell^2}}\;\rho V = \min \qquad (3)$$

In our simplification we neglected the interaction of quarks with the gluon field since detailed calculations show that the confining ℓ field contributes to the binding more than the local B_μ fields. We also neglected cubic and quartic terms in ℓ since it turns out that ℓ remains close to zero. In the variational calculation we restrict the field $\ell(\mathbf{r})$ to choose a constant value ℓ. Then the uncorrelated Fermi gas is indeed the variational solution for quarks.

As input parameters we need the mass M of the quantum of the ℓ field which we tentatively identify with the lowest 0^{++} glueball; the current quark mass m whose model value may be different from the high-energy value; and

the parameter w related to the "bag constant" $B = U(1) - U(0) = M^2w^2/8$.
if we redefine the chromodielectric field as $\ell = mh$ we see that the result depends only on the combination $z^3 = Mmw$. The energy per quark reads

$$E = \frac{1}{2} z^6 h^2/\rho + \sqrt{k^2 + h^{-2}} = \min \tag{4}$$

The variation is performed with respect to ρ and h. As output we get the equilibrium quark density ρ and energy E.

In order to have a simple pedagogical presentation we use the approximation

$$\sqrt{k^2 + h^{-2}} \approx \frac{1}{\sqrt{2}} (k + h^{-1}) \tag{5}$$

In fact, the right hand side is a lower bound and it is quite close if k and h^{-1} differ within a factor of two; most contribution comes from such interval.

We express $\rho = 12.(2\pi)^{-3}.4\pi k_F^3/3$, $\overline{|k|} = \frac{3}{4}k_F = \frac{3}{4}(\pi^2\rho/2)^{1/3}$, and we get

$$E = \frac{1}{2} z^6 h^2\rho^{-1} + 2^{-17/6}.3.\pi^{2/3} \rho^{1/3} + 2^{-1/2} h^{-1} = \min \tag{6}$$

One can see that there is a rivalry between the price for the h field ($\propto h^2$) and the expense $\propto h^{-1}$ if the h field is too small and enhances the "effective quark mass" m/ℓ. Regarding the quarks, the compromise is between the kinetic energy ($\propto \rho^{1/3}$) and the volume energy needed to accomodate quarks ($\propto \rho^{-1}$). The virial theorem gives then the ratios between the three terms in energy (eq.6) as 1:3:2 . The solution is easily obtained:

$$h = 2^{7/12}\pi^{-1/3}z^{-1} = 1.02\ z^{-1}$$

$$\rho = 2^{9/4}\pi^{-1} \quad z^3 = 1.51\ z^3$$

$$3E = 2^{-13/12}.3^2.\pi^{1/3} \quad z = 6.22\ z$$

This results are close to those of a more detailed numerical calculation which does not use the approximation (5):

$$E = \frac{1}{2} z^6h^2\rho^{-1} + 2^{-7/3}.3\pi^{2/3}\rho^{1/3} \left\{(1+q^2/2)(1+q^2)^{1/2}-(q^2/2)\mathrm{Arsh}\ q^{-1}\right\} = \min \tag{7}$$

where $q = (hk_F)^{-1}$. One obtains

$$h = 0.99\ z^{-1},\quad \rho = 1.18\ z^3,\quad 3E = 6.32\ z.$$

The product $3E\ (\rho/3)^{-1/3} = 8.63$ is independent of model parameters and is within the range of values 8.3 to 9.0 for real nuclear matter (using the experimental 3E = 0.923 GeV and r_o between 1.1 and 1.2 fm).

One can get a realistic value $3E \sim 1$ GeV with meaningful model parameters, for example M = 1.2GeV, m = 0.020 GeV, w = 0.165GeV ($B^{1/4}$=0.26GeV),
or M = 1.2GeV, m = 0.066GeV , w = 0.050GeV ($B^{1/4}$=0.146GeV).

4. The relevance of the simple model for nuclear matter

I have discussed the basic mechanism which in the simple model binds quarks in quark matter. Further refinements will probably not change drastically the energy per quark and the equilibrium density even if quarks get clustered into "nucleons". The refinements will, however, bring new qualitative features.

If we allow the field $\ell(\mathbf{r})$ to vary with $\mathbf{r}$, ℓ will still remain uniform since it only costs energy to produce modulations in ℓ (note the $(\nabla \ell)^2$ term) and in ρ (additional quark kinetic energy). No clustering appears and only a trivial "phase transition" occurs at zero temperature: for an average density below the equilibrium density ρ_o part of the vessel contains the quark fluid at ρ_o and part of the vessel is empty. (See also Bayer's results in these Proceedings [5], using Wigner-Seitz cells; the results become interesting at nonzero temperature which we have not yet treated).

We "know" that quarks are clustered within nuclei since the traditional picture works well. In order to get clustering, one has to introduce residual chromoelectric and chromomagnetic interactions between quarks which will (i) stimulate quarks to cluster three by three in colour singlet clusters, and (ii) distinguish between nucleon and clusters leeding to nuclear matter in the ground state. Work is in progress [6].

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