FIZIKA 19, (1987), Supplement 2, 47-50
(Proc. Workshop on Mesonic Degrees of Freedom in Hadrons, Bled/Ljubljana, June/July 1987)

ON DIELECTRIC MODELS OF STRONG INTERACTIONS

M. Poljšak
Institute J. Stefan, E. Kardelj University, Ljubljana, Yugoslavia

Abstract: A discussion is given of a derivation from quantum chromodynamics of some long range properties of strong interactions

QCD in the region of validity of perturbative expansion in the coupling constant predicts $^{1)}$ that virtual gluons of the vacuum, behaving as permanent colour-magnetic dipoles $^{2)}$, respond to an applied external field by aligning themselves parallel to the field, thus increasing its magnitude and making the magnetic permeability of the vacuum larger than one and therefore its dielectric constant smaller than one.

The dielectric model of the QCD vacuum $^{3)}$ assumes that this kind of vacuum polarization is a valid picture for the complete theory. The model has had some phenomenological success in providing a rationale for the bag formation in hadrons and the ensuing confinment of quarks and gluons $^{4)}$.

There were several attempts $^{5-7)}$ to justify the model from first principles, i.e. QCD beyond perturbation theory. Here I collect some comments on these efforts.

In broad terms,the strategy of determining which elements of a field theory are important at low energies is the following. A Lagrangian may not be revealing about the long distance characteristics of the theory if cooperative phenomena occur. The long distance cooperative variables may be generated from the original Lagrangian by averaging out some of its short distance degrees of freedom. In this way we hope to be led to a Lagrangian more transparent (or at least less obscure) about the bulk properties of the system than the original one but with the same physical content at low energies. We now apply this strategy in the battle with QCD at long distances.

The essential role of the gauge potential A_μ of QCD is to provide a gauge invariant definition of equivalence of quark colour multiplets at neighbouring points. For two arbitrary points (not necessarily close to each other) such an identification is assumed to be still possible if we choose a definite path C from x to y. Such a parallel transport of matter multiplets may be carried out by

$$U(C) = P \exp ig \int_C A_\mu(z).dz^\mu \qquad (1)$$

Here P is the path ordering operator and g is the coupling constant. Parallel transports along two different paths joining x to y in general differ by multiplication with an element of the group. It is only the long-distance parallel transports that are manifestly important to the interactions between widely separated degrees of freedom. But since a long-distance parallel transport does not depend only on the direction from the point x to the point y but also on the path between them and since no particular path can be regarded as being preferred (in the quantum world) it has been suggested 6) that parallel transports along different paths should be averaged. The question now is which paths are to be included in the average. Should non-differentiable and self-intersecting curves be allowed in addition to the smooth ones?

To guarantee stability of the vacuum in the effective theory Nielsen and Patkos make the theory local, i.e. they define the coarse-grained gauge potential at x by

$$B_\mu(x).dx^\mu = \lim_{y \to x + dx} \frac{\partial}{\partial y^\mu} \langle U(C_{x \to y}) \rangle_C \cdot dx^\mu \qquad (2)$$

In order for this limit to exist and be non-trivial, i.e. not identical to A_μ of the original theory , one has to include in the average $\langle\ \rangle$ those curves C which are smooth at x + dx and at least non-differentiable but possibly also self-intersecting at x. In that case we get

$$B_\mu(x) = K(x)\, A_\mu(x) \qquad (3)$$

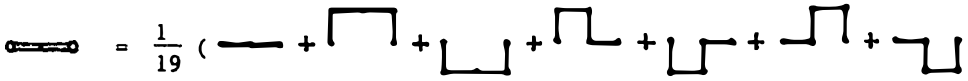
where

$$K(x) = \langle U(C_{x \to x}) \rangle_C \qquad (4)$$

is the average of the Wilson loops (closed paths) through x.

It is only natural in a quantum theory that simple non-smooth paths are taken into account (recall the definition of a path integral). In the framework of space-time lattice there is an argument to show that also self-intersecting paths should not be left out:

Infinitesimal closed loops measure the local distortion in the geometry of the state space of the colour structure of a quark produced by an external field. To find the effect of the field on a larger scale than given by an elementary plaquette of spacing a_0 we pick out a sublattice of spacing $a = n.a_0$. For n = 2 we define block links by 9)



+ 12 contributions in the other two space-time dimensions)

This definition avoids any need for gauge fixing, either globally or within specified blocks (unlike ref.7)) and thus retains the full symmetry of the original model. With this definition of the block link we have the following contributions to the new elementary Wilson loop

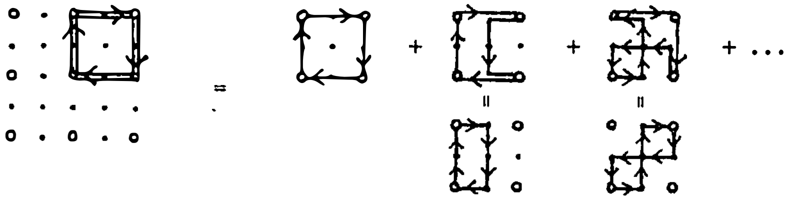


Fig. 1

The last diagram supports the claim that if we want to use B in a block link variable then this B should be given by eq.(3) where $C_{x\to x}$ is self-intersecting.

For self-intersecting contours the unrenormalized, dimensionally regularized Wilson loop functions tend to 0, 1 or ∞ when the length of the loop approaches 0 depending on the underlying gauge group and the angles between different branches of the loop at multiple points [10]. Therefore the gauge invariant field K(x), which is supposed [6] to be a good candidate for the dielectric field $\varepsilon(x) = \sigma(x)^4 = (\mathrm{Tr}\, K(x))^4$, is not renormalizable, i.e. it cannot embody properly the quantum fluctuations down to distance zero. Therefore we cannot define an action for K and B, effectively equivalent to S_{QCD}, through

$$\int [dA] \exp(-S_{QCD}) = \int [dK]\ [dB] \exp (- S_{eff}[K,B]) \tag{5}$$

by introducing $\int\delta($ eq.(3)$)[dB]$ and $\int\delta($ eq.(4)$)[dK]$ into the l.h.s. of eq.(5), as suggested by ref.[6]. In eq.(5) integration over B means integration over those potentials A whose fluctuations are larger than the scale that defines the extent of the loops considered in the definition of K.

A possible way out of the trouble is to replace K by

$$\tilde{K}(x) = \langle \phi(C_{x\to x}) \rangle_C \tag{6}$$

where

$$\phi(C) = P \exp \left(ig \int A_\mu(z(t))\, z^\mu(t)\, dt + g \int \frac{1}{2} [\gamma^\mu, \gamma^\nu] F_{\mu\nu}(z(t))\, dt \right) \tag{7}$$

and z(t) is a periodic function of period 1 whose values traverse the path $C_{x\to x}$ as t goes from 0 to 1. The operator (7) was proven [11] to be renormalizable even for self-intersecting paths in the case of QED and it is gauge invariant for both abelian and non-abelian fields.

Some difficulties about the coarse-grained parallel transports, which render the suggestions of Nielsen and Patkos strictly relevant only for the group SU(2), have been considered in ref.[12].

Next I turn my attention to ref.[7]. It sets out to construct the dielectric field entirely in the framework of lattice QCD and according to the same general strategy as outlined at the beginning of this comment.

The coarse-grained link variables are defined by requiring first a special gauge choice on the lattice sites such that the link variables of a specific set of plaquettes (marked with heavy line on Fig. 2) are as close to unity as possible.

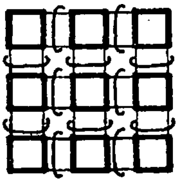


Fig. 2

Then the block-link variable between two neighbouring marked plaquettes is defined as the average of those gauge transformed elementary link variables that connect the two plaquettes (embraced on Fig. 2). It is shown for the gauge group SU(2) that the block link variables are elements of $R\otimes SU(2)$ and that the real factor χ of a block link variable is always smaller than 1. This factor is then identified with the value of the dielectric field (tensor ?) at the point of one of the two marked plaquettes, probably on the ground of the boundedness of χ between 0 and 1.

However, this does not warrant that the definition of the dielectric field is appropriate, as hinted by the example of an abelian gauge field in which case $0 \leq \chi \leq 1$ always because the arithmetic mean value of a set of unimodular complex numbers has the modulus ≤ 1, yet the vacuum of such a field is not paramagnetic.

There is nothing in this procedure that would discriminate between the abelian and the non-abelian case. In contradistinction, in the procedure of Nielsen and Patkos the difference between the two cases shows up in the choice of loops for the definition of K(x). Namely, for example, the coarse-grained Wilson loop on Fig. 1 comprises also the fine-grained loop of Fig.3 which is equivalent to the two disconnected loops on the r.h.s. of Fig.3 only for the abelian gauge group and it should be discarded then for it doesn't measure the field strength only at one of the indicated block lattice points.

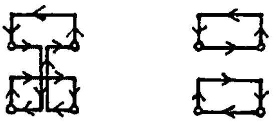


Fig. 3

Several further remarks concerning ref.$^{7)}$ are in order: (i) In the course of defining block-link variables by choosing a special gauge within specific blocks information is lost if the dimension of space-time exceeds 2. (ii) The dielectric field χ is a gauge-singlet field in the special gauge pertinent to the construction of χ. However, there is no gauge invariant definition of a colour singlet $^{13)}$.(Recall that K(x) from eq.(4) is gauge invariant). (iii) It is questionable whether an ansatz for the Lagrangian of the field χ in the form of a pure χ^4 theory makes sense because there is a compelling variety of evidence $^{14)}$ to suggest that such a theory is trivial or noninteracting in 4 space-time dimensions. Perhaps the coupling of χ to the coarse-grained gauge field rescues the theory from triviality as in the case of O(N) gauge model in the large-N limit $^{9)}$.

References:

1) D.J.Gross, F.Wilczek, Phys.Rev.Lett. 30(1973)1343, H.D.Politzer, ibid.1346
2) R.J. Hughes, Nucl. Phys. B186 (1981) 376.
3) J.B. Kogut, L.Susskind, Phys. Rev D9 (1974) 3501.
4) R. Friedberg, T.D. Lee, Phys. Rev. D15 (1977) 1694, D18 (1978) 2623.
5) S. Adler, Phys. Lett. 110B (1982) 302.
6) H. B. Nielsen, A. Patkos, Nucl. Phys. B195 (1982) 137.
7) H.J. Pirner, J. Wroldsen, M. Ilgenfritz, CERN preprint TH 4574/87.
8) V.S. Dotsenko, S.N. Vergeles, Nucl. Phys. B195 (1980) 572.
9) D.J.E. Callaway, R. Petronzio, Nucl. Phys. B267 (1986) 253.
10) R.A. Brandt, A. Gocksch, M.-A. Sato, F. Neri, Phys. Rev. D26 (1982) 3611,
H. Dorn, Fortschr. Phys. 34 (1986) 11.
11) S.G. Rajeev, Ann. Phys. 173 (1987) 249.
12) J.M. Drouffe, Nucl. Phys. B218 (1983) 89.
G. Mack, Nucl. Phys. B235 (1984) 197.
13) S. Mandelstam, Phys. Rev. D19 (1979) 2391.
14) D.J.E. Callaway, R. Petronzio, Nucl. Phys. B246 (1984) 577.
I.T. Drummond, S. Duane, R. Horgan, Cambridge preprint DAMTP 86-4 (1986).