

## STRANGE THINGS IN CHIRAL SOLITON MODELS

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### Abstract

We present an extension to three flavours of the chiral soliton model based on the linear  $\sigma$  model with quarks. Using the approximate projection method of Birse and Banerjee we calculate properties of the octet baryons. The magnetic moments are found to be in qualitative agreement with experiment, given that the kaons and pions are degenerate in this SU(3)-symmetric version of the model. The splitting between the octet and decuplet is calculated using a cranking method, exactly as in the SU(2) model. An extension of cranking to the full SU(3) group gives the energies of states which must involve mesonic excitations as well as three quarks.

### 1. The Model

A linear  $\sigma$  model,<sup>1</sup> involving quarks coupled to pion and scalar meson fields, has been proposed as a model for the nucleon and  $\Delta$ .<sup>2,3</sup> This model has provided a reasonably successful phenomenology of nucleon properties.<sup>4</sup> The motivation for this approach comes from the idea that QCD leads to a hidden chiral symmetry and that this is a dominant feature of quark interactions at low momenta.

Since the model embodies the ideas of current algebra and PCAC, it is natural to try to extend it to three flavours of quarks. After all, quarks were originally introduced to explain the "eightfold way" of current algebra<sup>5</sup> and low-energy hadron spectroscopy.<sup>6</sup>

The extension of the model to three flavours introduces a number of complications. The simplest representation of the chiral group SU(3)<sub>R</sub> × SU(3)<sub>L</sub> which includes pions and kaons is 18 dimensional! It includes<sup>7,8</sup> a scalar singlet field  $\sigma$ , a scalar octet  $\xi_a$ , a pseudoscalar singlet  $\eta$  and a pseudoscalar octet  $\phi_a$ . The Lagrangian for the model is

$$\mathcal{L} = \bar{\psi} [i\gamma^\mu \partial_\mu + g(\sqrt{\frac{2}{3}}(\sigma + i\eta\gamma_5) + \lambda^a(\xi_a + i\phi_a\gamma_5))] \psi + \frac{1}{6} \text{Tr}[\partial_\mu M^\dagger \partial^\mu M] - U, \quad (1.1)$$

where the meson interaction potential is

$$U = \frac{\lambda^2}{4} [(\frac{1}{3} \text{Tr}[M^\dagger M])^2 - \nu^2]^2 + \frac{\kappa^2}{4} [\frac{1}{3} \text{Tr}[(M^\dagger M)^2] - (\frac{1}{3} \text{Tr}[M^\dagger M])^2] + \sqrt{\frac{3}{2}} F_\pi m_\pi^2 \sigma, \quad (1.2)$$

and for convenience we define

$$M = \sigma + i\eta + \sqrt{\frac{3}{2}}(\xi_a + i\phi_a)\lambda^a. \quad (1.3)$$

The parameters  $\lambda$  and  $\kappa$  are related to the meson masses by

$$\lambda^2 = \frac{m_\sigma^2 - m_\pi^2}{3F_\pi^2}, \quad \kappa^2 = \frac{m_\xi^2 - m_\pi^2}{3F_\pi^2}, \quad (1.4)$$

and  $\nu$  is related to  $F_\pi$  by

$$\nu^2 = \frac{3}{2} \frac{F_\pi^2 m_\sigma^2 - 3m_\pi^2}{m_\sigma^2 - m_\pi^2}. \quad (1.5)$$

The dynamical quark mass is  $M_q = gF_\pi$ , as in the SU(2) model.

The final term in the meson potential (1.2) explicitly breaks<sup>9</sup> the chiral symmetry to SU(3)<sub>V</sub>. It gives all the pseudoscalar mesons the same mass,  $m_\pi$ . A similar term linear in  $\xi_8$  would give explicit SU(3) symmetry breaking. We have not yet investigated the effects of such a term.

As in the SU(2) model we start by finding a solution to the MFA Euler-Lagrange equations using the hedgehog ansatz. For  $\lambda = \kappa$  the solution is just an embedding of the old SU(2) hedgehog, with the new scalar fields given by

$$\begin{aligned} \sigma &= \sqrt{\frac{2}{3}}\sigma_{(2)} - \sqrt{\frac{1}{6}}F_\pi, \\ \xi_8 &= \sqrt{\frac{1}{3}}(\sigma_{(2)} + F_\pi), \end{aligned} \quad (1.6)$$

where  $\sigma_{(2)}$  is the scalar field of the SU(2) model. As before,<sup>2</sup> the energy of the soliton is 1119 MeV and the  $\pi$ -N  $\sigma$ -commutator is 92 MeV.

## 2. Baryon Properties

The hedgehog baryon is a state without good spin or isospin. As with the intrinsic state of a deformed nucleus, one needs to project it onto eigenstates of the symmetry operators.<sup>10</sup> Even the SU(2) projection is technically complicated.<sup>11</sup> The SU(3) projection will be much worse, involving an integral over  $D$ -functions which depend on eight Euler angles.<sup>12</sup>

Instead we use an approximate projection method proposed in Ref. 2. We assume that the quarks carry all the spin and flavour quantum numbers of the baryon. The quark contributions to observables can then be calculated using the standard quark-model wave functions.<sup>6</sup> The meson contributions are obtained with a coherent-state-like approximation (which is exact for operators linear in the pion field). They can be evaluated with the help of the Wigner-Eckart theorem and tables of SU(3) Clebsch-Gordan coefficients.<sup>13</sup> (One needs to be careful to use a consistent phase convention for the quark and meson parts of the calculation!) This approximation neglects the angular momentum, isospin, etc. carried by the mesons. In an exact projection these will tend to reduce the quark contributions to spin-dependent quantities.<sup>11</sup>

Results for the magnetic moments of the baryon octet are presented in the table. The quark contribution is denoted  $\mu_q$ , the meson isovector and isoscalar contributions  $\mu_{m1}$  and  $\mu_{m0}$ , and the total magnetic moment  $\mu_B$ . The parameter set used was  $m_\sigma = m_\xi = 1200$  MeV,  $m_\pi = 140$  MeV and  $M_q = 500$  MeV. The agreement with experiment is reasonably good. Note that the model used so far is SU(3)-symmetric and so the kaon tail is too long. As an estimate of what may be expected in a more realistic version of

the model we present results without the isoscalar meson contribution (which is purely kaonic). In general this changes the moments in the direction of the experimental ones.

We have also calculated  $|g_A/g_V|$  for various transitions (experimental values are given in parentheses):

$$\begin{aligned} n \rightarrow p &: 1.64 \quad (1.26) \\ \Lambda \rightarrow p &: 0.97 \quad (0.69) \\ \Sigma \rightarrow N &: 0.37 \quad (0.36 \pm 0.04) \end{aligned}$$

The  $\pi$ -N coupling is  $(m_\pi/2N)g_{\pi NN} = 1.24$ . Like the nucleon  $g_A$  it is too large, but this is consistent with the Goldberger-Treiman relation.

Baryon	Magnetic Moment (in n.m.)					expt.
	$\mu_q$	$\mu_{m1}$	$\mu_{m0}$	$\mu_B$	$(\mu_B - \mu_{m0})$	
p	1.743	0.791	0.113	2.646	(2.534)	2.79
n	-1.162	-0.791	0.113	-1.839	(-1.953)	-1.91
$\Lambda$	-0.581	0.0	-0.339	-0.920	(-0.581)	-0.61
$\Sigma^+$	1.743	0.565	0.339	2.646	(2.308)	2.48
$\Sigma^0$	0.581	0.0	0.339	0.920	(0.581)	-
$\Sigma^-$	-0.581	-0.565	0.339	-0.807	(-1.146)	-1.1
$\Xi^0$	-1.162	-0.226	-0.452	-1.839	(-1.388)	-1.25
$\Xi^-$	-0.581	0.226	-0.452	-0.807	(-0.355)	-0.69

### 3. Cranking

As mentioned the hedgehog baryon is interpreted as a mixture of symmetry eigenstates (mainly octet and decuplet). Its energy is the average of the energies of these states. A full projection calculation will yield energies for the various symmetry eigenstates.<sup>11</sup> One should also vary the wave function to obtain a stationary energy for each projected state. This was done in Ref. 11, but was restricted to grand-spin-zero variations in the fields. ("Grand spin" is angular momentum plus isospin.<sup>4</sup>) A more general variation has been performed in Ref. 14, with significant improvements in virial theorems and the Goldberger-Treiman relation.

An approximate way to calculate the energies of the symmetry eigenstates is cranking, often used in nuclear physics.<sup>10</sup> It has also been used to calculate the N- $\Delta$  splitting in the SU(2) chiral soliton model.<sup>15</sup>

In the self-consistent cranking approximation, moments of inertia are calculated by minimizing the energy in a rotating frame – the expectation value of

$$H' = H - \omega_a \Lambda^a, \quad (3.1)$$

where  $\Lambda^a$  are the SU(3) generators expressed in terms of intrinsic or "body-fixed" axes.

Provided the overlap between rotated hedgehog states is a sharply peaked function of the Euler angles, one can work to lowest order in the cranking frequencies  $\omega_a$ . The moments of inertia can be found from

$$\Lambda^a = \mathcal{I}_a \omega_a. \quad (3.2)$$

To this order the SU(3) rotational energy is

$$E_r = \frac{1}{2} \mathcal{I}_I (\omega_1^2 + \omega_2^2 + \omega_3^2) + \frac{1}{2} \mathcal{I}_S (\omega_4^2 + \omega_5^2 + \omega_6^2 + \omega_7^2), \quad (3.3)$$

where  $\mathcal{I}_I$  and  $\mathcal{I}_S$  are the moments of inertia for isospin ( $a = 1, 2, 3$ ) and strange rotations ( $a = 4, 5, 6, 7$ ) respectively.

On quantizing the rotations one gets a corresponding expression for the energies of the SU(3) eigenstates,<sup>16</sup>

$$E_r((p, q), J, Y_R) = \frac{1}{2}(\mathcal{I}_I^{-1} - \mathcal{I}_S^{-1})J(J+1) + \frac{1}{2}\mathcal{I}_S^{-1}C_2(p, q) - \frac{3}{8}\mathcal{I}_S^{-1}Y_R^2, \quad (3.4)$$

where  $(p, q)$  labels the SU(3) representation<sup>13</sup> and  $C_2(p, q)$  is the quadratic Casimir operator

$$C_2(p, q) = \frac{1}{3}(p^2 + q^2 + pq + 3(p + q)). \quad (3.5)$$

From the grand-spin-zero nature of the hedgehog the intrinsic isospin is just minus the total angular momentum  $J$ . The soliton is symmetric under intrinsic hypercharge rotations, and so no corresponding angular velocity  $\omega_8$  appears in (3.3). The intrinsic hypercharge is constrained<sup>16</sup> to be that of the  $N_c$  non-strange quarks in the hedgehog,

$$Y_R = \frac{1}{3}N_c. \quad (3.6)$$

The isospin cranking is very similar to that in Ref. 15. The allowed changes in the fields, to first order in  $\omega_i$  have grand spin one and even parity. They are further restricted by grand reversal symmetry.<sup>15</sup> Cranking generates a non-zero  $l = 1$  expectation value for the conjugate momentum to the pion field. It also mixes  $l = 0, 2$  grand-spin-one components into the quark wave functions. The only difference between the present model and the SU(2) one is that  $\xi_i$  fields with  $l = 0$  and  $\sqrt{2/3}\eta + \phi_8$  with  $l = 1$  are coupled to the quark functions.

The quark and meson contributions to the moment of inertia are  $\mathcal{I}_{Iq} = 0.495$  fm and  $\mathcal{I}_{Im} = 0.711$  fm respectively. The total  $\mathcal{I}_I = 1.21$  fm corresponds to an octet-decuplet splitting of 245 MeV.

It is interesting to note that for any  $N_c$  the SU(3) representations which can be formed from  $N_c$  quarks (with no antiquarks or mesons) have

$$C_2(p, q) = J(J+1) + \frac{1}{12}N_c(N_c + 2), \quad (3.7)$$

where  $J = p/2$  is the isospin corresponding to the maximum hypercharge in the multiplet, given by (3.6). Hence splittings between these states are determined by  $\mathcal{I}_I$  only. Other multiplets which satisfy (3.6) are possible but these must involve mesonic excitations, in addition to  $N_c$  quarks. Their energies involve both moments of inertia.

The strange cranking mixes into the intrinsic state components with strangeness  $\pm 1$  and grand-spin- $\frac{1}{2}$ . However there is no analogue of grand reversal. If we chose to crank with  $\Lambda_6$ , conjugate momenta are generated for  $\phi_a$   $a = 4, 5, 7$  and  $\xi_7$ . Also a spin-up strange quark component is produced, coupled to  $\phi_a$   $a = 4, 5, 6$  with  $l = 1$  and  $\xi_6$  with  $l = 0$ .

A problem arises in the calculation of these changes in the fields. In the linearized approximation the cranking equations are just the zero-energy RPA equations with a source term.<sup>10</sup> For cranking about  $\Lambda_6$ , the homogeneous equation has a zero-mode solution corresponding to rotations about  $\Lambda_7$ . Since the zero-mode is not orthogonal to the inhomogeneous term, equation has no finite solution. However the zero mode is just a rotation of the intrinsic state. In a full projection calculation, the energy of the

projected states must be independent of such rotations. Hence we propose to keep only the pieces of the solution which are orthogonal to the zero-mode. These are finite and correspond physically to coupling of intrinsic excitations to the collective rotation.

The quarks and meson moments of inertia from this calculation are  $\mathcal{I}_{S_q} = 0.027$  fm and  $\mathcal{I}_{S_m} = 0.407$  fm. Note the dominance of the meson contribution, as expected from the fact that strange cranking gives the energies of mesonically excited states (at least for a pure hedgehog). The total strange moment of inertia  $\mathcal{I}_S = 0.434$  fm, less than half that for isospin cranking. The energies of the not-purely-quarkish states lie at correspondingly higher energies. For example there is a  $J = \frac{1}{2}$  anti-decuplet at 681 MeV above the nucleon, and a  $J = \frac{5}{2}$  35-plet at 878 MeV. Even if these are not just artifacts of the semiclassical approximation, they are unlikely to show up as well-defined resonances.

### References

1. M. Gell-Mann and M. Lévy, *Nuovo Cim.* **16** (1960) 705.
2. M. C. Birse and M. K. Banerjee, *Phys. Lett.* **136B** (1984) 284; *Phys. Rev.* **D31** (1985) 118.
3. S. Kahana, G. Ripka and V. Soni, *Nucl. Phys.* **A415** (1984) 351.
4. For a review see: M. K. Banerjee, W. Broniowski and T. D. Cohen, University of Maryland preprint 86-155 (1986), to be published by World Scientific.
5. M. Gell-Mann and Y. Ne'eman (eds.), *The Eightfold Way*, (Benjamin, New York, 1965);  
S. L. Adler and R. F. Dashen, *Current Algebras*, (Benjamin, New York, 1968).
6. F. E. Close, *An Introduction to Quarks and Partons*, (Academic, New York, 1979).
7. M. Lévy, *Nuovo Cim.* **52A** (1967) 23.
8. For a review see: H. Pagels, *Phys. Rep.* **16C** (1975) 219.
9. S. Glashow and S. Weinberg, *Phys. Rev. Lett.* **20** (1968) 228;  
M. Gell-Mann, R. Oakes and B. Renner, *Phys. Rev.* **175** (1968) 2195.
10. See, for example: P. Ring and P. Schuck, *The Nuclear Many-Body Problem*, (Springer, New York, 1980);  
J.-P. Blaizot and G. Ripka, *Quantum Theory of Finite Systems*, (MIT, Cambridge, 1986).
11. M. C. Birse, *Phys. Rev.* **D33** (1986) 1934;  
B. Golli and M. Rosina, *Phys. Lett.* **165B** (1985) 347.
12. D. F. Holland, *J. Math. Phys.* **10** (1969) 531.
13. P. Carruthers, *Introduction to Unitary Symmetry*, (Wiley, New York, 1966).
14. M. Fiolhais, A. Nippe, K. Goeke, F. Grümmer and J. N. Urbano, University of Coimbra preprint (1987), submitted to *Phys. Lett.*
15. T. D. Cohen and M. K. Banerjee, *Phys. Lett.* **167B** (1986) 21;  
T. D. Cohen and W. Broniowski, *Phys. Rev.* **D34** (1986) 3472.
16. E. Guadagnini, *Nucl. Phys.* **B236** (1984) 35;  
A. P. Balachandran, F. Lizzi, V. G. J. Rogers and A. Stern, *Nucl. Phys.* **B256** (1985) 525;  
H. Yabu and K. Ando, Kyoto University preprint KUNS 851 (1986).