FIZIKA 19 (1987), Supplement 2, 61-63
(Proc. Workshop on Mesonic Degrees of Freedom in Hadrons, Bled/Ljubljana June/July 1987)

Comparison of Exact and Approximate Solutions of a Simple Model for Pion Cloud Around a Static Source

Marko Čibej
Jožef Stefan Institute, Ljubljana, Yugoslavia

Abstract

A simple model for pion cloud around a static source containing three quarks is solved exactly in the small and large coupling limits. The solutions are compared to an approximate solution using the hedgehog ansatz. The hedgehog solution is better for J=T=1/2 than for J=T=3/2 state.

Introduction

Effective models of hadrons are at present the most promising way to approach low energy phenomenology, but they suffer from two drawbacks. Due to the lack of a complete derivation based on QCD, the evidence for their appropriateness in the regime in which they are applied is somewhat circumstantial. Furthermore, since they cannot be solved exactly, one is sometimes unsure of whether discrepancies are due to the model or to the method of solving it. Here I give what may be a handle for a part of the second problem. I solve an extremely simplified model exactly in two regimes of coupling, and compare the exact solutions to an approximate one obtained by variation of the hedgehog ansatz.

The Model

I study the model used by Fiolhais and Rosina [1] which is derived from the sigma model with quarks. The model treats three valence quarks with only spin and isospin degrees of freedom (with frozen spatial wavefunctions) and p-wave pions with frozen radial degrees of freedom. There are four S=T=1/2 three quark states, called the bare nucleon, and sixteen S=T=3/2 states called the bare delta. The Hamiltonian is

$$H = \sum_{i=1}^{3} \sum_{\alpha=1}^{3} \{ a^{+}_{i\alpha} a_{i\alpha} - g \sum_{n=1}^{3} \sigma^{n}_{i} \tau^{n}_{\alpha} [a^{+}_{i\alpha} + a_{i\alpha}] \} . \qquad (1)$$

where σ^n_i and τ^n_α are Pauli matrices acting on the n-th quark, and $a^+_{i\alpha}$ and $a_{i\alpha}$ create and annihilate p-wave pions with angular momentum in the direction i, and isospin in the direction α. The Hamiltonian can also be written in spherical components, as in (1), giving

$$H = \sum_{t=1}^{3} \sum_{m=1}^{3} \{ a^{+}_{tm} a_{tm} - g B_{tm} [a_{tm} + (-)^{t+m} a^{+}_{-t-m}] \} . \quad (1a)$$

Here m and t are z projections of angular momentum and isospin. The three quarks are treated as a single object with twenty different states (the bare nucleon and delta states), and B_{tm} is a generalized Pauli matrix.

The Hedgehog Solution

The model (1) has been solved variationally using a slightly generalised hedgehog ansatz by Rolhais and Rosina. Their results can be expressed as a power series in the coupling constant (for the perturbative region), or in the inverse coupling constant for the asymptotic region. The series are, for perturbative region:

$$E_N = -57 g^2 + 849 g^4 + 89827.04 g^6 , \quad (2)$$

$$E_\Delta = -33 g^2 + 514 g^4 -13347 g^6 , \quad (3)$$

and for the asymptotic region

$$E_N = -27 g^2 - 3/2 + 1/(64 g^2) - 1/(48 g^2) , \quad (4)$$

$$E_\Delta = -27 g^2 - 3/2 + 7/(48 g^2) - 1/(48 g^2) . \quad (5)$$

The meaning of the two $1/g^2$ terms is explained in the next section.

Perturbative Solution for Low g Region

For small coupling constants the average number of pions in the pion cloud is very small, so the solution can be obtained by perturbation around the zero-pions state. The first part of the Hamiltonian that counts the number of pions in the cloud can be solved exactly. The solutions are simply configurations with a fixed number of pions and quarks coupled to one of the twenty bare states. The zero pion state is then perturbed by the interaction part of the hamiltonian. Depending on whether the zero pion state has quarks coupled to N or Δ states, one obtains perturbative series for N and Δ state:

$$E_N = -57 g^2 + 804 g^4 - 37224 g^6 . \quad (2a)$$

$$E_\Delta = -33 g^2 - 12 g^4 - 355073 g^6 . \quad (3a)$$

Both series converge to about g=0.05.

Perturbative Solution for the Asymptotic Region

The problem of obtaining an exact series solution in the limit of large coupling constants was treated in 1984 by Parmentola [2] for a model similar to the present one, but containing only one spinor-isospinor in the source (only bare nucleon degrees of freedom). His derivation can be repeated using the more general model. For the lowest lying states one finds that the difference of the models reduces to a simple rescaling of the coupling constant g by a factor of 3. One can then take over his results:

$$E_N = -27\ g^2 - 3/2 + 1/(144\ g^2) - 1/(48\, g^2)\ , \tag{4a}$$

$$E_\Delta = -27\ g^2 - 3/2 + 1/(24\ g^2) - 1/(48\, g^2)\ . \tag{5a}$$

The positive $1/g^2$ terms are rotational energy, while the negative ones come from the anharmonic part of the Hamiltonian. The $1/g^2$ terms in the variational result are separated in the same way, showing that the the hedgehog reproduces the anharmonic terms exactly to the $1/g^2$ order. Further details of this comparison can be found in [3].

Discussion

In both the low and the high g regimes the the hedgehog result is surprisingly good. In the low g region the the delta result of course leaves much to be desired, but in the asymptotic region both the nucleon and the delta results are comparably accurate. Since the only discrepancy in that region is in rotational energy, variation of the hedgehog ansatz, supplemented perhaps with a semiclassical method for determining rotational energy, seems to be a good method forthe region of large g. The low g region is probably best left to perturbative methods, except for qualitative calculations, where variation of the hedgehog can give a good idea of the overall behaviour.

I wish to thank Mitja Rosina for helpful discussions.

References

[1] M. Fiolhais and M. Rosina, *Portgal. Phys.* 17 (1986) 49

[2] J. A. Parmentola, *Phys. Rev.* D 29 (1984) 2563

[3] M. Čibej, M. Fiolhais and M. Rosina, to be published.