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The B = 2-system in the Chiral σ-Model with Quarks[§]

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Abstract

It is shown that the B=2 system of lowest energy in the sigma model with quarks has winding number n=1 and occupied 0^+, 1^+ orbitals.

We investigate the σ-model Lagrangian including massless quarks in the semiclassical or mean-field approximation[1,2].

$$\mathcal{L}_\sigma = i\,\bar{\Psi}\,\partial\!\!\!/\,\Psi + \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\,\partial_\mu\vec{\pi}\,\partial^\mu\vec{\pi}$$
$$+\, g\,\bar{\Psi}\left[\,\sigma(x) + i\,\vec{\tau}\cdot\vec{\pi}\,\gamma_5\,\right]\Psi$$
$$-\,\frac{\lambda^2}{4}\left[\,\sigma^2 + \vec{\pi}^2 - \nu^2\,\right]^2 - F_\pi\, m_\pi^2\,\sigma$$

We assume the hedgehog ansatz for the meson fields

$$\vec{\pi}^2(\vec{r}) = \hat{r}\,h(r) \quad\text{and}\quad \sigma(\vec{r}) = \sigma(r)$$

and thus for consistency construct quark spinors as eigenfunctions of the grandspin operator $\vec{G} = \vec{I} + \vec{J}$ and the parity π, labeling them by G^π. Using the method of Post et al.[3] we have solved the resulting equations of motion for different B = 2-configurations[4]. Alternatively we have investigated a B = 2system with vanishing pionfield $\vec{\pi} = 0$ (SLAC Bag-ansatz) by occuping the 1s orbital with six quarks.

Fig.1 shows that the B = 2-system with lowest energy is obtained for the $G^\pi = 0^+$ and the 1^+-orbitals, each filled with three quarks. The relatively large energy gap between the $(0^+,1^+)$, $(0^+,1^-)$-systems and the $(0^+,0^-)$-solution is due to the different field configurations. This can be seen most obviously in the nonlinear limit ($\lambda \longrightarrow \infty$), where the fields can be parameterized by the chiral angle $\theta(r)$

$$\sigma(r) = -F_\pi \cos\theta(r),\quad \vec{\pi}(r) = -\,\hat{r}\,F_\pi \sin\theta(r)$$

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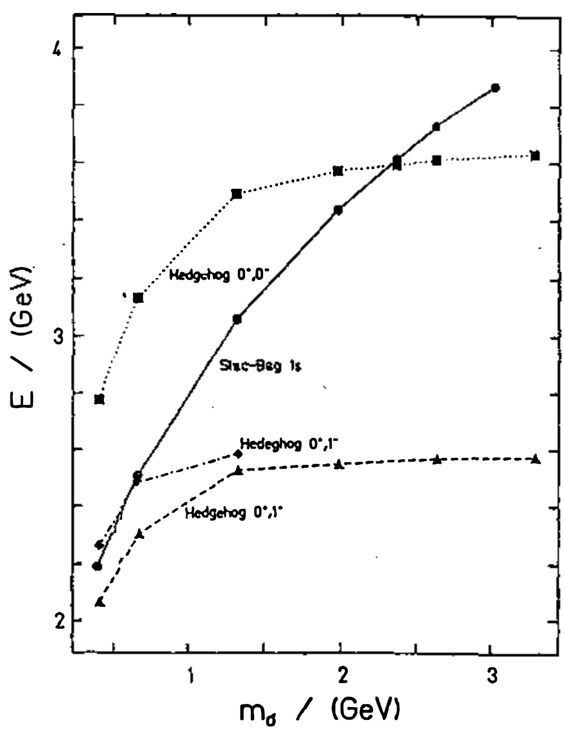
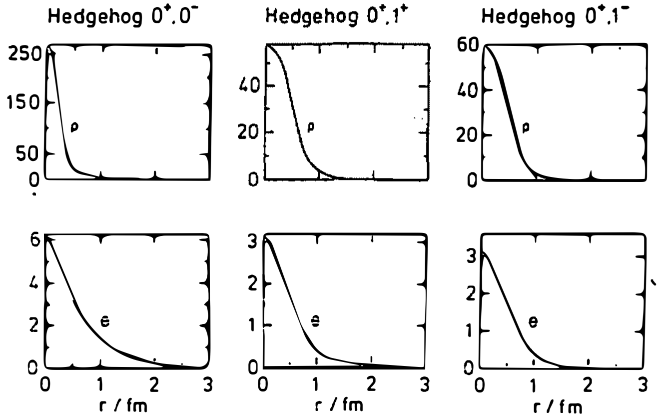
and can be labeled by their winding number n given via $\theta(r = 0) = - n\pi$.

In Fig.2. we see that the fields of the $(0^+,1^+)$- and $(0^+,1^-)$-solutions have winding number n = 1 whereas the $(0^+,0^-)$-configuration leads to meson fields with n = 2. Therefore the $(0^+,0^-)$-system has a significantly higher mesonfield energy. This also shows that in contrast to the pure skyrmion model, where



Figur 1: Total energies of the different B = 2systems as a function of m_σ for g = 6 and m_π = 0.



Figur 2: Quarkdensities $\rho(r)$ and chiral angles $\theta(r)$ in the nonlinear σ-model (λ = 1000 i.e. m_σ = 131.5 GeV) for g = 8 and m_π = 139.6 MeV.

$n = B$, the model with explicit quark degrees of freedom allows for solutions with $n \neq B$. Fig.2 further shows that the chiral field of the $(0^+,0^-)$-system reaches out very far, whereas the quark density is concentrated in the interior of the bag. Therefore, the chiral field should dominate the long range behaviour and is therefore the relevant field in the low energy region. In contrast the non skyrmion-like systems $(0^+,1^+;\ 0^+,1^-)$ have quark densities and chiral angles which reach out equally far and cannot be effectively described by the mesonfields alone. This is further supported by the investigation of the quark energies as a function of g. Here we find that only for the $(0^+,0^-)$-system both orbitals acquire negative energy and enter the Dirac sea, whereas the 1^+ and 1^--orbitals of the $n \neq B$ solutions remain positive. Thus appling the interpretation of Kahana and Ripka[1], which says that the skyrmion contains an effective description of the Dirac sea polarized by the intruder states, we see that for the $0^+,1^+$- and $0^+,1^-$- systems only the part of the baryon number coming from the 0^+- quarks can be simulated by considering the mesonic sector. The effects of the 1^+- and 1^--quarks however, cannot be mapped onto a corresponding meson field. In contrast, as we have seen above, the $0^+,0^-$-system for large g may be effectively described by the pure mesonic skyrmion-model, since both occupied orbitals have entered the Dirac sea.

In conclusion, we have found that the σ-model with explicit quark degrees of freedom leads to non skyrmion-like solutions for the B = 2-system which are lower in energy than the skyrmion-like solutions. These non skyrmion-like n ≠ B-systems seem to be a unique feature of the model with explicit quark degrees of freedom, that is not contained in the pure skyrmion-model. Further work is likely to focus on the question whether this feature of the explicit quark degrees of freedom is needed to describe experimental results. More effort should also be invested in the treatment of the Dirac sea, since these corrections become important for large coupling constants.

References

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