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QUARK STRUCTURE AND BARYON-BARYON INTERACTION

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Abstract

This talk will deal with an exploratory study of baryon - baryon interaction based on a model where quarks interact by exchanging σ and π fields. Six quarks are placed in two spinors separated by a specified distance R which is ensured with a constraint. The problem is treated at the mean field level and the state is obtained by minimizing the energy functional including the constraint terms. When separated by a large distance each three-quark group is described by the Birse-Banerjee soliton. Two main results emerge. When the separation is 1 fm or less the notion of two solitons becomes meaningless. The system forms a single six-quark soliton. For separations larger than 1 fm the spatial polarization of the solitons is the most dominant effect. Unlike previous calculations we obtain some attraction in the central potential at intermediate distances.

In this talk, based on the Ph. D. thesis of Fernando J. Pineda [1], I will describe a study of the behaviour of two interacting solitons, each made up of three quarks, when they are held at a certain distance apart

The work is based on the chiral quark model of the baryon which Michael Birse and I [2] developed. It uses the Gell-Mann - Lévy linear σ model [3] with the quark as the fermion. We place three quarks, red, blue and green, in the spinor $q_{\mathbf{LEFT}}$ and three others in the spinor $q_{\mathbf{RIGHT}}$. Using a suitable constraint we separate the centers of the orbits by a prescribed distance, R. We then obtain the stationary state of this system in the *mean field approximation*. We vary R from a maximum value of 2.8 fm to 0 fm in steps of 0.1 fm. Our most significant result is that for R ≤ 1 fm the notion of separated baryons fail. The system merges into a single six-quark soliton. We extract a NN potential for the external region, R > 1 fm,in the spirit of the Born-Oppenheimer approximation using the SU(4) ansatz.

There are several similar studies based on the skyrmion theory [4]. In all these studies, including ours, the spin-flavor degrees of freedom are kept fixed. However, we allow for full spatial polarization and study more spin-flavor sets than other authors. The result is that we are the first to obtain some attraction in the central NN potential. Despite its purely academic nature the results are still very interesting.

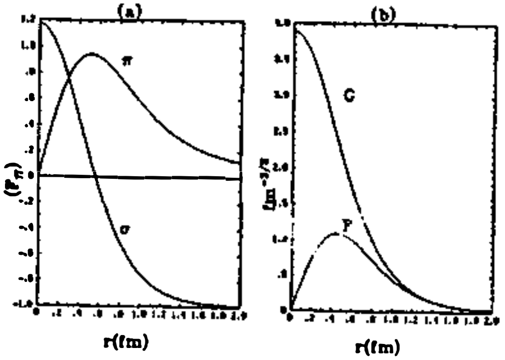
The Gell-Mann - Lèvy linear σ model lagrangian is

$$\mathscr{L} = \bar{\psi}\,\{i\gamma^{\mu}\partial_{\mu} + g\,(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\}\,\psi + \frac{1}{2}\,(\partial_{\mu}\sigma\partial^{\mu}\sigma + \partial_{\mu}\vec{\pi}\cdot\partial^{\mu}\vec{\pi}) - \frac{\lambda^2}{4}\,(\sigma^2 + \vec{\pi}^2 - \nu^2\,)^2 - F_{\pi}m_{\pi}^{\,2}(\sigma + F_{\pi}) + \frac{m_{\pi}^{\,2}}{4\lambda^2}\,, \tag{1}$$

with $\lambda^2 = \dfrac{m_{\sigma}^{\,2} - m_{\pi}^{\,2}}{2F_{\pi}^{\,2}}$ and $\nu^2 = F_{\pi}^{\,2} - \dfrac{m_{\pi}^{\,2}}{\lambda^2}$. We use m_{σ} = 1200 MeV and gF_{π} = 500 MeV with F_{π} = 93 MeV. The model yields a three-quark hedgehog soliton in the mean field approximation.

The hedgehog solution produces a baryon with three quarks, antisymmetric in color, occupying identical spin-flavor state of the form

$$q(\vec{r}) = \left[\frac{1}{4\pi}\right]^{1/2}\begin{bmatrix} G(r)\chi_h \\ i\vec{\sigma}\cdot\hat{r}F(r)\chi_h \end{bmatrix}, \tag{2}$$



(a) (b)

Fig.1 (a) G(r) and F(r), (b) $\sigma(r)$ and $\pi(r)$ in units of F_π

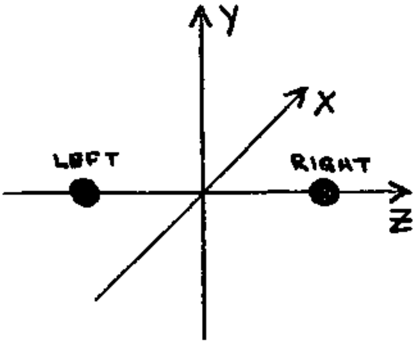


Fig.2 Geometry of the two-soliton system. The centers of the two spinors are on the Z-axis equidistant from the origin.

$$\text{where } \chi_h = \sqrt{1/2}\,[u_\downarrow - d^\uparrow], \qquad (3)$$

$$\text{and } (\vec{\sigma} + \vec{\tau})\chi_h = 0. \qquad (4)$$

Fig. 1a shows G(r) and F(r) vs. r. The hedgehog meson fields have the form

$$\sigma(\vec{r}) = \sigma(r), \quad \pi_\alpha(\vec{r}) = \pi(r)\hat{r}_\alpha, \qquad (5)$$

and their distributions are shown in fig 1b. It should be remembered that the hedgehog solution is obtained in a particular frame. In this frame the baryonic state has only three quarks – no mesons or qq pairs. The act of projection of spin or isospin produces mesons and qq pairs. In the SU(4) ansatz we ignore the role of these bosons and the hedgehog baryon will is taken to have the composition

$$|B\rangle = \sqrt{1/8}\ \{\,|\Delta^{++}, -3/2\rangle - |\Delta^{+}, -1/2\rangle + |\Delta^{0}, 1/2\rangle - |\Delta^{-}, 3/2\rangle\}$$
$$- (1/2)\{|p, -1/2\rangle - |n, 1/2\rangle\}. \qquad (6)$$

When the separation R between the solitons is very large the spinors have the form

$$q_{\text{RIGHT,LEFT}} = \begin{bmatrix} G(|\vec{r} \mp \vec{R}/2|)\chi \\ \\ i\vec{\sigma}\cdot\vec{r}\ F(|\vec{r} \mp \vec{R}/2|)\chi \end{bmatrix}, \qquad (7)$$

The forms of spin-flavor spinor χ used and the labels of the baryons, when formed with three quarks in such spinors are listed below.

$$(1)\ \chi = h = \sqrt{1/2}\,[u_\downarrow - d^\uparrow], \qquad |q^3\rangle \equiv |H\rangle, \qquad (8a)$$

$$(2)\ \chi = h_x = -i\sigma_x h = -i\sqrt{1/2}[u^\uparrow - d_\downarrow],\ |q^3\rangle \equiv |H_x\ \rangle, \tag{8b}$$

$$(3)\ \chi = h_y = -i\sigma_y h = -\sqrt{1/2}[u^\uparrow + d_\downarrow],\ |q^3\rangle \equiv |H_y\ \rangle, \tag{8c}$$

$$(4)\ \chi = h_z = -i\sigma_z h = i\sqrt{1/2}[u_\downarrow + d^\uparrow],\ |q^3\rangle \equiv |H_z\ \rangle. \tag{8d}$$

The choice of coordinate system is shown in fig. 2. These form a complete set of orthogonal spin-flavor spinors and their use simplifies the calculations considerably. We consider the systems HH, HH_x and HH_z. The remaining 7 configurations which can be made out of the set (8) are obtainable from these three with some suitably chosen symmetry operation. For example, H_zH_y may be obtained from HH_z by a rotation about the z axis through π.

The mean field calculation involves finding the stationary point (not the minimum) of the constrained energy function

$$\begin{aligned} \mathcal{E}(q_{\mathrm{RIGHT}}, q_{\mathrm{LEFT}}, \sigma, \vec{\pi}) &= E(q_{\mathrm{RIGHT}}, q_{\mathrm{LEFT}}, \sigma, \vec{\pi}) \\ &- \lambda_{\mathrm{RIGHT}}\int d^3r\ q^\dagger_{\mathrm{RIGHT}}q_{\mathrm{RIGHT}} - \lambda_{\mathrm{LEFT}}\int d^3r\ q^\dagger_{\mathrm{LEFT}}q_{\mathrm{LEFT}} \\ &- \lambda\int d^3r\ [q^\dagger_{\mathrm{RIGHT}}zq_{\mathrm{RIGHT}} - q^\dagger_{\mathrm{LEFT}}zq_{\mathrm{LEFT}}], \end{aligned} \tag{9}$$

and

$$\begin{aligned} E(q_{\mathrm{RIGHT}}, q_{\mathrm{LEFT}}, \sigma, \vec{\pi}) &= \int d^3r q^\dagger_{\mathrm{RIGHT}}\{-i\vec{\alpha}\cdot\vec{\nabla} - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\}q_{\mathrm{RIGHT}} \\ &+ \int d^3r q^\dagger_{\mathrm{LEFT}}\{-i\vec{\alpha}\cdot\vec{\nabla} - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\}q_{\mathrm{LEFT}} + \int d^3r\ U(\sigma,\vec{\pi}), \end{aligned} \tag{10}$$

where $U(\sigma,\vec{\pi})$ is the mesonic potential given eq (1). The last lagrange multiplier term in (9) ensures the separation between the solitons. At the stationary point we require that

$$\lambda\int d^3r\ [q^\dagger_{\mathrm{RIGHT}}zq_{\mathrm{RIGHT}} - q^\dagger_{\mathrm{LEFT}}zq_{\mathrm{LEFT}}] = R. \tag{11}$$

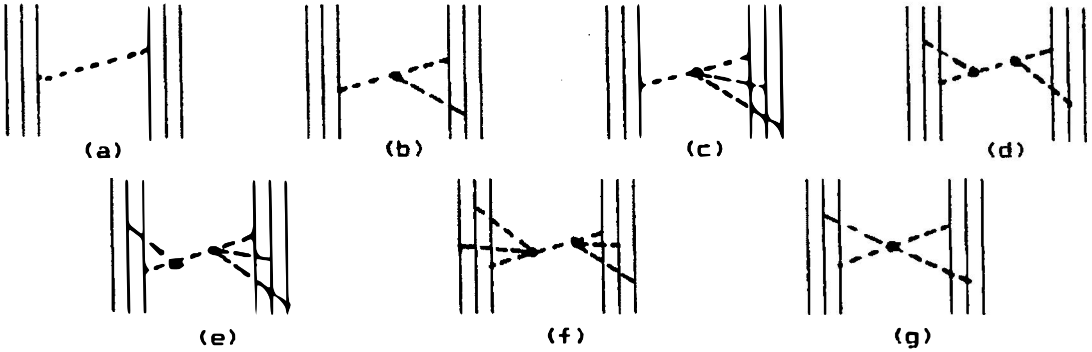
The stationary point is found by freezing the spin-flavor states to that chosen for very large R, i.e., HH, HH_x or HH_z, and varying the space functions. The are 12 space functions in the problem, 8 associated withe the spinors, viz,

$$q_{\mathrm{RIGHT,LEFT}} = \begin{bmatrix} G_{\mathrm{RIGHT,LEFT}} \\ \\ i\sigma_x F^x_{\mathrm{RIGHT,LEFT}} + i\sigma_y F^y_{\mathrm{RIGHT,LEFT}} + i\sigma_z F^z_{\mathrm{RIGHT,LEFT}} \end{bmatrix},$$

and 4 with the meson fields, viz, $\sigma(x,y,z)$, $\pi_x(x,y,z)$, $\pi_y(x,y,z)$ and $\pi_z(x,y,z)$. For each value of R a two-centered harmonic oscillator potential is chosen with the centers separated by a distance close to but not equal to R. All space functions are expanded in terms of the eigenfunctions of this two-centered harmonic oscillator.

Calculations for the HH and HH_z configurations could be carried out on a VAX with an FPS array processor. But for the HH_x

configuration it was neccesary to use a CRAY XMP.



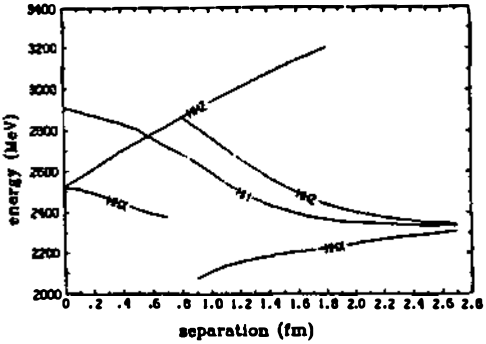
Figs.3 Meson exchanges between two baryons.

Before presenting the results a few remarks are in order. A mean field calculation includes only tree graphs. The resulting meson exchanges between two baryons are of 7 types shown in figs.3 a thru' g. Fig. 3a is the ordinary one-meson exchange with form factor due to spatial extension of the quark distributions. Figs. 3b thru' 3f show contribution to the form factors due to the meson clouds around the baryons. Fig. 3g does not represent one-meson exchange and has some interesting features when 4 pions are involved. Together with figs 3d it shows how the 'σ exchange' potential is mainly due to contact interaction bewteen the pion cloud sources of σ. This explains the low (500 MeV) mass of the σ in nuclear physics. Fig 3g also generates some ρ exchange force.

The preceding remarks are useful and have some relevance to our calculations. However, the overwhelmingly important feature of our investigations is the polarizability of the baryons, i.e., the dynamical aspects of the quark structure. Of course, color polarizability is simply not allowed in a mean field calculation. By putting 3 quarks in the same orbit we ensure that the wavefunctionis color singlet not just globally but locally, even though the effective lagrangian is invariant only under global color gauge transformation. For practical reasons we choose and then freeze the spin-flavor degree of freedom. But we do allow full spatial degree of freedom. As the following results will show, the spatial polarization determines the main features of our results.

The energies of the HH, HH$_z$ and HH$_x$ systems for$0 < R < 2.8$ fm are shown in fig. 4. For HH$_z$ and HH$_x$ there are clear breaks at $R \sim 1$ fm. The solution for R=2.8 fm, regarded as asymptotic solution for our purpose, deforms continuously to yield solutions for smaller R down to the breakpoint, R_{CRIT}. For $R < R_{CRIT}$ no solution can be found which is on the outerbranch. For these lower R's solutions can be generated by starting from R=0, i.e., the 6 quarks are put in two spinors having the same center, and then increasing R while generating solutions which are continuously connected. For HH$_x$ the inner branch also stops at $R \sim R_{CRIT}$. The HH$_z$ inner branch appears to touch the HH$_z$ outer branch at R_{CRIT}. But we have verified that

R_{CRIT} is not a bifurcation point.



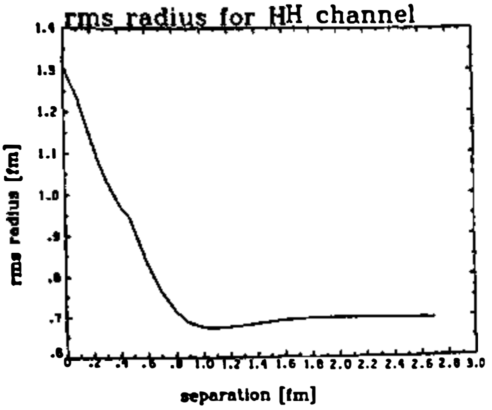


Fig. 4 The energies of the 3 channels for $0 < R < 2.8$ fm.

Fig. 5 The rms radius of q_{RIGHT} for HH channel for $0 < R < 2.8$ fm.

While the energy curve for HH shows no break study of other features of the solutions do show sharply different behaviour as R decreases below a $R_{CRIT} \sim 1$ fm. As an example, the rms radius of the right orbital as a function of R is shown in fig 5. The dramatic increase in the size as R decreases below R_{CRIT} can be understood by examining the potential, $-g(\sigma + F_\pi)$, seen by the quarks due to the σ field. This potential is deep and attractive at the centers of the scalar density which are close to the centers of the spinors when they are well separated. The potential goes to zero as $z \to \mp \infty$. For large R the potential is zero at z=0 also. Since the quark energy (eigenvalue) is negative the σ potential forms a barrier between the two centers keeping the spinors localized. As the solitons come close together the potential at z=0 starts to decrease. At R = 1.2 fm it falls below the quark energy allowing for the spinors to spread. The spreading increases rapidly as R is decreased further.

We do not have any explanation for the breaks in the HH_z and HH_x cases. For the former we do find an associated change in the topology of the meson fields but its causal connection to the occurence of the break is not clear.

The results of the calculation show clearly that inside of 1 fm the baryon-baryon interaction may not be describable in terms of potentials. It is more natural to invoke the idea of a compound system of 6 quarks. We may call it a six-quark soliton. Of course, this idea has been advocated very forcefully by Kisslinger [5]. Lomon and his collaborators [6] have used this idea, which is a natural extension of the Feshbach-Lomon approach , to analyze the NN phaseshifts.

We now turn to the exterior region. With the limited information, viz, only three spin-flavor configurations, available we can extract some information about the baryon-baryon interaction only if we make drastic assumptions. We must assume that results of the mean field calculation can be described by a sum of local potentials between the constituents quarks of the two baryons and the baryon-baryon potential can be obtained by folding these quark-quark potentials. We write the quark-quark potential in the following form;

$$\mathcal{V}(R) = \sum_{a,b} [\frac{1}{9} V_c(R)\ I^a I^b + \frac{9}{25} V_{ss}(R)\ \vec{\sigma}^a\cdot\vec{\sigma}^b\ \vec{\tau}^a\cdot\vec{\tau}^b + \frac{9}{25} V_t(R)\ \vec{\tau}^a\cdot\vec{\tau}^b\ \{ \vec{\sigma}^a\cdot\hat{R}\ \vec{\sigma}^b\cdot\hat{R} - \frac{1}{3}\vec{\sigma}^a\cdot\vec{\sigma}^b \}. \quad (12)$$

Here a and b label the quarks in the two baryons. The numerical factors are so chosen that upon folding (12) gives for the NN potential

$$V(R) = V_c(R) + V_{ss}(R)\ \vec{\sigma}_1\cdot\vec{\sigma}_2\ \vec{\tau}_1\cdot\vec{\tau}_2 + V_t(R)\ \vec{\tau}_1\cdot\vec{\tau}_2\ \{ \vec{\sigma}_1\cdot\hat{R}\ \vec{\sigma}_2\cdot\hat{R} - \frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2 \}, \quad (13)$$

where the subscripts 1 and 2 stand for the two nucleons. The potentials $V_c(R)$, $V_{ss}(R)$ and $V_t(R)$ for R > 1 fm, extracted from our calculations are shown in fig. 6. The central potential shows a shallow attractive part. Small as it is, this is the first time any attraction has been seen in a study of soliton-soliton interaction.

Fig. 7 compares the central potential, $V_c(R)$, obtained in this calculation with the one σ exchange potential between two solitons which are kept frozen in the form they have when they are far apart. Of course, this is what is often done in a conventional calculation of OBE potential. The overwhelming role of spatial polarization of the solitons is very clear.

We get attraction where others had not because (a) we include the HH_x channel and (b) we allow spatial polarization. In fig.8 we plot the various components of the total energy as functions of R for 1 fm < R < 2.8 fm. The quark kinetic energy is given by K_Q. $K_\sigma = \frac{1}{2}\int d^3r\ (\vec{\nabla}\sigma)^2$ and K_π is the similar quantity for the pion field. $H_{Q\sigma}$ and $H_{Q\pi}$ are the quark meson interaction energies and U the meson-meson interaction energy. As functions of R five of these six quantities behave similarly for the three channels. The exception is the quark kinetic energy. It shows a rapid decrease with decrteasing R for the HH_x channel, but not for the other channels. It is clearly the source of the attraction in V_C thus reconfirming the very important role of spatial polarization.

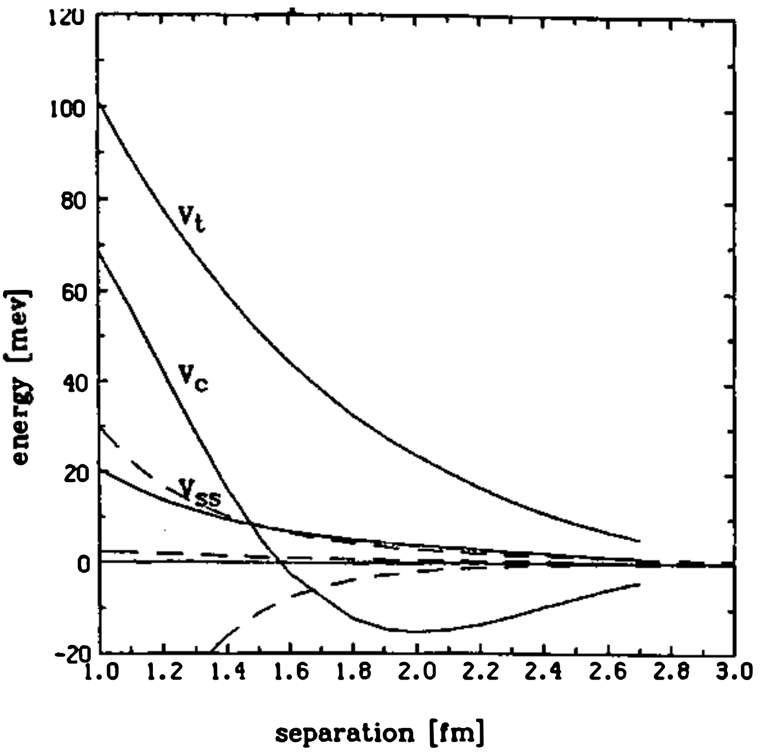


Fig 6 V_c, V_{ss} and V_t as functions of R. The dashed line represents the Bonn potential.

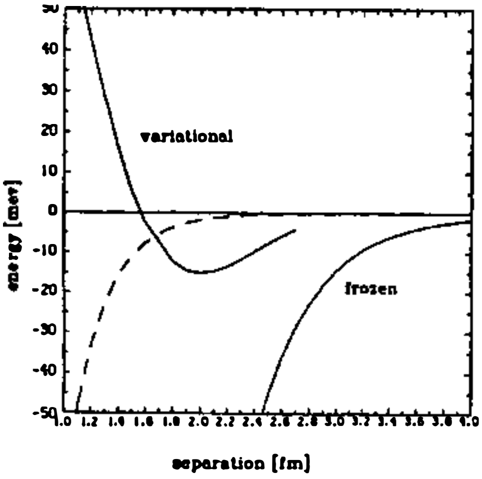


Fig. 7 $V_c(R)$ and the 'frozen' σ-exchange potential.

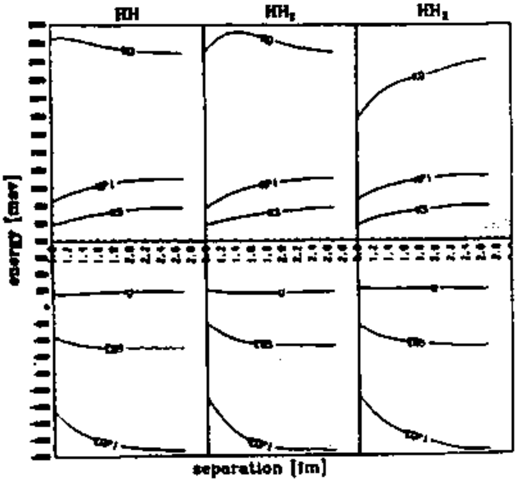


Fig. 8 Various contributors to the energy in the 3 channels.

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