FIZIKA 19, (1987), Supplement 2, 86-92
(Proc. Workshop on Mesonic Degrees of Freedom in Hadrons, Bled/Ljubljana, June/July 1987)

QUARK-MESON INTERPLAY IN FLAVOUR-CHANGING PROCESSES

Short-Distance vs. Long-Distance Contributions

I. Picek
Rudjer Bošković Institute, Zagreb, Croatia, Yugoslavia

Strange-particle decays are ideally suitable for studying an interplay of weak and strong forces. Because of the short-range nature of the weak force acting between quarks and leptons, strong interactions are not expected to have a considerable impact on these decays. However, because of the colour confinement, the quarks and gluons emitted in a decay must recombine and again form hadrons. We present part of growing evidence for the importance of such soft processes, culminating in a long-standing puzzle of the $\Delta I=1/2$ rule.

1. Hidden scales of QCD and their manifestation in $\pi^0 \to 2\gamma$ and $K^0 \to 2\gamma$

Let us start by listing the scales involved in strange-particle decays and the different appearances of QCD on these scales*:

(i) Short-distance (SD) scales of the order of $1/M_W \sim 10^{-16}$ cm for W exchange, triggering a flavour change. Here QCD is simple: chirally symmetric (χS), owing to negligible quark masses on this scale and asymptotically free (AF), allowing for perturbative calculations and quantitative predictions.

(ii) Long distances (LD) of $\sim$(1/10-1) fm, experiencing the recombination of quarks and gluons into hadrons. In this case, QCD is a rich theory, exhibiting chiral-symmetry breaking (χSB) and confinement.

(iii) Distances of $\sim$1 cm of detectors, where QCD is an empty theory.

Issues of modelling the confinement and χS aspects of QCD are the main topic of this Workshop and have already been presented[2]. Therefore, let us focus on another aspect of QCD - the appearance of scales arising from quantum-mechanical symmetry breaking.

---

*It has been recognized since the time of Galileo that physical laws depend on the change of scale[1].

1.1. Hidden QCD scale

In the χS limit, QCD is a theory without any parameter with dimension. However, as illustrated by the quadratically divergent vacuum polarisation,

$$\Pi^{\mu\nu}(q) \sim g^2 \int \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr}\left(\gamma^\mu \frac{1}{\not k} \gamma^\nu \frac{1}{\not k - \not q}\right) ,$$

the parameter with dimension enters via an ultraviolet (UV) cut-off, $\Lambda_{cut\text{-}off}$. Effectively, by fitting the bare coupling constant g to some measurable quantity at energy $\sim\mu$, $\Lambda_{cut\text{-}off}$ disappears from the expressions for measurable quantities. Instead, a scale parameter of QCD, Λ_{QCD}, appears as an integration constant when integrating the renormalisation-group β function, giving

$$\frac{1}{g(\mu^2)} = \frac{11N_c - 2N_f}{24\pi^2} \ln \frac{\mu}{\Lambda_{QCD}} .$$

Alternatively, in a diagrammatic approach, an infra-red cut-off, μ_{IR}, may be introduced, ensuring that the perturbative evaluation is really meaningful.

1.2. 1^{st} QCD anomaly and the U(1) problem

There is another quantum-mechanical symmetry breaking known as anomaly. In the limit of massless quarks, the QCD Lagrangian

$$\mathcal{L} = \bar\Psi(i\not\partial + \not G)\Psi \; ; \qquad \Psi = \Psi_L + \Psi_R , \tag{1}$$

exhibits the global chiral symmetry under independent rotations among left- and right-handed quarks separately,

$$\Psi_L \to e^{i\alpha}\Psi_L , \qquad \Psi_R \to e^{i\beta}\Psi_R .$$

For an axial transformation ($\alpha = -\beta$) of the bispinor, $\Psi = \binom{\Psi_L}{\Psi_R} \to e^{i\beta\gamma_5}\Psi$, there is a naively conserved U(3)-singler Nöther current, $\partial_\mu j_5^\mu = \partial\mathcal{L}/\partial\beta(x) = 0$,

$$j_5^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\beta(x))} = \bar\Psi\gamma^\mu\gamma_5\Psi = \bar u\gamma^\mu\gamma_5 u + \bar d\gamma^\mu\gamma_5 d + \bar s\gamma^\mu\gamma_5 s . \tag{2}$$

Actually, owing to the anomaly (a QCD version of the ABJ anomaly[3])

$$\mathcal{A} = N_f \frac{g^2}{32\pi^2} G^{\mu\nu}\tilde G_{\mu\nu} \; ; \qquad \tilde G_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$$

and small quark masses, $m_u \simeq m_d = m$, and a relatively large mass $m_s = 150$ MeV, we have

$$\partial_\mu j_5^\mu = 2im + im_s + \mathcal{A} . \tag{3}$$

The large mass of η' (possessing the quantum numbers of the singlet current (2)), $m_{\eta'}/m_\eta = 960$ MeV/550 MeV, was stated as the U(1) problem[4]. The mass m_s is not sufficient to explain η' (η' is not a pseudo-Goldstone boson). A non-perturbative QCD condensate and the instanton structure of the QCD vacuum are involved.

1.3. 2nd QCD anomaly and $\pi^o \to 2\gamma$

Another anomaly is related to chiral QCD in an external electromagnetic field A_μ

$$\mathcal{L} = \bar{\Psi} i(\not{\partial} + \not{G} + Q\not{A})\Psi \; ; \quad Q = \begin{pmatrix} 2/3 & \\ & -1/3 \end{pmatrix} . \tag{4}$$

The axial transformation

$$\Psi = \binom{u}{d} \to e^{i\beta\tau_3\gamma_5}\Psi$$

leads to the Nöther current

$$j_5^{\mu(3)} = \bar{\Psi}\gamma^\mu\gamma_5\tau^3\Psi = \bar{u}\gamma\,\gamma_5 u - \bar{d}\gamma\,\gamma_5 d ,$$

which is anomalous (by the ABJ anomaly[3]):

$$\partial_\mu j_5^{\mu(3)} = 3(Q_u^2 - Q_d^2)\,\frac{e^2}{8\pi^2}\,F^{\mu\nu}\tilde{F}_{\mu\nu} . \tag{5}$$

With the aid of the PCAC relation

$$\phi_\pi^{(i)}(x) = \frac{1}{m_\pi^2 f_\pi}\,\partial_\mu j_5^{\mu(3)}(x) ,$$

Eq. (5) leads to an excellent prediction of the decay width:

$$\Gamma(\pi^o \to 2\gamma) = \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{64\pi}\,\frac{m_\pi^3}{8\pi^2} \simeq 7.25\ \text{keV} \quad (\text{expt.}\ \ 7.3 \pm 0.2\ \text{keV}).$$

Besides solving the puzzle that, approximately (in the chiral limit), π does not decay, the anomaly predicts $N_c=3$ numbers of colour.

1.4. SD vs. LD in $K^o(\bar{K}^o) \to 2\gamma$

A natural expectation for $K_L \to 2\gamma(-)$ (CP odd $F^{\mu\nu}\tilde{F}_{\mu\nu}$ final state) would be that it proceeds by the SD effect of the anomaly (SD as given by the ultraviolet cut-off). Indeed, in the limit of heavy W bosons, the simple W-exchange graph reduces to the anomaly triangle, as shown in Fig. 1. However, because of the presence of the GIM cancellation, there is effectively no UV cut-off, and Fig. 1 accounts only for $\sim$20%[5] of the $K_L \to 2\gamma$ rate*. The LD contributions of the two-stage process (Fig. 2) turn out to be dominant. This presents an illustration that going from a a simple $\pi^o \to 2\gamma$ to a flavour-changing $K^o \to 2\gamma$ process involves a

---

* A significant SD effect might be expected for CP-violating amplitudes originating from heavy-quark contributions[6].

non-trivial dynamics of the $K \to \pi$ transition, and thus a long-standing problem of $\Delta I=1/2$ enhancement.

2. $\Delta I=1/2$ problem and its possible resolution

The $\Delta I=1/2$ problem refers to a large ratio

$$\frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K^+ \to \pi^+\pi^0)} \simeq 450 \,, \tag{2.1}$$

while the bare $\Delta S=1$ interaction

$$H_W \sim G_F s_c c_c (\bar{u}\gamma_\mu Ls)(\bar{d}\gamma^\mu Lu)$$

gives $\Delta I=1/2$ and $\Delta I=3/2$ of comparable magnitude. The problem started by realising that a simple vacuum insertion approximation (VSA) reproduces the $K^+ \to \pi^+\pi^0$ amplitude[7], but in no way $K_S \to 2\pi$.

In an attempt to locate the origin of the problem, one can observe that the "flavour-annihilation quark diagrams" for $K^+ \to \pi^+\pi^0$ are cancelled (because of the Bose symmetry of final states), as shown in Fig. 3, so that only "flavour-decay diagrams" (Fig. 4) are of importance. This might give some hint about the difficulties avoided by the cancellation shown in Fig. 3.

2.1. Early SD attempts

Let us present a survey of an attempt to (perturbatively) resolve the $\Delta I=1/2$ rule by using SD dynamics[8]. It started by observing[8] that the SD corrections to the bare current-current interaction led to the effective interaction with the enhanced $\Delta I=1/2$ part

$$H_{eff} \sim C_+(\mu) O_+(\Delta I=1/2,3/2) + C_-(\mu) O_-(\Delta I=1/2) \,,$$

$$O_\pm = (\bar{d}\gamma^\mu Lu)(\bar{u}\gamma_\mu Ls) \pm (\bar{d}\gamma^\mu Ls)(\bar{u}\gamma_\mu (u)) \,, \tag{2.2}$$

with $C_+ \simeq 0.7$ and $C_- \simeq 2.5$.

Note that the evaluation of the physical $K \to 2\pi$ amplitudes is converted by the soft-pion reduction into the evaluation of the off-shell $K \to \pi$ transition. In the latter we may distinguish two classes of $K \to \pi$ transitions (Fig. 5). Obviously, a dynamical enhancement of class II (pure $\Delta I=1/2$) over class I has the best chance to resolve the $\Delta I=1/2$ problem. Two attempts have been made to perturbatively evaluate class II, namely a penguin-diagram approach and an off-diagonal self-energy approach.

2.2. Penguin diagrams

A more careful treatment of the SD QCD corrections discussed above revealed[9] the so-called penguin part to be added to (2.2):

$$C_5(\mu) O_5 ; \quad O_5 = (\bar{d}\gamma^\mu \lambda^a Ls)(\bar{u}\gamma_\mu \lambda^a Ru + \bar{d}\gamma_\mu \lambda^a Rd) \,. \tag{2.3}$$

After the initial optimism[9], the lessons ensuing from penguin operators are the following:

(a) Owing to a small coefficient, $C_5(\mu) \sim -0.1$, the only chance is to enhance the matrix element.

(b) Since the parameter of direct CP violation, ε', is given by the same matrix element, the existing experimental bound rules out[10] the perturbative penguin explanation of the $\Delta I=1/2$ problem.

2.3. Off-diagonal self-energy

After a long controversial history[11] a novel impetus to such an explanation has recently been raised by Shabalin[12]. Chia[11] showed that after proper subtraction the power-like GIM cancellation $(m_c^2-m_u^2)/M_W^2$ made the self-energy numerically unimportant. However, Shabalin's observation that gluonic corrections changed the GIM cancellation into the logarithmic cancellation has revived the interest in such a mechanism[13,14]. That has already happened in the case of penguin operators, so the lesson following from these operators, as mentioned above, should be kept in mind. Actually, this has resulted in a criticism by Guberina, Peccei and myself[13], culminating in the observation that, in a strict SD treatment, Shabalin's operator as a part of the gauge-invariant operator,

$$\pi^3 = \bar{d}\, \not{\pi}\, \not{\pi}\, \not{\pi}\, Ls \;, \qquad \pi_\mu = p_\mu - gT^a G_\mu^a \;, \tag{2.4}$$

which vanishes by the QCD equations of motion, cannot give rise to any physical effects.

2.4. LD non-perturbative resolution

Eliminating the most recent attempt to resolve the $\Delta I=1/2$ problem with SD dynamics, let us list different proposals for doing this using various pieces of LD physics. Of course, all of them cannot be viable. In particular, this has turned out to be the case with the QCD sum-rule approach[15]. The large-N_c attempt[16] has also been subjected to certain criticism[17]. The possibility that a penguin might not be SD-dominated has been considered by Eeg[18], while new, instanton-induced operators, with an undetermined coefficient have been deduced by Konishi and Ranfone[19] (see the contribution to this Workshop by D. Horvat). Finally, much hope has been raised by an attempt[20] to calculate the matrix elements of the local operators $O_i(\mu)$ using lattice techniques. However, we wish to suggest an additional check (for example, a comparison of Figs. 3 and 4) upon the origin of the LD effect required for the explanation of the $\Delta I=1/2$ puzzle.

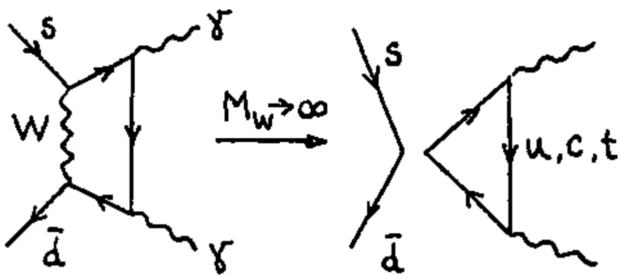

Fig.1

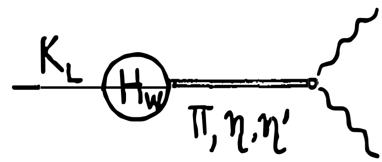

Fig.2

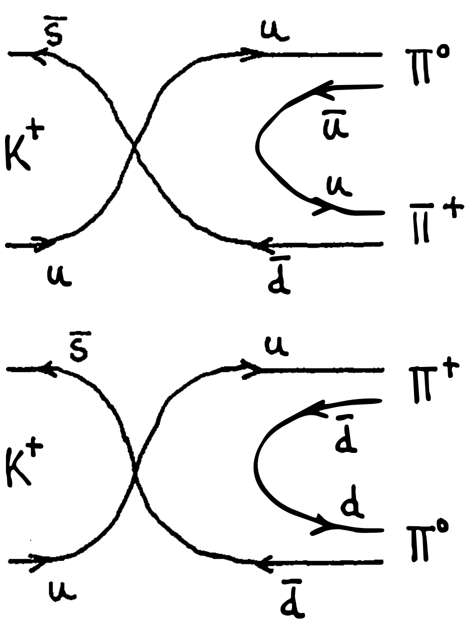

Fig.3

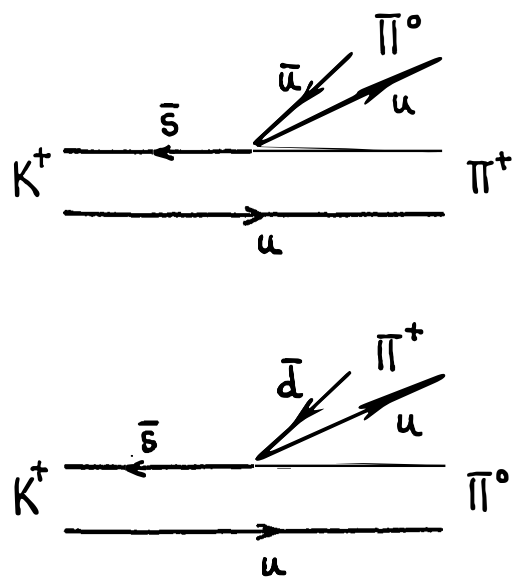

Fig.4

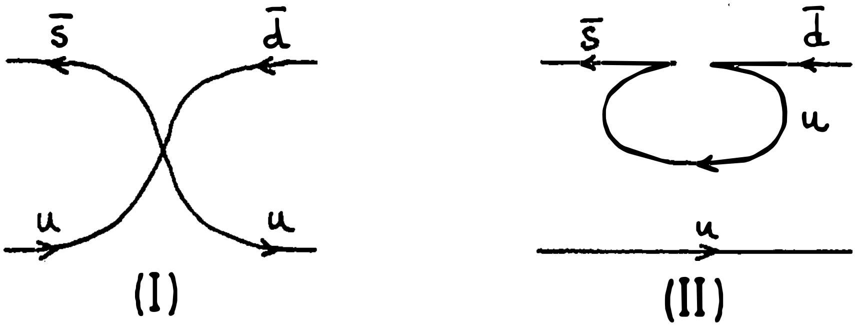

Fig.5

References

1. R.P. Feynman, The Character of Physical Law, M.I.T. Press (1967) p.91, translation into Croatian, Školska knjiga 1986 .
2. I. Picek, Acta Phys. Polonica B16 (1985) 353.
3. L. Rosenberg, Phys. Rev. 129 (1963) 2786;
   S.L. Adler, Phys. Rev. 177 (1969) 2426;
   J.S. Bell and R. Jackiw, Nuovo Cim. 60 (1969) 47.
4. E. Witten, Nucl. Phys. B156 (1979) 269.
5. M.K. Gaillard and B.W. Lee, Phys. Rev. D10 (1974) 987;
   E. Ma and A. Pramudita, Phys. Rev. D24 (1981) 2476;
   G. Eilam and M.D. Scadron, Phys. Rev. D31 (1985) 2263.
6. J.O. Eeg and I. Picek, Oslo Inst.Phys.Rep. 87-19/Zagreb RBI-TP-1-87.
7. R.P. Feynman, Proc. 1964 Int. School "E. Majorana", A. Zichichi ed., Academic Press, NY 1965).
8. M.K. Gaillard and B.W. Lee, Phys. Rev. Lett. 33 (1974) 109;
   G. Altarelli and L. Maiani, Phys. Lett. B52 (1974) 351.
9. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B120 (1977) 361; Sov. Phys. - JETP 45 (1977) 670.
10. F.J. Gilman and M.B. Wise, Phys. Lett. B83 (1979) 83.
11. M.A. Ahmed and G.G. Ross, Phys. Lett. B61 (1976) 287 ;
    P. Pascual and R. Terrach, Phys. Lett. B87 (1979) 64;
    M.D. Scadron, Phys. Lett. B95 (1980) 123.
    B.H.J. McKellar and M.D. Scadron, Phys. Rev. D27 (1983) 157 ;
    S.P. Chia, Phys. Lett. B147 (1984) 361.
12. E.P. Shabalin, ITEP preprint ITEP 86-112.
13. B. Guberina, R.D. Peccei and I. Picek, Phys. Lett. B188 (1987) 258.
14. J.O. Eeg, Oslo Report 87-15.
15. A. Pich, B. Guberina and E. de Rafael, Nucl. Phys. B277 (1986) 197;
    A. Pich and E. de Rafael, Phys. Lett. B189 (1987) 369.
16. W.A. Bardeen, A.J. Buras and J.-M. Gerard, MPI-PAE/PTh 37/86.
17. R.S. Chivukula, J.M. Flynn and H. Georgy, Phys. Lett. B171 (1986) 453;
    J. Bijnens and B. Guberina, DESY 87-067.
18. J.O. Eeg, Phys. Lett. B155 (1985) 115; B171 (1986) 103.
19. K. Konishi and S. Ranfone, Phys. Lett. B172 (1986) 417.
20. C. Bernard et al., Phys. Rev. Lett. 55 (1985) 2770;
    Phys. Rev. D32 (1985) 2343;
    N. Cabibbo, G. Martinelli and R. Petronzio, Nucl. Phys. B244 (1984) 381;
    L. Maiani, G. Martinelli and M. Testa, CERN TH.4517/86.